Design Criteria for Composite Masts

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ABSTRACT

Composite materials such as fiberglass have been used in recreational marine craft for over 30 years. Only within the last decade however have composites found applications in mast construction. This application to masts is due to the lighter weight possible by using composites such as carbon fiber/epoxy that have higher stiffness-to-weight ratios than the traditional aluminum and wood mast materials. Composites use in masts has not been without problems however as designers and manufacturers learn the techniques necessary to produce reliable structures. As with any maturing technology, empirical developments will eventually result in acceptably reliable structures.

This paper uses a reliability-based design criterion to aid composite mast development. The basic approach characterizes the uncertainties and variabilities of mast design (loads, materials, analytical models, etc.) to predict a total uncertainty called "the probability of failure." The total probability of failure is then compared to "acceptable" probabilities of failure generated from existing aluminum mast designs to determine an appropriate factor of safety.

This approach takes the guesswork out of factor of safety selection, giving the mast designer an important tool. By using reliability-based methods the influence of each part on the overall probability of failure can be determined. This allows for intelligent selection of design areas to improve that provide the biggest potential gain in reliability, cost or weight, and decreases the likelihood of either an overly-conservative, heavy mast, or an under-designed mast.

The paper begins with the causes of mast failure and then presents the impact of load modeling, material properties and structural modeling on predicted mast reliability. An example mast design is used to illustrate the method, with the result that a composite mast can be designed at nearly the same factor of safety as an aluminum mast.

NOTATION

β = safety index
B = bias
D = accumulated damage ratio
$\Phi[\ ]$ refers to the cumulative area to the left of the ordinate β on a Gaussian "normal" distribution curve
m = slope of the S-N curve
n = mean or central factor of safety
n(Sfi) = number of stress cycles at stress fi
N(Sfi) = number of cycles to failure at stress fi
P_f = probability of failure
R = strength
Rm = mean strength value
Rn = nominal strength value
S = load
σ_X = standard deviation of X
σ_X^2 = variance of X
V_c = COV of manufacturing variances
V_{mr} = COV of Miner's Rule
V_n = COV of reduced strength due to fatigue
Vr=COV of strength evaluation
V_s = COV of stress (loading) evaluation
V_{sn} = COV of S-N data
V_X = coefficient of variation (COV) of X,
X_m = mean value of the X distribution

1.0 INTRODUCTION - Statement of the problem and the causes.

Although composites have been used in mast construction for sailboards, sailing dinghies, and specialized cruising boats for over a decade, the popularization of composite masts is a relatively recent development due to their use in the International America's Cup Class boats. With a carbon/epoxy laminate's higher specific stiffness than aluminum and wood, a significant reduction in mast weight is possible resulting in increased righting moment and reduced pitch gyradius, both leading to increased performance. Table 1 shows the specific stiffness and strength of three common mast materials relative to aluminum. The composite laminate is superior in both specific axial stiffness and specific compressive strength.

Mast Material	Density lbs/in^3	Specific Axial Stiffness*	Specific Comp. Strength**
Wood (Sitka Spruce) (Gougeon, 1985)	0.015	1.05	0.83
Alum. (Spotts, 1978)	0.10	1.00	1.00
Carbon / Epoxy (Tsai, 1988)	0.058	1.89	2.70

TABLE 1: Typical Mast Material Properties

* The Specific Axial Stiffness is taken as [(Young's modulus of material)/(Density of material)]/[(Young's modulus of aluminum)/(Density of Aluminum)]. The Specific Compressive Strength is similarly calculated, without considering section buckling.

** Based on a 50% 0°, 40% ±45°, and 10% 90° laminate.

The technology used in composite mast construction is still developmental, resulting in a lack of documented reliability, and in turn, extra conservativeness by spar designers and makers. This is not simply due to the challenges of mast construction, as the lack of documented reliability of advanced composites is prevalent in other marine applications (NRC, 1990). Reliability is another way of saying "the lack of failures" and a reliability analysis begins with determining the causes of failure.

The causes of mast failure can be categorized many ways, and the definition of "failure" differs with the intended function. For a cruising boat "failure" may be a loss of function that cannot be repaired while underway, while for a racing boat "failure" is the loss of competitiveness. Figure 1 shows a distribution of mast failure causes estimated from information provided by marine industry experts. The five categories include:

Operator errors: These include failure due to broaching, failure to adjust rigging correctly, incorrect selection or installation of fittings by the operator, and lack of maintenance.

Structural: Problems caused by improper modeling of loads or material capacities, insufficient manufacturing quality control, and improper specification or installation of fittings by the designer or mast maker.

Corrosion: This includes corrosion or electrolysis in rigging wire or fittings.

Wear and Fatigue: Rope, wire and fittings failure from wear or fatigue.

Other: These include non-operator error collisions, groundings and other failures not falling in the above categories.

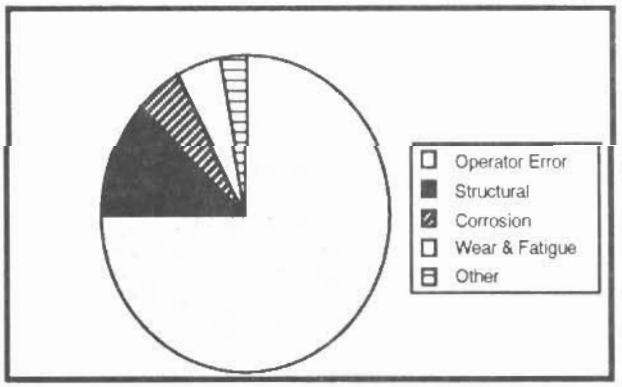

Fig. 1: Estimated Causes of Aluminum and Composite Mast Failures

Although the lack of data allowed only an estimate, the distribution is typical of failure distributions in other marine industries (Bea, 1990; Lamb, 1993). Section 2 describes some aspects of operational and human error and their influence on mast failures. The remaining sections focus on structural issues, with some discussion of corrosion, wear and fatigue.

