MECHANICAL ENGINEERING DEPARTMENT
UNITED STATES NAVAL ACADEMY

EM423 - INTRODUCTION TO MECHANICAL VIBRATIONS

SINGLE DEGREE OF FREEDOM SYSTEMS DATA SHEET

Viscous damping – unforced motion:

$$x = Xe^{-zw_n t}\sin\left(\sqrt{1-z^2}\,w_n t + j\right)$$

Logarithmic decrement:

$$d_N = \ln\left(\frac{x_O}{x_N}\right) = \frac{2Npz}{\sqrt{1-z^2}} \approx 2Npz \quad (\text{for light damping})$$

Harmonic force excitation:

$$H(w) = \frac{X}{F_O} = \frac{1}{m\left\{\left(w_n^2 - w^2\right) + 2izww_n\right\}}$$

$$\left|Hw\right| = \left|\frac{X}{F_O}\right| = \frac{1}{m\left\{\left(w_n^2 - w^2\right)^2 + 4z^2w^2w_n^2\right\}^{1/2}} \qquad \text{phase of } H(w) = j = \tan^{-1}\left(\frac{2zww_n}{\left(w_n^2 - w^2\right)}\right)$$

Rotating unbalance:

$$\frac{X}{me} = \frac{w^2}{M\left\{\left(w_n^2 - w^2\right) + 2izww_n\right\}}$$

$$\left|\frac{X}{me}\right| = \frac{w^2}{M\left\{\left(w_n^2 - w^2\right)^2 + 4z^2w^2w_n^2\right\}^{1/2}} \qquad \text{phase } = j = \tan^{-1}\left(\frac{2zww_n}{\left(w_n^2 - w^2\right)}\right)$$

Displacement transmissibility:

$$\frac{X}{Y} = \frac{\left(w_n^2 + 2izww_n\right)}{\left(w_n^2 - w^2 + 2izww_n\right)}$$

$$\left|\frac{X}{Y}\right| = \frac{\sqrt{w_n^4 + 4z^2w_n^2w^2}}{\sqrt{\left(w_n^2 - w^2\right)^2 + 4z^2w_n^2w^2}} \qquad j = \tan^{-1}\left(\frac{2zw^3}{w_n\left(w_n^2 - w^2\right) + 4z^2w_nw^2}\right)$$