ROTATIONAL MOTION DIAGRAMS

To describe linear motion, the following kinematic quantities were used: position x, displacement Δx, velocity $v = \Delta x/\Delta t$, and acceleration $a = \Delta v/\Delta t$. To describe rotational motion, a similar set of quantities can be used. Now, instead of starting with the position of an object along an axis, we use the angle θ in the counter clockwise direction from the positive x axis to a line drawn from the origin of coordinates to the position of a point on the rotating object (Fig. 6.4). Three kinematic quantities evolve from this angular position coordinate: angular displacement $\Delta\theta$, angular velocity $\omega = \Delta\theta/\Delta t$, and angular acceleration $\alpha = \Delta\omega/\Delta t$.

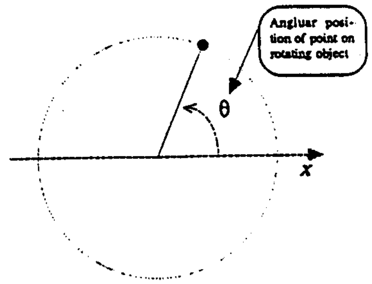


Fig. 6.4 The angular position θ of a point on a rotating object is the angle between the positive x axis and a line from the axis of rotation to the point.

Read in your textbook the material that introduces the rotational motion kinematic quantities θ, ω, and α—see pages 227-230.

On the next pages, motion diagrams for several different types of rotational motion are provided. In each diagram, we show a view of a point on the outer edge of the rotating object as seen along the axis of rotation. This "axis" view includes dots that indicate the changing positions of the point at times separated by equal time intervals. Also shown are the sign and magnitude of the object's angular velocity ω (positive for counterclockwise and negative for clockwise rotation) and angular acceleration α.

In some cases a series of "perspective" views appear below the axis view. The perspective views indicate more visually the vectors that represent the angular velocity and the angular acceleration. Notice that the angular velocity vector points along the axis of rotation and perpendicular to the plane in which the object turns. If the fingers of your right hand curled around in the direction of rotation, your right thumb would point in the direction of the angular velocity [above the plane for counterclockwise rotation and below for clockwise rotation (see Fig. 6.5)]. The angular acceleration is a vector that equals the *change* in angular velocity during a time interval divided by the time interval.

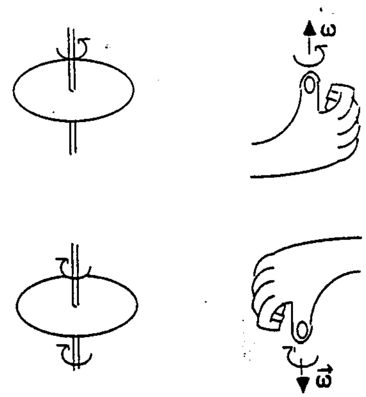


Fig. 6.5 The direction of the angular velocity vector $\vec{\omega}$ points perpendicular to the plane of rotation and in the direction of your right thumb if your right fingers curl in the direction of motion of the rotating object.

A Stationary, Extended Body

The motion diagram at the right is for a stationary, extended body. The dot represents the unchanging position of one point on the body. Because the body does not turn, its angular velocity is zero. Because the angular velocity does not change, its angular acceleration is zero.

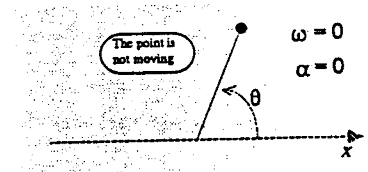


Fig. 6.6 Motion diagram for an extended body that is not rotating.

Rotation at Constant Angular Velocity

The motion diagram below represents an extended body that rotates in the xy plane in the counterclockwise direction at constant angular velocity (it has a positive angular velocity in the z direction perpendicular to and out of the paper). The dots represent the changing position of one point on the rotating body at times separated by equal time intervals. Notice that equal distances separate adjacent dots. Also, the *change* in the angular position $\Delta\theta$ between adjacent dots is the same [constant angular velocity ($\omega = \Delta\theta/\Delta t$)]. At the side of the axis view motion diagram are three perspective views of the position of the dot at the three different times. Notice that the angular velocity vector points up perpendicular to the plane of the motion and does not change. Because ω is constant, the angular acceleration vector is zero.

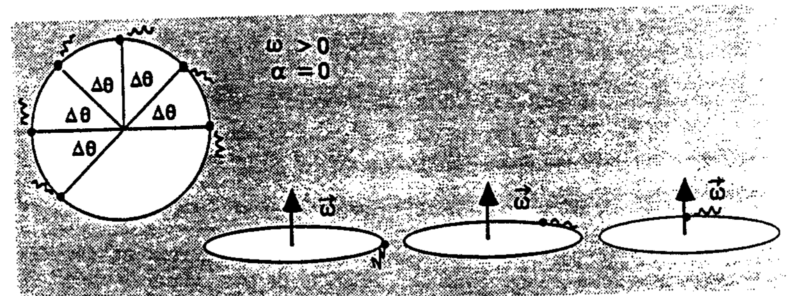


Fig. 6.7 A motion diagram for an extended body that rotates at constant angular velocity. The angular acceleration is zero because the angular velocity does not change.

