MECHANICAL ENGINEERING DEPARTMENT
UNITED STATES NAVAL ACADEMY

EM423 - INTRODUCTION TO MECHANICAL VIBRATIONS

DESIGN OF A TUNED VIBRATION ABSORBER

INTRODUCTION

The starting point for this handout is the work presented in the 2DOF handout. The current handout develops some of the ideas that are necessary in designing a tuned vibration absorber. Only the dynamics aspects of the design are considered; detailed design and implementation are beyond the scope of this handout. A single case study is considered, and it is assumed that two-degree-of-freedom theory is adequate.

PROBLEM STATEMENT

A rotating pump has a mass of 80 kg, and is suspended from the deckhead on a fixture that has a mass of 25 kg. The pump's misbalance means that when it is running at 1200 revolutions per minute, it produces a peak force of 20 N. There is a 4 mm clearance around the pump that could be used for a vibration absorber.

Design the absorber!

GENERAL DESIGN STRATEGY

There are basically three different ways of approaching the design:

1) Choose a secondary mass and calculate the required stiffness. This approach may be adequate for a situation where mass is critical. However, a "one-shot" design of this type is unlikely to be an optimum design.

2) Choose the stiffness and calculate the required mass. This approach may be adequate for a situation where space is limited. As for method 1), though, this approach cannot easily be used to optimize a design.

3) Leave both mass and stiffness as variables, and generate a family of solutions. Using this family of solutions, use some criteria to optimize the design. Designs made using this philosophy can usually be optimized.

In this handout, we will take the third, "general" approach. Since we seem to be working on a space-limited design, we will base the design on the motion of the secondary mass. There are, of course, other bases that could be used (e.g., mass, stiffness), in which case the overall approach to the design would be similar to this one, but would develop functions of (e.g.) mass rather than displacement.

THE STIFFNESS EQUATIONS

The problem statement gives us a 4 mm clearance. We will assume that this is available on both sides, and that it is just the "rattle-space" – i.e. the space additional to the space required for the secondary mass itself. However, the clearance is only the maximum allowed motion of the secondary mass; we can always have a design where the secondary mass moves less. Letting the motion of the secondary mass be X_2, with the restriction that $X_2 < 4$ mm, we can determine the required stiffness as a function of the motion and misbalance force. From the 2DOF handout, we found that:

$$X_2 = \frac{-F_1}{k_2}$$

Rearranging, we can get:

$$\boxed{k_2 = \left|\frac{F_1}{X_2}\right|}$$

THE MASS EQUATIONS

The controlling mass equation is the one that brings the frequency of the secondary mass/spring combination into tune with the excitation frequency, which we will call w. Hence:

$$w^2 = \frac{k_2}{m_2} \qquad \text{or} \qquad m_2 = \frac{k_2}{w^2}$$

Substituting in the stiffness relationship we get:

$$\boxed{m_2 = \frac{F_1}{X_2 w^2}}$$

THE NATURAL FREQUENCIES EQUATIONS

The final consideration we have is to determine the new natural frequencies of the system. These frequencies are related to the ratio of secondary mass to primary mass. As we saw in the 2DOF handout, the larger the mass ratio, the larger the difference between the two new natural frequencies of the modified system. The relevant equation from the 2DOF handout is:

$$w_{1,2}^2 = \left(\frac{k_1}{2m_1}\right)\left(1 + q^2(1+m) \pm \sqrt{q^4(1+m)^2 - 2(1-m)q^2 + 1}\right)$$

with

$$q = \frac{\text{target frequency}}{\text{original natural frequency}} = \frac{\sqrt{k_2/m_2}}{\sqrt{k_1/m_1}} \qquad \text{and} \qquad m = \text{ mass ratio } = \frac{m_2}{m_1}$$

We are not given enough information in the problem statement to solve these equations exactly as written. Thinking about the physical problem of the design, we are asked to reduce the vibration of a pump. This must be because the current level of vibration is unacceptably large. The chances are, therefore, that the pump is operating close to resonance. For a real design we would have to investigate this aspect further. However, for this paper exercise, let's assume the pump is operating close to resonance, and set q = 1.0. This also means that $\frac{k_1}{m_1} = w^2$. We can now rewrite the frequency equation with these restrictions to determine the two new natural frequencies of the modified system:

$$w_1^2 = \frac{w^2}{2}\left(2 + m - \sqrt{m\left(4+m\right)}\right)$$
$$w_2^2 = \frac{w^2}{2}\left(2 + m + \sqrt{m\left(4+m\right)}\right)$$

Or, substituting in the mass equation:

$$\boxed{\begin{aligned} w_1^2 &= \frac{w^2}{2}\left(2 + \frac{F_1}{m_1 X_2 w^2} - \sqrt{\frac{F_1}{m_1 X_2 w^2}\left(4 + \frac{F_1}{m_1 X_2 w^2}\right)}\right) \\ w_2^2 &= \frac{w^2}{2}\left(2 + \frac{F_1}{m_1 X_2 w^2} + \sqrt{\frac{F_1}{m_1 X_2 w^2}\left(4 + \frac{F_1}{m_1 X_2 w^2}\right)}\right) \end{aligned}}$$

CLOSING IN ON THE DESIGN

We now have a series of equations to help with the design. The design equations are all based on the permitted displacement, X_2, with $X_2 < 4$ mm. Recall that for this specific design, the excitation frequency is $w = 1200$ revs/min = 125.7 rad/s.