1.1 Solution Approach / Loads and Strengths/ Types of Uncertainties

The approach used in this paper is to develop design criteria using engineering reliability methods used in other marine applications (DNV, 1991). These criteria include designing the mast to meet the normal loads associated with daily use and some abnormal loads caused in "extreme" conditions. The load magnitudes of each case are tied to the intended environmental conditions. For example, a sailboard or sailing dinghy will not see the same conditions as those found by round-the-world racing yachts and therefore the sailboard's "extreme" loads must be defined differently.

The criteria are then looked at in terms of reliability with the quantified lack of reliability called the probability of failure. The probability of failure values are chosen on the basis of what is acceptable to the operator. It is clearly impossible to design for every contingency, and "over design" can negate any performance advantage. In many cases a good approach is to estimate an acceptable probability of failure by comparison with similar structures.

The reliability assessment looks at each component of design and manufacturing and determines its influence on the final probability of failure. The options involved with each piece can be looked at in terms of cost, performance, likelihood of failure or other criteria and can then be subjectively evaluated. Part of the evaluation includes considering the variability of each component, as greater variability increases the probability of failure, but also probably decreases the initial cost. By using reliability methods designs may be improved by either identifying the areas where variability can be controlled and reliability increased, or by saving money in areas where reliability is adequate or excessive.

Structural reliability analyses are traditionally categorized by their "level" (Lewis, 1988). *Level 1* reliability methods use a single "safety factor" to describe uncertainties. Three deterministic design methods are described in Section 1.4. *Level 2* reliability methods use two values to describe the uncertainties of each variable; for instance the mean and standard deviation of a distribution. *Level 3* methods use a jointly calculated probability of failure for the system to characterize its reliability. This paper uses the Level 2 method. The first step in a reliability analysis is to define the design criteria.

A flowchart showing the reliability-based mast design criteria used in this paper is shown in Figure 2. The approach used is similar to that in offshore structures (Bea, 1990). The flowchart starts with three inputs, a definition of the desired reliability, characterization of the environment, and a preliminary design concept. The design method is then followed and revised until the desired performance and reliability are met. Section 3 of the paper deals with the loads and load uncertainties, and Section 4 details the material properties and structural uncertainties. Section 5 combines the probabilistic results of Sections 3 and 4 to determine the probability of failure of the system.

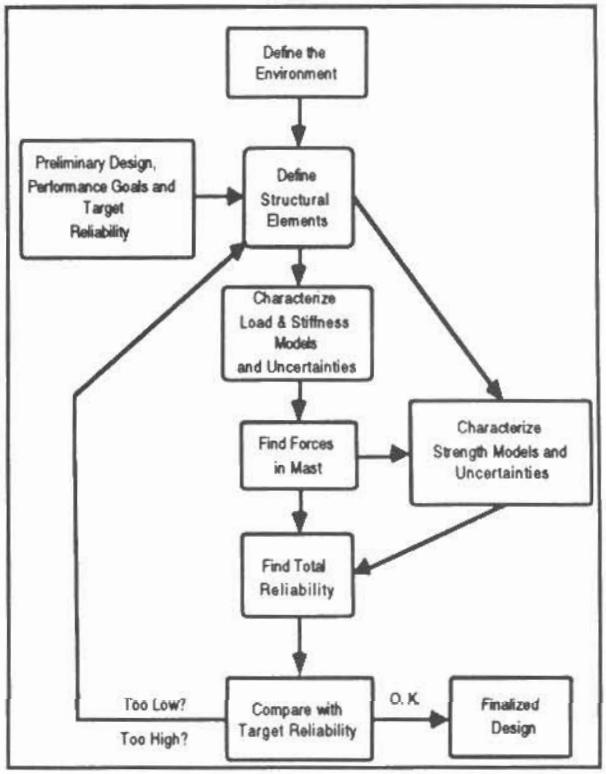

Fig. 2: Flowchart of Reliability-Based Mast Design Criteria

Loads and strengths will vary during the life of the vessel and can be characterized using statistical (historical) and probabilistic (predicted) distributions. By knowing the historical trend of the load or strength, future performance can be predicted with some confidence depending on the variability of the data. Two common statistical distributions useful in marine applications are the normal and lognormal distributions. Distributions like these are used to characterize the loads and strengths analyzed in Sections 3 and 4. In general, three methods are available to generate a statistical or probabilistic distribution. They are:

Historical. This is the traditional approach of gathering large amounts of data and is the most accurate.

Discretize. To characterize a complex system, the system is broken down into smaller subsystems which are then analyzed separately.

Judgment. The experience of experts is used to form an estimated distribution. Figure 1 for example was generated from the judgment of experienced sailors and designers.

The range of the values in the distributions indicate a lack of certainty in any one value of load or strength being the best for design purposes. These uncertainties are typically categorized two ways:

Type 1 Uncertainties: Those uncertainties inherent in nature. These include variations in wind strength, tested values of material properties, and the number of sunny days in a given month.

Type 2 Uncertainties: These are caused by modeling inaccuracies, with one example being the errors caused by the plane stress and linear elastic assumptions in Euler beam theory.

Both sets are used to calculate the probability of failure by characterizing the variability of each uncertainty. In general, type 2 uncertainties have smaller variabilities and are easier to control.