Rotation at Increasing Angular Velocity

The motion diagram shown in Fig. 6.8 represents an extended body that rotates with increasing angular velocity in the counterclockwise direction. It might represent the motion of a Merry-Go-Round as its rotational velocity increases at the start of a ride, or the increasing rotational velocity of a centrifuge at the start of a spin. This rotational motion is the analog of the situation involving linear motion in which a car's velocity increases—its acceleration points in the direction of the increasing velocity.

The dots below and to the left represent the changing position of one point on the rotating body at times separated by equal time intervals. Notice that the distances between adjacent dots increase as it moves around the circle. Also, note that the change in the angular position $\Delta\theta$ between adjacent dots increases—the extended body rotates at increasing angular velocity. At the side of the axis view motion diagram are three perspective views of the position of the dot at three different times. The angular velocity vector ω points up perpendicular to the plane of the motion and increases in length as the object rotates faster. Because ω increases in the upward direction, the angular acceleration vector α points up. It is as though we are adding to the angular velocity vector as time progresses.

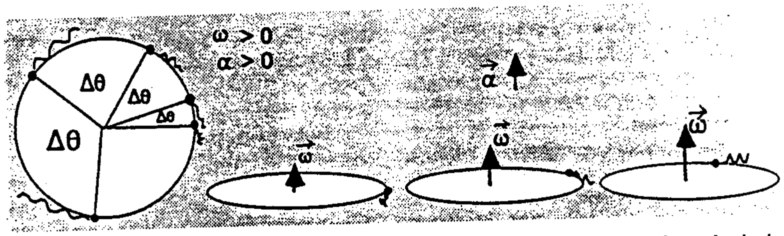


Fig. 6.8 The motion diagram for an extended body whose angular velocity is increasing. The angular acceleration is in the direction of the increasing angular velocity.

Rotation at Decreasing Angular Velocity

The motion diagram shown in Fig. 6.9 represents an extended body that rotates with decreasing angular velocity in the counterclockwise direction. It might represent the motion of a Merry-Go-Round as its rotational velocity decreases at the end of a ride, or the decreasing rotational velocity of a centrifuge at the end of a spin. This rotational motion is the analog of the situation involving linear motion in which a car's velocity in the positive direction decreases—its acceleration points opposite the velocity.

The dots below and to the left represent the changing position of one point on the rotating body at times separated by equal time intervals. Notice that the distances between adjacent dots decrease as it moves around the circle. Also, note that the change in the angular position $\Delta\theta$ between adjacent dots decreases—the extended body rotates at decreasing angular velocity. Beside the axis view motion diagram are three perspective views of the position of the dot at three different times shown in the motion diagram. The angular velocity vector points up perpendicular to the plane of the motion and decreases in length as the object rotates slower. Because the angular velocity ω decreases in the upward direction, the angular acceleration vector α points down in the negative direction. It is as though we are subtracting from the angular velocity vector as time progresses.

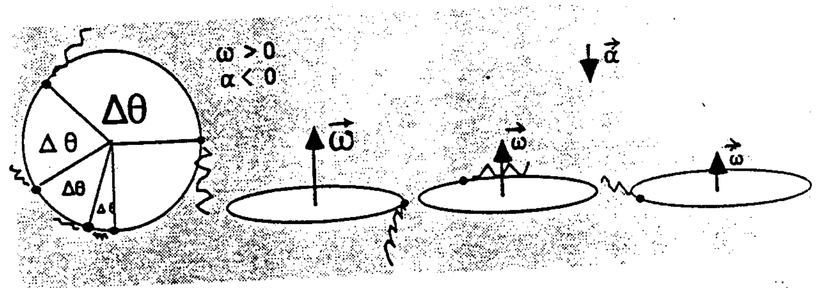


Fig. 6.9 The motion diagram for an extended body whose angular velocity ω is decreasing. The angular acceleration α points opposite the direction of the decreasing angular velocity.

Problems

6.17 (a) Construct a motion diagram for the left rear wheel of a car tire that starts at rest and spins on an icy surface at increasing angular velocity as the car tries to inch forward. (b) In which direction does the angular velocity point? (c) In which direction does the angular acceleration point (if it is not zero)?

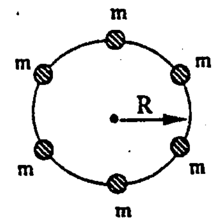
6.18 (a) Construct a motion diagram for the turntable of a hi-fi system as it starts to rotate. (b) In which direction does the angular velocity point? (c) In which direction does the angular acceleration point (if it is not zero)?