The equations (all derived above) are:

Secondary mass:

$$m_2 = \frac{F_1}{X_2 w^2}$$

Secondary stiffness:

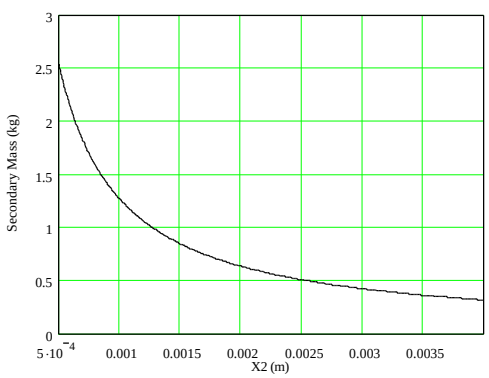
$$k_2 = \frac{F_1}{X_2} = m_2 w^2$$

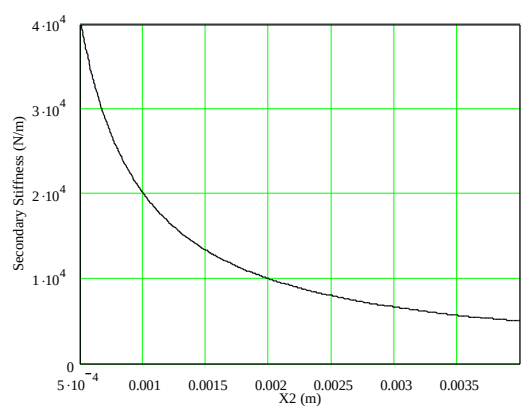
The two new natural frequencies:

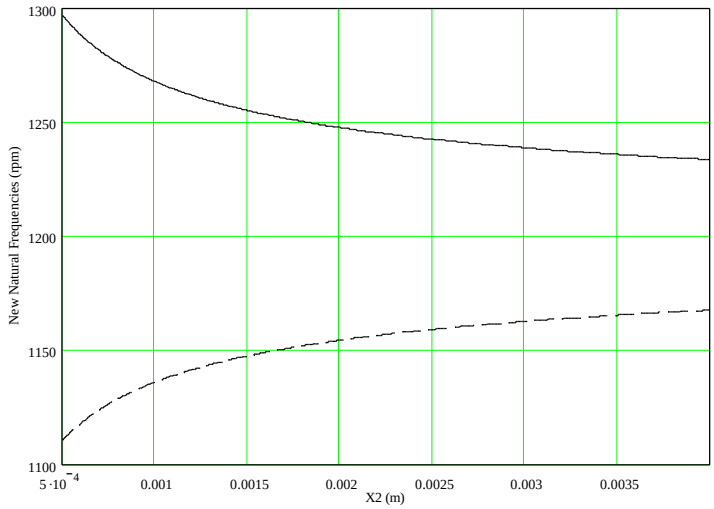
$$w_1^2 = \frac{w^2}{2}\left(2 + \frac{F_1}{m_1 X_2 w^2} - \sqrt{\frac{F_1}{m_1 X_2 w^2}\left(4 + \frac{F_1}{m_1 X_2 w^2}\right)}\right)$$

$$w_2^2 = \frac{w^2}{2}\left(2 + \frac{F_1}{m_1 X_2 w^2} + \sqrt{\frac{F_1}{m_1 X_2 w^2}\left(4 + \frac{F_1}{m_1 X_2 w^2}\right)}\right)$$

Let's see the graphical results:







As with most design situations, there is no unique “perfect” answer. The following discussion, therefore, comes to one of many possible conclusions. Your description of your solution, and explanation as to how you arrived at that solution, are often as important as the actual solution itself.

The pump is running at a nominal 1200 rpm, but its actual running speed most probably varies with load and other effects. For a real design, you should investigate this variation from other literature or from experiments. However, for this paper exercise we will assume the variation is less than ±50 rpm. Looking at the natural frequency curves for speeds of 1200+50 rpm and 1200-50 rpm, we see we are constrained to have the displacement of the secondary mass to be less than about 0.0015 m (1.5 mm) to avoid hitting one of the new resonances. Thus, we see that we will not be using the full available rattle space.

Using this X_2 value, we see that the secondary mass must be at least about 0.8 kg, with a secondary stiffness of about 13 kN/m. To be safe, we would probably select a secondary mass of about 1.5 kg, which really isn’t that large when compared to the total 105 kg of the pump and its fixture. Using m_2 = 1.5 kg gives us an X_2 value of about 0.85 mm, a secondary stiffness of about 23 kN/m, and two new natural frequencies of about 1130 and 1275 rpm.