STATISTICS DATA SHEET FOR
EM375 STATISTICS AND MEASUREMENT UNCERTAINTY

Parameters and Statistics

$$m = \frac{\sum x_i}{N} \qquad \bar{x} = \frac{\sum x_i}{n}$$

$$s^2 = \frac{\sum (x_i - m)^2}{N} \qquad S^2 = \frac{\sum (x_i - \bar{x})^2}{(n-1)}$$

Parameter estimation

Standard error of the mean: $s_{\bar{x}} = \frac{s}{\sqrt{n}}$

Confidence interval of the mean, d : $\left.\begin{array}{c} m = \bar{x} \pm d \\ \bar{x} - d \le m \le \bar{x} + d \end{array}\right\} d = t s_{\bar{x}} = \frac{tS}{\sqrt{n}}$

Degree of Confidence $= P\left(\bar{x} - d \le m \le \bar{x} + d\right)$

Level of significance, a = (1 – Degree of Confidence) is the probability the mean will fall outside the confidence interval, i.e., there is a significant event (failure).

Probabilities

$P(A) + P(\bar{A}) = 1$

If events A and B are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$

If events A and B are independent, probability that both will occur, $P(AB) = P(A).P(B)$

Probability of A or B or both, $P(A \cup B) = P(A) + P(B) - P(AB)$

Normal Distribution

If $x \sim N(m, s)$ and $z = \frac{(x-m)}{s}$ then $z \sim N(0,1)$

$P(z < z_T)$ is tabulated for values of z_T as the Cumulative Normal Distribution Function

$$P(z_1 < z < z_2) = P(z < z_2) - P(z < z_1)$$

Binomial Distribution

$$P(r) = \binom{n}{r} p^r (1-p)^{n-r} \qquad \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$m = n.p \qquad s = \sqrt{n.p(1-p)}$$

Poisson Distribution

$$P(x) = \frac{e^{-l}\, l^{x}}{x!} \qquad s = \sqrt{l}$$

$$P(x \le k) = \sum_{i=0}^{k} \frac{e^{-l}\, l^{i}}{i!} \qquad P(x > k) = 1 - P(x \le k) = 1 - \sum_{i=0}^{k} \frac{e^{-l}\, l^{i}}{i!}$$

Chi-squared Distribution, C^2

The X^2 statistic for a goodness of fit test is calculated as:

$$X^2 = \sum_{i=1}^{k} \left\{ \frac{(O_i - E_i)^2}{E_i} \right\}$$

where O_i is the observed number of events in each of the *k* bins, and E_i is the expected number of events in the bins, based on the distribution function being tested.

Student-t Distribution

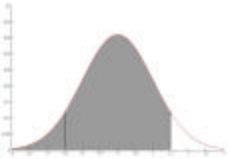
	Level of Significance, α						
	10%	5%	2.5%	1%	0.5%	0.05%	1-Sided
dof	20%	10%	5%	2%	1%	0.1%	2-Sided
1	3.07768	6.31375	12.70620	31.82052	63.65674	636.61925	
2	1.88562	2.91999	4.30265	6.96456	9.92484	31.59905	
3	1.63774	2.35336	3.18245	4.54070	5.84091	12.92398	
4	1.53321	2.13185	2.77645	3.74695	4.60409	8.61030	
5	1.47588	2.01505	2.57058	3.36493	4.03214	6.86883	
6	1.43976	1.94318	2.44691	3.14267	3.70743	5.95882	
7	1.41492	1.89458	2.36462	2.99795	3.49948	5.40788	
8	1.39682	1.85955	2.30600	2.89646	3.35539	5.04131	
9	1.38303	1.83311	2.26216	2.82144	3.24984	4.78091	
10	1.37218	1.81246	2.22814	2.76377	3.16927	4.58689	
11	1.36343	1.79588	2.20099	2.71808	3.10581	4.43698	
12	1.35622	1.78229	2.17881	2.68100	3.05454	4.31779	
13	1.35017	1.77093	2.16037	2.65031	3.01228	4.22083	
14	1.34503	1.76131	2.14479	2.62449	2.97684	4.14045	
15	1.34061	1.75305	2.13145	2.60248	2.94671	4.07277	
16	1.33676	1.74588	2.11991	2.58349	2.92078	4.01500	
17	1.33338	1.73961	2.10982	2.56693	2.89823	3.96513	
18	1.33039	1.73406	2.10092	2.55238	2.87844	3.92165	
19	1.32773	1.72913	2.09302	2.53948	2.86093	3.88341	
20	1.32534	1.72472	2.08596	2.52798	2.84534	3.84952	
25	1.31635	1.70814	2.05954	2.48511	2.78744	3.72514	
30	1.31042	1.69726	2.04227	2.45726	2.75000	3.64596	
40	1.30308	1.68385	2.02108	2.42326	2.70446	3.55097	
50	1.29871	1.67591	2.00856	2.40327	2.67779	3.49601	
60	1.29582	1.67065	2.00030	2.39012	2.66028	3.46020	
70	1.29376	1.66691	1.99444	2.38081	2.64790	3.43501	
80	1.29222	1.66412	1.99006	2.37387	2.63869	3.41634	
90	1.29103	1.66196	1.98667	2.36850	2.63157	3.40194	
100	1.29007	1.66023	1.98397	2.36422	2.62589	3.39049	
infinity	1.28157	1.64488	1.96001	2.32642	2.57593	3.29072	