1.2 Scope and Assumptions

The paper's goal is to describe a method to characterize and control the inherent uncertainties in mast design, and to illustrate a method of developing a design factor of safety for composite masts. The reliability method does not replace traditional deterministic methods, rather it allows an engineer the opportunity to select an appropriate "factor of safety". Proper selection of loading and strength models for a specific mast is obviously crucial to the success of any design criteria, but is not fully developed in this paper. For example, the best critical buckling model for composites is not covered.

The reliability approach assumes certain characteristics that limit the approach's accuracy. These assumptions include the need to fit the data to a standardized distribution and the required use of correct physical models when analyzing the data. In the first case the error is small as most data fit at least one standard distribution well. An example of the second assumption is that in addition to the variability of test data there also exists an error in using the ASTM D3039 specified strain rate for tensile tests of composites that are strain sensitive and see different strain rates in the field. If known, this error can be included as an uncertainty, if unknown, the error continues as an uncertainty in the final result.

To illustrate the reliability-based design criteria approach an example mast design is followed. The example is the mast of an International 10 Square Meter Sailing Canoe (IC) designed for San Francisco Bay. This design was selected due to its simplicity of rigging and the eleven years of experience using composite masts in the class. Table 2 compares dimensions of three mast designs for the IC. The masthead is approximately 20.5 feet above the deck.

	Wood (Williamson, 1985)	**Aluminum** (Proctor, 1987)	**Composite**
Dimensions (in)	3.25 x 2.3	2.64 x 2.16	2.5 x 2.25
Unrigged Weight (lbs)	10	12	7.5
Wall thickness (in)	0.25	0.06	0.07

TABLE 2: Typical Dimensions of International Canoe Masts

1.3 Probability of Failure Description and Separation by Function

As stated earlier, the probability of failure is the likelihood that the mast fails its functional purpose. Since masts are structures, this could also be stated as the likelihood of the loads exceeding the strength. As an equation this is written as:

$$P_f = P[S>R]$$

where:
P_f is the probability of failure
P[ ] is read as "the probability of..."
R is the strength
S is the load

Assuming the likelihood of failure follows a trend that is normally distributed (i.e., follows a "bell" curve), the probability of failure is the area under the tail of the distribution that falls to the right of a particular ordinate. That ordinate is referred to as the "safety index" and is labeled β. Therefore,

$$P_f = 1 - \Phi[\beta]$$

where:

Φ[ ] refers to the cumulative area to the left of the ordinate β on the normal distribution curve, and

β is the safety index

β can be calculated from the known ities, as will be done in Sections 3 and 4 per or, based on known reliabilities of ctures, a target β can be selected and ainties controlled to get an acceptable at minimum cost or weight.

he selection of the target safety index n the desired structural reliability. For small β might be chosen for an cer followed by a tender, while a large d be appropriate for a world cruiser. onally, different mast components may have different target β's. A component with a small impact on mast function may have a very small β to reduce weight, while a critical item will have a higher β.

1.4 Design Methodologies

Three design methodologies are currently used in structural engineering to produce satisfactory service (Bea, 1990). These methodologies are:

Working Stress Design (WSD): This is the traditional method of using a nominal design value for loads and strengths. After structural analysis is completed (usually a linear elastic analysis) a factor of safety is applied to cover load and strength uncertainties not considered in the original analysis.

Load and Resistance Factor Design (LRFD): This method uses separate factors for loads and strengths. The load "correction" factor, γ, is greater than one to raise the loads, and the strength factor, ϕ, is less than one to lower the strengths. This method offers certain advantages over WSD as changes in the variability of *either* strengths or loads can be handled by adjusting the appropriate factor. An example of this is switching materials in a proven application. The loads remain the same, but the variability in material properties may have changed. This will be used later in the paper.

Limit State Design (LSD) or Ultimate State Design: This method analyzes the structure for ultimate strength by using nonlinear ultimate (plastic for metals, last ply failure for composites) analysis. Loads and strengths are factored as in LRFD.

2.0 Operational Errors

Figure 1 showed that somewhere near 75% of mast failures are caused by "human error" not related to design or manufacturing. Some example causes are severe broaching where unintended design loads are imposed, failure to tighten backstays, failure to install cotter pins in turnbuckles, improper use of hardware or recklessness in racing situations. In many instances a "gray area" exists about what the mast designer should include as design loads. *Defining this area was the first step in the mast* design criteria flowchart, Figure 2. Effectively communicating the load bounds is therefore required to reduce human error.

2.1 Ignorance/Training

Human errors are basically inevitable, and therefore the goal is to reduce their

likelihood. Studies of the *Piper Alpha* oil platform and *Exxon Valdez* accidents indicated complex systems combined with lack of communication, human fatigue and lack of training were primary causes (Bea and Moore, 1992). In mast failures caused by human error many of these same problems are common. Solutions include thorough crew training, communication of design loads and organized maintenance.

Although communications and training are not normally considered the realm of designers, the designer should be careful to communicate any information about the mast's operation that may not be "common" information. In addition to the design loads, examples with carbon/epoxy masts include effects of ultraviolet degradation, ways to prevent aluminum/carbon electrolysis and correct ways to install or move fittings.

2.2 Human Limitations/Human Factors Design

The level of system complexity is a proven factor in human errors. In masts the reduction of adjustable rigging reduces the likelihood of human error. Unfortunately, it may also reduce performance. One solution is to design adjustable systems with control bounds so that adjustment outside safety limits is not possible. This may simply mean marks on ropes, or "red lines" on hydraulic gauges.

A good solution is to design the mast and its controls with "human factors" in mind. This means designing for easy control (i.e. foot braces near winches), lack of confusion in controls (i.e., different colored control lines), and easy-to-release controls in emergency situations (i.e., a quick-release vang).

As human errors contribute the largest source of failures their reduction can contribute the greatest impact on reducing the probability of failure. Reliability analyses can also be used to quantitatively reduce human errors in engineering structures (Bea, 1990; Bea and Moore, 1992). One example is performance-based statistical results by intended operators of government technical manuals.

