HW #1
SP211 Vanhoy

Due Mon 31 August
(at the beginning of class)

Eleven Book problems

Serway Ch 1 Problems: *** none ***

Ch 2 Problems: 1, 5, 13, 15, 23, 27, 33, 39, 51

Nine multiple choice – only the answer needs to be given for the multiple choice

1. During a short interval of time the velocity v in m/s of an automobile is given by $v = at^2 + bt^3$, where the time t is in seconds. The units of a and b are respectively:
   A) $m \cdot s^2$; $m \cdot s^4$
   B) s^3/m; s^4/m
   C) m/s^2; m/s^3
   D) m/s^3; m/s^4
   E) m/s^4; m/s^5

2. The average speed of a moving object during a given interval of time is always:
   A) the magnitude of its average velocity over the interval
   B) the distance covered during the time interval divided by the time interval
   C) one-half its speed at the end of the interval
   D) its acceleration multiplied by the time interval
   E) one-half its acceleration multiplied by the time interval.

3. The coordinate of a particle in meters is given by $x(t) = 12 - 3.0t^2$, where the time t is in seconds. The particle is momentarily at rest at $t =$
   A) 2.0 s
   B) 3.0 s
   C) 4.0 s
   D) 5.0 s
   E) 0.0 sec

$v = \frac{dx}{dt} = -6t$

$v = 0$ when $t = 0$

4. A drag racing car starts from rest at $t = 0$ and moves along a straight line with velocity given by $v = bt^2$, where b is a constant. The expression for the distance traveled by this car from its position at $t = 0$ is:
   A) bt^3
   B) $\boxed{bt^3/3}$
   C) $4bt^2$
   D) $3bt^2$
   E) $bt^{3/2}$

$x = \int v\,dt = \int bt^2\,dt = \frac{1}{3}bt^3 + \text{const}$

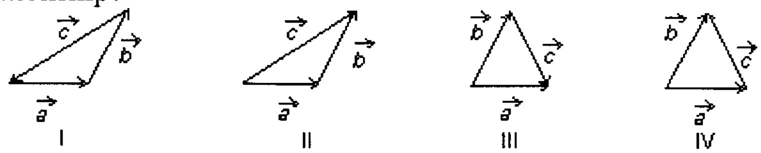
5. Each of four particles move along an x axis. Their coordinates (in meters) as functions of time (in seconds) are given by
   particle 1: $x(t) = 3.5 - 2.7t^3$
   particle 2: $x(t) = 3.5 + 2.7t^3$
   particle 3: $x(t) = 3.5 + 2.7t^2$
   particle 4: $x(t) = 3.5 - 3.4t - 2.7t^2$
   Which of these particles have constant acceleration?
   A) All four
   B) Only 1 and 2
   C) Only 2 and 3
   D) $\boxed{\text{Only 3 and 4}}$
   E) None of them

Constant acceleration formulae look like
$x = x_0 + v_0 t + \frac{1}{2}at^2$

6. Each of four particles move along an x axis. Their coordinates (in meters) as functions of time (in seconds) are given by
   particle 1: $x(t) = 3.5 - 2.7t^3$ neg accel
   particle 2: $x(t) = 3.5 + 2.7t^3$ $\rightarrow v = 8.1t^2$
   particle 3: $x(t) = 3.5 + 2.7t^2$ $\rightarrow v = 8.1t^2$
   particle 4: $x(t) = 3.5 - 3.4t - 2.7t^2$ neg accel
   Which of these particles is speeding up for $t > 0$?
   A) All four
   B) Only 1
   C) $\boxed{\text{Only 2 and 3}}$
   D) Only 2, 3, and 4
   E) None of them

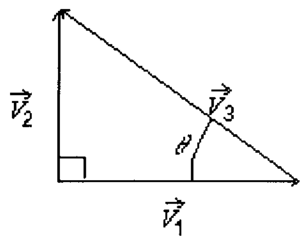
$v = \frac{dx}{dt}$

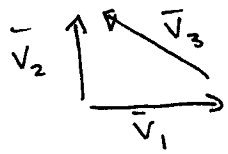
7. The vectors **a**, **b**, and **c** are related by **c** = **a** + **b**. Which diagram below illustrates this relationship?