6.19 (a) Construct a motion diagram for the turntable of a hi-fi system as it slows to a stop. (b) In which direction does the angular velocity point? (c) In which direction does the angular acceleration point (if it is not zero)?

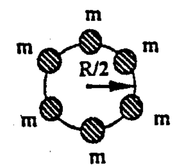
6.20 (a) Construct a motion diagram for a point on the equator of the earth. (b) In which direction does the angular velocity point? (c) In which direction does the angular acceleration point (if it is not zero)?

ROTATIONAL INERTIA

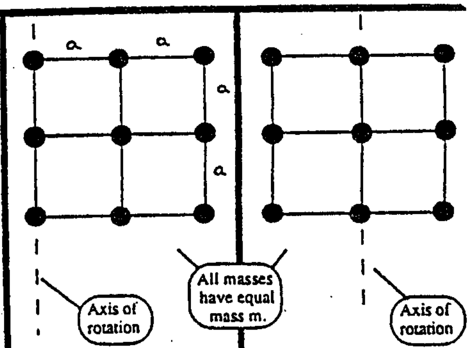
Write an expression for the rotational inertia of each group of masses shown below.



About the axis through the center and perpendicular to the page.



About the axis through the center and perpendicular to the page.



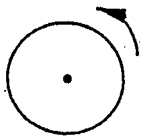
Which ring is easiest to start (or stop) rotating? Explain.

About which axis is it easiest to (start or stop) the rotation of the above masses? Explain.

ROTATIONAL KINEMATICS

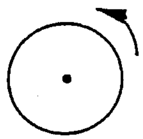
For each situation below, indicate the direction of the angular velocity $\vec{\omega}$ and of the angular acceleration $\vec{\alpha}$. [Note: in = into paper, out = out-of-papter, and 0 = zero.]

Disc turning at constant angular velocity in ccw direction.



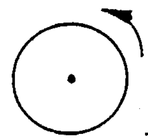
$\vec{\omega}$ ____ $\vec{\alpha}$ ____

Increasing ω in ccw direction.



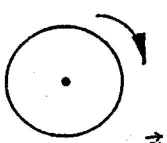
$\vec{\omega}$ ____ $\vec{\alpha}$ ____

Decreasing ω in ccw direction.



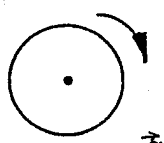
$\vec{\omega}$ ____ $\vec{\alpha}$ ____

Constant ω in the cw direction.



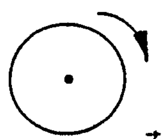
$\vec{\omega}$ ____ $\vec{\alpha}$ ____

Increasing ω in the cw direction.



$\vec{\omega}$ ____ $\vec{\alpha}$ ____

Decreasing ω in the cw direction.



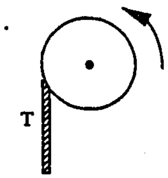
$\vec{\omega}$ ____ $\vec{\alpha}$ ____

ROTATIONAL FORM OF NEWTON'S SECOND LAW

For each situation shown below, determine the direction of the angular velocity, the direction of the angular acceleration, and the directions of the resultant torque. Place these directions (in = into the paper, out = out of the paper, or 0 = zero) in the table.

Initially rotating ccw

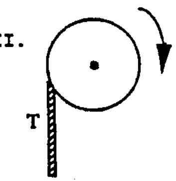
I.



$\vec{\omega}_o$ ____ $\vec{\alpha}$ ____ $\Sigma\vec{\tau}$ ____

Initially rotating cw

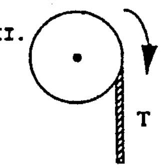
II.



$\vec{\omega}_o$ ____ $\vec{\alpha}$ ____ $\Sigma\vec{\tau}$ ____

Initially rotating cw

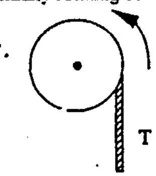
III.



$\vec{\omega}_o$ ____ $\vec{\alpha}$ ____ $\Sigma\vec{\tau}$ ____

Initially rotating ccw

IV.



$\vec{\omega}_o$ ____ $\vec{\alpha}$ ____ $\Sigma\vec{\tau}$ ____

(b.) Describe in words how the angular velocity changes.

I.

II.

III.

IV.

(c.) Complete the table.

	Direction $\vec{\omega}$	$\vec{\alpha}$	$\Sigma\vec{\tau}$
I.			
II.			
III.			
IV.			

(d.) Bases on the information in the table, is $\Sigma\vec{\tau}$ proportional to $\vec{\omega}$? Explain.

(e.) Is $\Sigma\vec{\tau}$ proportional to $\vec{\alpha}$?