3.0 Loads and Loading Uncertainties

Due to the variabilities of wind and waves the greatest uncertainty in engineering analysis of marine structures is the loading (S). In sailboat masts these uncertainties increase the likelihood of loads exceeding a nominal or "static" design load. Unfortunately it is beyond the scope of this paper to fully review or develop new load models. Instead, the factors involved in loading will be discussed and the reliability method demonstrated.

An important point is that to realize benefits from reliability analyses the most accurate models should be used, rather than overly conservative models which include "bias" (see Section 4.1) By including the accurate models' lower uncertainties the final probability will maintain its accuracy, and the final β will more closely represent the physical reality. At that point intelligent decisions can be made to adjust the factor-of-safety.

3.1 Wind Loading

Various models have been proposed to simulate sail loads on masts (Hoste, 1978). These include simple rectangular, quadrilateral, triangular and parabolic distributed loads. Current state-of-the-art methods using computational fluid dynamics have increased the accuracy, but are generally beyond the financial capacity of most designers. To determine the variability, one good way to get a representative loading is to start with aerodynamic first principles.

Prandtl's lifting line theory presents a reasonable starting point (Abbott and Doenhoff, 1959). Using this approach a triangular sail plan would produce a parabolic loading, although a triangular loading pattern is thought to be more plausible (Coakley, 1989).

Another approach credits empirical development toward the "ideal", which would conceivably yield the best performance. Prandtl's theory suggests the highest lift/drag ratio will occur with an elliptical loading. Another justification for using an elliptical sail loading model may be that for small aspect ratios loading tends toward an elliptical pattern (Marchaj, 1979). As loading bounds, a comparison between triangular and rectangular loading distributions showed a maximum 9% difference in mast compression on a 40' sailboat (Coakley, 1989). An elliptical distribution will fall in between, resulting in a Type 2 uncertainty of 5%, which will be considered the *coefficient of variation* (COV) for wind load *modeling*. Analytically, this can be represented as a dimensionless number:

$$V_x = \frac{\sigma_x}{X_m}$$

where V_X is the coefficient of variation of X,

σ_X is the standard deviation of X and,

X_m is the mean value of the X distribution

The *population standard deviation* is found from:

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{\sum \frac{(X - X_m)^2}{n}}$$

where σ_X^2 is the *variance*

In addition to the variability in modeling, a larger uncertainty exists in the wind speed. This results from gusts, long-term speed variations and changes in apparent wind. In the first two cases, studies have shown the wind speed can be characterized by normal and Weibull distributions, respectively (DNV, 1991). Note that windspeed distributions may be "capped" or "truncated" by local conditions.

In the example case, the operator is not likely to take the vessel out when the average wind speeds exceed 30 knots as class rules do not permit racing in winds of this magnitude. Using a normal distribution the estimated COV for gusts is 25%, resulting in a standard deviation of 7.5 (DNV, 1991). The long-term speed variation is considered insignificant as the operator can adjust for the changing conditions.

The apparent wind is the true wind modified by the boat's motion. In 30 knots of wind the International Canoe's upwind boatspeed is about 10 knots at a nearly constant angle of 45° to the true wind, resulting in an apparent wind speed of 37 knots. Rolling due to waves is considered negligible for the IC, so the apparent windspeed uncertainty is also considered insignificant.

The standard deviation of the combined wind loading and modeling can be found from the algebra of normal functions (Bea, 1990). For two independent functions (i.e., the accuracy of the wind loading model is not dependent on the windspeed) the combined standard of deviation is:

$$\sigma_{wl} = X_m Y_m \sqrt{V_X^2 + V_Y^2 + (V_X^2 V_Y^2)}$$

If the variations are known, the combined COV is simply:

$$V_{wl} \cong \sqrt{V_X^2 + V_Y^2} \text{ (if Vx and Vy are <<1)}$$

For the example problem, as lift is a function of windspeed to the second power, the combinded COV is:

$$V_{wl} \cong \sqrt{0.05^2 + 2(0.25^2)} \cong 35.7\%$$

3.2 Righting Moment Variations

Mast loading is directly related to the righting moment developed (Marchaj, 1979). In many boats this is controlled by the amount of ballast and hull shape and may be assumed to be a "fixed" quantity at any given heel angle. In boats using a large proportion of crew weight the variability of crew weight needs to be considered.

The International Canoe has minimal form stability and crew weights range from about 115 to 230 pounds. Assuming a normal distribution, a mean of 170 pounds, and the "ends" of the range representing the 5th and 95th percentile values, the standard deviation and COV are therefore approximately:

$$\sigma_{rm} \cong 27.5$$

$$V_{rm} \cong 15.7\%$$

3.3 Dynamic and Wave Loads

Transient inertial loads caused by rolling, pitching and slamming increase mast loads. These transient loads are dependent on the vessel geometry, heading, speed, and location, and can significantly increase mast loads over the "steady state" values determined by using sail load distributions discussed in Section 3.1. By using appropriately derived equations of motion (i.e., based on F=ma+dv) the rigging and mast loads can be determined. See (DNV, 1991) for a similar problem solution.

The uncertainty in these loads are based on variabilities in accelerations (caused by changes in wave slope), damping (aero- and hydrodynamic), and virtual mass. This requires further analysis of intended operating conditions. In the example calculation the significant wave height for San Francisco Bay corresponding to 30 knots wind speed is about 2.5 feet (Chou, 1974). Using a JONSWOP spectrum a standard deviation of 0.3 feet and a COV of 0.12 was estimated (Johnson, 1953). The damping factors and mass were assumed constant, leading to a transient standard deviation and COV of:

$$\sigma_{tr} = 0.3$$

$$V_{tr} = 12\%$$

Note that the wave loading is a function of wave height squared. If the boat was used in other locations other standard deviations and COV's would need to be calculated, and the results combined using the algebra of normal functions.

