MECHANICAL ENGINEERING DEPARTMENT
UNITED STATES NAVAL ACADEMY

EM423 - INTRODUCTION TO MECHANICAL VIBRATIONS

CONTINUOUS SYSTEMS SUMMARY
STRINGS, RODS and SHAFTS

SYMBOLS

r	mass density
A_X	Cross-sectional area
T	Tension (string)
T	Internal torque (shaft)
P	Internal longitudinal force (varies along the rod)
x	Distance along the string, rod or shaft
y	Lateral deflection from equilibrium position
u	Longitudinal deflection from equilibrium position (rod)
q	Slope of string relative to the x-axis (string)
q	Angle of twist (shaft)
E	Young's Modulus of elasticity
G	Shear modulus of elasticity
I_P	Polar moment of inertia

ASSUMPTIONS

See the separate handouts for detailed assumptions. The main ones to remember are linearity, and that these derivations assume the component is uniform in all respects. If that is not the case, the equations must be derived from first principles.

BASIC SOLUTIONS

	STRING	ROD	SHAFT
Wave Equation	$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$	$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$	$\frac{\partial^2 q}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 q}{\partial t^2}$
Wave Speed	$c = \sqrt{\frac{T}{r A_x}}$	$c = \sqrt{\frac{E}{r}}$	$c = \sqrt{\frac{G}{r}}$

Using the symbol a to represent y, u, or q, the general solution is:

$$a(x,t) = \left\{A.\sin\left(\frac{wx}{c}\right) + B.\cos\left(\frac{wx}{c}\right)\right\}\sin(wt)$$

$$\frac{\partial a(x,t)}{\partial x} = \left\{A.\cos\left(\frac{wx}{c}\right) - B.\sin\left(\frac{wx}{c}\right)\right\}\left(\frac{w}{c}\right)\sin(wt)$$

COMMON BOUNDARY CONDITIONS

	STRING	ROD	SHAFT
Fixed end	$y = 0$	$u = 0$	$q = 0$
Free end	N/A	$\frac{\partial u}{\partial x} = 0$	$\frac{\partial q}{\partial x} = 0$