Student's-t: $t = \dfrac{\bar{x} - m}{S / \sqrt{n}}$

Cumulative Normal Distribution Function

$$z = \frac{(x-m)}{s} \approx \frac{(x-\bar{x})}{S}$$

The tabulated values show



z	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998

Example: $P(z < 2.86) = 0.99788 = 99.788\%$

Percentage Points for the Chi-squared Distribution

Example: $P\left(X^2 < 18.3074\right) = 5\%$ for dof = 10

	Level of Significance, α						
dof	99%	95%	90%	10%	5%	2%	1%
1	0.00016	0.00393	0.01579	2.70554	3.84146	5.41189	6.63490
2	0.02010	0.10259	0.21072	4.60517	5.99146	7.82405	9.21034
3	0.11483	0.35185	0.58437	6.25139	7.81473	9.83741	11.34487
4	0.29711	0.71072	1.06362	7.77944	9.48773	11.66784	13.27670
5	0.55430	1.14548	1.61031	9.23636	11.07050	13.38822	15.08627
6	0.87209	1.63538	2.20413	10.64464	12.59159	15.03321	16.81189
7	1.23904	2.16735	2.83311	12.01704	14.06714	16.62242	18.47531
8	1.64650	2.73264	3.48954	13.36157	15.50731	18.16823	20.09024
9	2.08790	3.32511	4.16816	14.68366	16.91898	19.67902	21.66599
10	2.55821	3.94030	4.86518	15.98718	18.30704	21.16065	23.20918
11	3.05348	4.57481	5.57778	17.27501	19.67506	22.61786	24.72492
12	3.57057	5.22603	6.30380	18.54935	21.02601	24.05389	26.21693
13	4.10692	5.89186	7.04150	19.81190	22.36199	25.47146	27.68823
14	4.66043	6.57063	7.78953	21.06412	23.68475	26.87273	29.14122
15	5.22935	7.26094	8.54676	22.30711	24.99576	28.25947	30.57790
16	5.81221	7.96165	9.31224	23.54181	26.29620	29.63315	31.99992
17	6.40776	8.67176	10.08519	24.76902	27.58709	30.99503	33.40866
18	7.01491	9.39046	10.86494	25.98941	28.86928	32.34614	34.80530
19	7.63273	10.11701	11.65091	27.20356	30.14351	33.68741	36.19086
20	8.26040	10.85081	12.44261	28.41197	31.41042	35.01961	37.56623
21	8.89720	11.59131	13.23960	29.61508	32.67056	36.34344	38.93217
22	9.54249	12.33801	14.04149	30.81328	33.92443	37.65949	40.28936
23	10.19572	13.09051	14.84796	32.00689	35.17245	38.96830	41.63840
24	10.85636	13.84843	15.65868	33.19624	36.41502	40.27035	42.97982
25	11.52398	14.61141	16.47341	34.38158	37.65248	41.56607	44.31410
30	14.95346	18.49266	20.59923	40.25602	43.77297	47.96180	50.89218
40	22.16426	26.50930	29.05052	51.80506	55.75848	60.43613	63.69074
50	29.70668	34.76425	37.68865	63.16712	67.50481	72.61325	76.15389
60	37.48485	43.18796	46.45889	74.39700	79.08194	84.57995	88.37942
70	45.44172	51.73928	55.32894	85.52704	90.53122	96.38753	100.42518
80	53.54008	60.39148	64.27785	96.57820	101.87947	108.06934	112.32879
90	61.75408	69.12603	73.29109	107.56501	113.14527	119.64845	124.11632
100	70.06490	77.92947	82.35814	118.49800	124.34211	131.14168	135.80672

Approximate formula for dof = $f > 30$: $X_p^2 \simeq \frac{1}{2}\left(\sqrt{2f-1} - z_p\right)^2$ where z_p is the z-statistic (Normal Distribution) for probability $p = 1-a$. Example: α=5%; p = 95% gives $z_p = 1.64485$. For f=50; $X_p^2 \simeq \frac{1}{2}\left(\sqrt{2\times 50-1} - 1.64485\right)^2 = 34.4867$ compared to 34.76425 exact value.