The total uncertainty in loading can be found by combining the different elements using the algebra of normal functions. Table 3 summarizes the uncertainties of loading.

Uncertainty	Estimated COV
Wind Loading	35.7%
Righting Moment	15.7%
Wave Loading	12.0%

TABLE 3: Coefficients of Variation for IC Mast Loading

For the example problem the variables were assumed independent, so the standard deviation of the loading is:

$$\sigma_s = \sqrt{\sigma_{wl}^2 + \sigma_{rm}^2 + 2\sigma_{tr}^2}$$

Or, as used earlier, if the coefficients of variation are known and are small,

$$V_s = \sqrt{V_{wl}^2 + V_{rm}^2 + 2V_{tr}^2}$$

So the uncertainty in the loading is,

$$V_s = \sqrt{(0.357)^2 + (0.157)^2 + 2(0.12)^2} = 42.5\%$$

As noted, in this case the variables were considered independent. This could be assumed as in the sheltered bay waters wind speed and wave height are loosely correlated. Where reasonable correlations exist the equations must be modified to include the correlation effects.

4.0 Strengths (Capacities) and Structural Modeling Uncertainties

After the preliminary design and environment characterization are complete, the structural capacity uncertainties are calculated. These uncertainties are typically much lower than those for loading, for both type 1 and type 2 uncertainties. This is due to the greater ability to control material properties and the successful application of accurate structural analysis tools.

The definition of structural capacity (R), is dependent on the operational goal of the structure. Two structural capacities are typically defined; serviceability and ultimate. The serviceability limit is where function is impaired, but not completely removed. For a mast this might mean the loss of a halyard. The ultimate limit is where all function is lost. In this paper's example the ultimate limit will be analyzed; the approach for serviceability is the same, although other components would be included.

To determine the uncertainties in an ultimate limit state analysis, the pieces involved in the analysis are characterized. These include the materials, analytical models for initial strength and models for determining reduced service-life due to corrosion, fatigue, and wear.

4.1 Mast Materials and Analytical Models

Table 1 listed the densities and specific stiffness and strength of common mast materials. The values used to calculate the specific stiffness and strength were taken from published tables of "nominal" values used for design purposes. The "nominal" values however, do not represent the mean, mode or median of the material value distribution. Rather it represents a "conservative" number that helps increase the factor of safety. When using reliability methods this needs to be taken into consideration. When a nominal value differs from the actual median value the value is said to be biased. The analytical representation is:

$$Bias = B = \frac{\text{true median value}}{\text{nominal or predicted value}}$$

and $R_m = R_n \bullet B$

where

Rm= the mean strength value and

Rn= the nominal strength value

To get an accurate safety index for the structural capacity, the bias must be taken out. Material values listed in specifications such as AISC or MILSPEC are usually biased. For example, A36 steel typically has a bias of about 1.2. The best way to remove bias is to test a sufficient amount of the mast material to statistically characterize the material property, and then use the standard deviation or COV of the test data.

Current composite masts are made of a wide variety of materials (Greene, 1990). A typical combination is T300 grade carbon with a 150-250° F cure epoxy resin combined in a prepreg system. Sample tensile test data provided by Hexcel for a T300/250°-Epoxy are shown in Table 4.

Coupon #	Panel #	Tensile Str. [ksi]	Tensile Mod. [msi]
1	HF131	284.8	18.2
2	HF131	271.2	18.6
3	HF131	305.4	19.3
4	HF131	293.5	19.3
5	HF232	299.5	19.3
6	HF232	293.4	19.0
7	HF232	289.8	18.9
8	HF332	269.9	18.9
9	HF332	307.7	19.5
10	HF332	303.5	19.3
	Median=	293.45	19.15
	Mean=	291.87	19.03
	St. Dev.=	13.32	0.40
	Coef. of Var.=	4.56%	2.09%

TABLE 4: Material Test Results for Uni-Carbon/Epoxy

Table 4 shows two interesting points. First is that the mean and median values are slightly different, which means the distribution does not strictly follow a "normal" distribution. The lack of data points is one likely reason. The difference is small however, indicating the assumption is acceptable.

The second point is that the COV is similar to typical steel and aluminum values. A generally accepted average COV for ASTM D3039 results is 8%. The limited number of data points indicates the importance of sufficient data. Table 5 shows tensile strength results based on each panel. The conclusion is that using only 3 or 4 data points is insufficient to adequately characterize the uncertainties in a material.

Panel#	Mean Ten Str	Coef of Var	Resin Content	Fiber Volume
HF131	288.73	4.99%	39.7	52.05
HF232	294.23	1.67%	41.1	50.51
HF332	293.70	7.05%	40.0	51.76

TABLE 5: Material Property Statistics by Panel

Another consideration is the duplication of manufacturing techniques. As masts are typically laid-up over two or three days, the use of three different panels better represents the actual variabilities in the manufacturing process. If "laboratory" test data is used the uncertainty from the actual mast construction methods must be included. These uncertainties may range from 1-30% depending on the techniques and quality control.

The test conditions must also be considered. The ASTM D3039 test specifies a strain rate of 0.01-0.02 in/in-min (ASTM, 1978). This strain rate is smaller than that occurring during gusts and waves. Studies of similar materials indicate an increase of strength of 11% for strain rates approximating aero- and hydrodynamic loadings (Smith, 1990). This increase can be treated as a bias.

The last point about the test data is that the test should accurately represent the type of failure indicated by the structural analysis. In most masts this would be a compression failure, so compression data would be more appropriate. Unfortunately suitable compression data representing the 150°F cured, vacumn-bagged laminates typical of IC mast construction was not available. Due to the failure mode, compressive data typically has higher coefficients of variation than tensile data. For the example problem, non-vacuum bagged, room temperature cured carbon/epoxy material properties from earlier tests by the author were used, and had:

$$V_{mp} \cong 12\%$$

A presentation of other composite material properties anduncertainties are given in (Chamis and Shiao, 1993).

Other uncertainties exist in the analytical models used to predict failure. Various modeling techniques are used to predict deflections, stresses and failures in masts. These include Euler beam analysis, stiffness matrix analysis (Hoste, 1978) and finite element analysis (Miller and Dillon, 1994). One specialized non-linear FEA program designed for quick mast analysis is GEMSPAR (Coakley, 1989).

The uncertainties of each method differ depending on the validity of the assumptions used. In general, the slenderness of the masts allows the Euler method to maintain uncertainties within about 10%, the stiffness matrix within 5%, and the finite element method within 2-3% (Ochoa and Reddy, 1992). For the example problem the COV will be taken as 3% as typical failures in IC masts are due to non-buckling compressive failures. For buckling calculations the uncertainty is higher, on the order of 10-30% for finite element analysis (Ochoa and Reddy, 1992).

4.2 Rigging Materials

In addition to the mast materials other materials included in the rig must be included in the analysis. These include rigging wire, tangs, rivets, bolts, spreader brackets and turnbuckles. As shown earlier the best way to determine the coefficients of variation is testing. Another way is to estimate a COV using known values from other components. Two ways of estimation are: 1) find a similar part with a known coefficient, and 2) compile a coefficient from known subparts.

To demonstrate the methods, a COV for the rigging wire will be estimated. Using the first method, from similar wire an estimated COV to use is 10% (DNV, 1991). Using the second method, the 1X19 rigging wire is made up of 19 parallel strands of 316L stainless steel. The manufacture of cold drawn stainless steel is tightly controlled, resulting in a COV between 2 and 3% (Spotts, 1978). Taking V_{316L} as 2.5% the combined V_{wr} is found from:

$$V_{wr} \cong \sqrt{\sum_{i=1}^{N} V_i^2} \cong \sqrt{\sum_{1}^{19} V_{316L}^2}$$

or,

$$V_{wr} \cong \sqrt{19 \bullet 0.025^2} = 10.9\%$$

If the components are highly correlated (i.e., not independent), then for systems in series rather than parallel, the COV can be closely approximated by using the variation of the part with the greatest uncertainty. This is a reasonable approximation for marine structures as the loadings usually correlate the components (Bea, 1990). Table 6 lists the estimated coefficients of variation for the example.

Uncertainty	Estimated Coefficient of Variation
Carbon/Epoxy	12.0%
Analytical Model	3.0%
Rigging Wire*	10.9%
Turnbuckles*	6.0%
Tangs*	4.0%
Bolts*	4.0%
Swages*	13.0%

TABLE 6: Coefficients of Variation for IC Mast Structural Capacity

The items with asterisks are in a series system, the remaining items are parallel. Using the rules mentioned above, this results in a COV for initial structural capacity of:

$$V_r \cong \sqrt{(0.12)^2 + (0.03)^2 + (0.13)^2}$$

$$V_r \cong 17.9\%$$

V_r = COV of strength

4.3 In-Service Changes in Structural Capacity

The uncertainties of strength characterized in Sections 4.1 and 4.2 were based on initial material properties. In-service environmental or operational conditions may modify those properties over time. Examples include corrosion, wear, fatigue and ultraviolet degradation. These conditions all reduce material properties, but in rare cases such as long-term postcuring of some resins (or

concrete), material properties may rise. Any changes need to be included in the reliability analysis if they are likely to become important during the structure's lifetime.

Masts are subjected to numerous degrading effects that are alleviated by proper inspection, maintenance and repair (IMR). In the case of masts and rigging designed for long service lives in conditions where IMR may be difficult, the reliability analysis needs to include fatigue effects caused by cyclic wind and wave loads. This may include reduction of properties over the service-life. In the case of inshore vessels which have easily replaceable components and an efficient IMR plan, fatigue considerations are less important.

Most fatigue problems result from either manufacturing flaws or operational damage. To reduce fatigue-based strength reduction three steps are usually taken (Bea, 1990):

1. Avoid stress concentrations: Proper design significantly reduces fatigue problems. Examples include designing for the correct number, size, and distribution of fasteners, and using "softening" strips and quasi-isotropic laminates around hardware.

2. Minimize flaws: This means maintaining good quality control during manufacturing and hardware installation. Examples include aligning hardware with the load, installing the correct material, size and type of fasteners and minimizing voids in the lamination.

3. Minimize degradation: This is oriented at both the local element and system level. At the local level this includes selecting good materials and hardware, protecting them from hostile environments whenever possible, and proper IMR. At the system level this means designing the mast and rigging system with sufficient capacity so that after detection, repairs can be made before the mast fails.

Designing for fatigue in metal structures subjected to the marine environment is well-documented (Wirsching and Chen, 1967; Geyer and Stahl, 1986; Bea, 1990). Some fatigue and degradation data also exists for design of composite structures for the marine environment (Macander and Silvergleit, 1977; Loveless, Ingle et al., 1981; Springer, 1981; Jakubowski, 1990; Olsson and Lonno, 1990; Smith, 1990; Cable, 1991). For typical mast materials carbon/epoxy laminates seem to perform better than aluminum in fatigue, with wood performing better than either (Reichard, 1988).

Fatigue analysis includes (Bea, 1990; Smith, 1990):

1. Characterize the environment. This involves extending the Figure 2 flowchart's first step to include the number of anticipated cycles the mast will see. The major factors are the wave-induced, and gust-induced loads. If there are significant loads during transportation those loads would also be included. For the IC mast the parameters might be taken as: a service life of 10 years, used on 80 days per year for 5 hours a day, hitting a 1' wave each second, and encountering a gust every 10 seconds. This works out to roughly 1.6×10^7 cycles.

2. Evaluate the stresses and strains using an accurate analysis tool such as finite element analysis. Unfortunately this was not possible within the scope of this paper.

3. Find the strength degradation (the remaining strength) for the desired cycles by referencing appropriate S-N fatigue curves. For a thorough discussion of fatigue, see (Collins, 1993).

4. Find the "fatigue life" through the use of a cumulative damage model, and check that the fatigue life is not less than the design life. For metals the linear Palmgren-Miner model is often used:

$$D = \sum_i \frac{n(S_{fi})}{N(S_{fi})}$$

where,

D = the accumulated damage ratio

Usually failure is assumed to occur when D=1, although for some FRP this is taken as less than 1 (Smith, 1990). For composites the Hashin-Rotem logarithmic equation is sometimes used (Hashin, 1985):

$$1 = \sum_i \frac{\log(n(S_{fi}))}{\log(N(S_{fi}))}$$

n(Sfi)= the number of stress cycles at stress fi

N(Sfi)= the number of cycles to failure at stress fi

Unfortunately comparable data on cyclic stress to failure could not be found for laminates similar to those used in IC mast construction. For unidirectional carbon/epoxy laminates the degradation after $5X10^6$ cycles was approximately -16%, and for quasi-isotropic laminates it was approximately -29% (Smith, 1990).

To find the fatigue analysis uncertainties the Munse model can be used:

$$V_n = \sqrt{(V_{sn}^2 + V_{mr}^2 + m^2 V_s^2 + V_c^2)}$$

where,

V_n = COV of reduced strength due to fatigue

V_{sn} = COV of S-N data

V_{mr} = COV of Miner's Rule

m = slope of S-N curve

V_s = COV of stress (loading) evaluation

V_c = COV of manufacturing variances

For comparison, in ships these are estimated as (Bea, 1990): V_{sn} =0.62, V_{mr} =0.15, V_s =0.10, V_c =0.4 and with m =|-6.16|, $V_n \cong 96\%$., which is substantially higher than that normally calculated for initial capacities.

Other in-service changes in material properties, and their uncertainties, can be similarly calculated and used in the reliability analysis.

5.0 Probability of Failure

Once the uncertainties in the loads and strengths (initial or in-service modified) are calculated, the next step is to calculate the probability of failure of the system (see Figure 2). Recall from Section 1.3 that:

$$P_f = P[S>R]$$

and

$$P_f = 1 - \Phi[\beta]$$

Using the coefficients of variation for *normally* distributed functions (since the mast loads and strengths were found to be normally distributed), β can be found from (Bea, 1992):

$$\beta = \frac{n-1}{\sqrt{n^2 Vr^2 + Vs^2}}$$

where,

$$n = \frac{R_m}{S_m}$$

Vr=COV of strength evaluation
V_s = COV of stress (loading) evaluation

The value "n" is considered the mean or *central factor of safety* based on the mean strength and load, where the subscript m refers to the mean value of the load (S) or strength (R). Note that "n" is typically a larger value than the "nominal" factor of safety, as the bias has been removed.

5.1 Example Prediction

Table 7 shows the values of β and P_f for the IC mast for different values of n, using the coefficients of variation for loading and *initial* strength. The P_f values are annually based as the loads were calculated on an annual basis. The table shows a rapid decrease in the probability of failure for a steady increase in n.

n	β	Pf
1.0	0.00	50.000%
1.5	0.99	16.026%
2.0	1.80	3.617%
2.5	2.43	0.762%
3.0	2.92	0.178%
3.5	3.30	0.049%
4.0	3.60	0.016%

TABLE 7: Safety Index and Probability of Failure for Composite IC Masts Versus Central Factor of Safety

The table indicates for instance, that for a central factor of safety of 2, there is a 3.6% annual probability of failure. This table can be used two ways. First, if the operator desires a mast designed for strength only, and knows their

acceptable probability of failure, the appropriate n or β can be used in the design process.

The second way is if the mast is designed for stiffness. Then the n can be used to directly find the P_f. If P_f is lower than necessary, the coefficients of variation may be intentionally raised to reduce cost or weight.

5.2 Acceptable Probability of Failure Determination

Choosing an acceptable probability of failure for the mast is ultimately the owner's and operator's responsibility, with the understanding that the only way to completely eliminate the probability of failure is not to use the mast. Too low a probability of failure (i.e. too conservative) results in an uncompetitive or costly mast, while too high a probability of failure may result in failure at an unfortunate time.

The designers role is to assist in selecting the appropriate risk for the application. Some factors included in the decision might be: safety, cost, publicity, possible litigation, competitiveness, and crew experience.

Three basic approaches are used to help select an acceptable probability of failure for marine systems (Bea, 1990) Summarized for mast analysis they are:

1. Historical: This approach uses data from similar structures in similar environments. For the example problem a similar structure might be a successful aluminum mast. The probability of failure could be calculated for the aluminum mast and used as the baseline.

2. Code Calibration: This approach uses current design practices or codes. In-house semi-empirical methods are used to develop a comparable structure which then has a reliability analysis performed to be used as a baseline for new design practices. For example, probabilistic methods for steel and concrete offshore platforms have resulted in annual P_f's of 0.1 to 0.001, with 0.01 a typical value (Stahl, 1986).

3. Cost Utility: This uses initial and expected future costs as a basis to determine what probability of failure will yield the best results.

For the example problem, the simplest approach would be the historical approach, using a successful aluminum mast as a guide.

Comparing the COV's for the composite mast, the loading remains unchanged, only the material property COV changes for the aluminum mast. The COV for carbon/epoxy in compression was assumed as 12%. If the similar COV for aluminum is taken as 4% (Spotts, 1978), then the comparison values for the composite and aluminum masts are as shown in Table 8.

	Carbon/Epoxy		Aluminum	
n	β	Pf	β	Pf
1.0	0.00	50.000%	0.00	50.000%
1.5	0.99	16.026%	1.06	14.568%
2.0	1.80	3.617%	1.97	2.460%
2.5	2.43	0.762%	2.73	0.318%
3.0	2.92	0.178%	3.35	0.040%
3.5	3.30	0.049%	3.86	0.006%
4.0	3.60	0.016%	4.28	0.001%

TABLE 8: Comparison of Safety Index and Probability of Failure for Carbon and Aluminum IC Masts

Using the chart, if a structural analysis of the aluminum mast revealed the acceptable probability of failure as 1% (the aluminum mast's central factor of safety (n) would be 2.22), the corresponding central factor of safety for the carbon/epoxy mast would be 2.41. If other than mean strength or load values were used then the bias involved with the nominal values would need to be removed.

6.0 Areas to Address and Improve

A reliability analysis can be used to select areas to improve. The areas with the highest coefficients of variation can be targeted first as they contribute the most to the probability of failure. Unfortunately the areas with the highest uncertainty are also the most difficult to control, the loads. Assumining the loads are not controllable, the target areas can focus on strength and modeling variabilities. For the example mast a few target areas will show the impact of increased component reliability on the overall system reliability. Figure 3 shows the changes on the central factor of safety for the composite mast. The baseline case assumes an annual P_f of 1%, with n=2.41.

Righting Moment Uncertainty Reduced: If the mast design were targeted to a specific person, the coefficient of variation for righting moment might be reduced to 2%. This would decrease n to 2.3.

Material Property Uncertainty Changes: If the material properties COV change, the required central factor of safety will also change. The COV might be changed by using higher technology lamination or curing processes, or by specifying tighter quality control. Compressive strength data provided by Hexcel for 250°F cure unidirectional carbon/epoxy for example had COV's of 2.4-3.4%. If this material was used for mast construction the central factor of safety for the composite mast would be 2.18, slightly lower than for the aluminum mast. If "low" technology composites were used which had a COV of 20%, n would be 2.87. Figure 3 shows the impact of varying the carbon/epoxy material property COV's from 1 to 35%. The graph shows that low COV's have little impact on the central factor of safety, but COV's greater then 20% drive the required factor of safety.

Changes in Analytical Models: The finite element model was assumed to have a 3% COV. If less accurate models are used greater factors of safety are required. Figure 3 also shows the impact of using less accurate models. To maintain a probability of failure of 1%, the model needs to have a COV of less than 20% to maintain a reasonable factor of safety.

Changes in rigging uncertainties: The shroud and stay systems are a major factor in the overall system uncertainty. As the rigging systems are essentially a series system, the largest uncertainty drives the probability of failure. Figure 3 also shows the impact of changing the largest uncertainty in the rigging system. The changes might be made by increasing quality control in swaging, using rod rather than wire rigging, or by using oversized turnbuckles or other hardware. If changes are made that might change the importance of other loads, or fatigue resistance, the reliability analysis might need revision.

The changes illustrate the importance of keeping all controllable uncertainties to values less then about 20%, and the first areas to address are those with large COV's. The trade-off in a reliability analysis approach is the cost of characterizing the distributions through testing or other methods versus the potential benefits of reduced material and manufacturing costs, or increased performance. If the assumed COV of an individual component is high or unknown, the cost is likely justified.

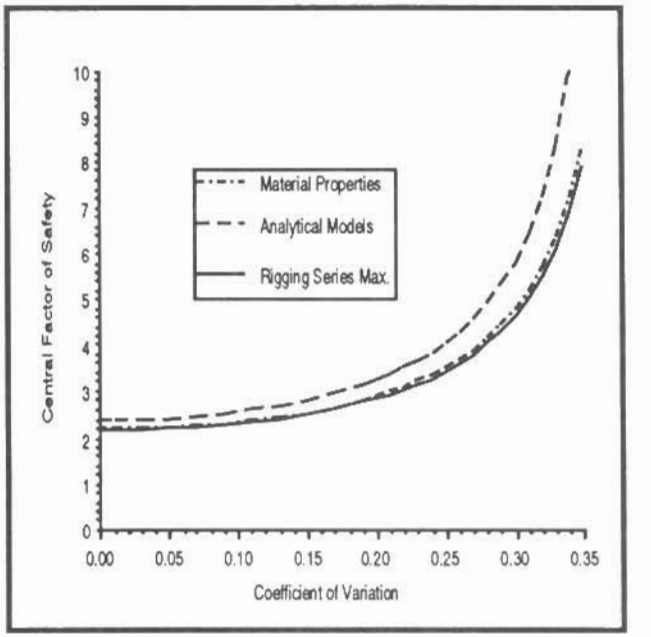

FIGURE 3: Effect of COV Changes on Central Factor of Safety

Although the reliability analysis does not relieve engineers of the need to thoroughly analyze each structure, it does provide a tool to make intelligent decisions regarding selection of appropriate factors of safety to meet the owners and operators goals. In the case of replacing aluminum masts with carbon/epoxy masts, reliability analysis indicates that to maintain the same level of risk, the designer should select a material with the same COV in compression as aluminum. Alternatively, they could either slightly increase the factor of safety or reduce other elements of uncertainty.

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