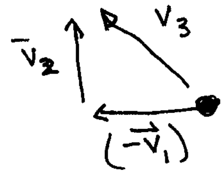


A) I.
B) II.
C) III.
D) IV.
E) None of these

8. The vector $\mathbf{v}_3$ in the diagram is equal to:

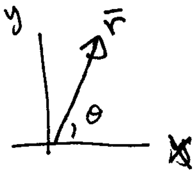






A) $\mathbf{v}_1 - \mathbf{v}_2$
B) $\mathbf{v}_1 + \mathbf{v}_2$
C) $\mathbf{v}_2 - \mathbf{v}_1$
D) $v_1 \cos\theta$
E) $v_1/(\cos\theta)$

9. The angle between **r** = (25 m)**i** + (45 m)**j** and the positive x axis is:
A) 29°
B) 61°
C) 151°
D) 209°
E) 241°



$\tan\theta = \frac{y}{x} = \frac{45}{25}$

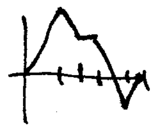
$\theta = 61°$

Ch 2

1. Average velocity during various time intervals

$$\vec{v}_{AVE} = \frac{\bar{r}_2 - \bar{r}_1}{\Delta t} = \frac{\Delta \bar{r}}{\Delta t}$$

$$\bar{v}_x = \frac{\Delta x}{\Delta t}$$



a) $\bar{v}_x = \frac{10-0}{2} = +5 \text{ m/s}$

b) $\bar{v}_x = \frac{5-0}{4} = +5/4 \text{ m/s}$

c) $\bar{v}_x = \frac{5-10}{2} = -\frac{5}{2} \text{ m/s}$

d) $\bar{v}_x = \frac{-5-5}{3} = \frac{-10}{3} \text{ m/s}$

e) $\bar{v}_x = \frac{0-0}{8} = 0 \text{ m/s}$

5.

a) $\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{2-8}{4-1.5} = \frac{-6}{2.5} = -2.4 \text{ m/s}$

b) Estimate instantaneous by slope of tangent line

v_x at $t=2$ $\approx \frac{\Delta x}{\Delta t} = \frac{0-13}{3.5-0} = -3.71 \text{ m/s}$

c) $v_x = 0$ when $\frac{dx}{dt} = 0$

which occurs at $t=4$

13. $x = 2+3t-t^2$

Find quantities at $t=3$

a) Position

$$x = 2+3t-t^2$$
$$= 2+3(3)-3^2 = 2+9-9 = 2 \text{ m}$$

b) Velocity

$$v_x = \frac{dx}{dt} = 3-2t$$
$$= 3-2(3) = -3 \text{ m/s}$$

c) Acceleration

$$a_x = \frac{dv_x}{dt} = -2 \text{ m/s}^2$$

15. Position given by

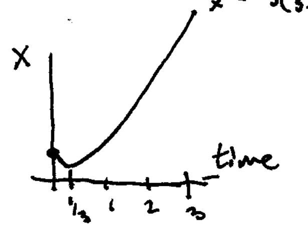
$$x = 3t^2 - 2t + 3$$

make a plot of this just to be sure I know what I'm doing.

min occurs at $0 = \frac{dx}{dt} = 6t - 2$

$t = 1/3$

Value at min $x = 3(\frac{1}{3})^2 - 2(\frac{1}{3}) + 3 = 2.67$



Now onto answering some questions

a) Average Speed = $\frac{\text{total dist traveled}}{\text{time}}$

$x(t=3) = 3 \cdot 3^2 - 2 \cdot 3 + 3 = 24$

$x(t=2) = 3 \cdot 2^2 - 2 \cdot 2 + 3 = 11$

Ave Speed = $\frac{24-11}{1\,\text{sec}} = 13$ m/s

b) Instantaneous Velocity (speed)

$x = 3t^2 - 2t + 3$

$V = \frac{dx}{dt} = 6t - 2$

at $t=2$ $\quad V = 6 \cdot 2 - 2 = 10$ m/s

at $t=3$ $\quad V = 6 \cdot 3 - 2 = 16$ m/s

c) Ave Acceleration = $\frac{\text{change in velocity}}{\text{time}}$

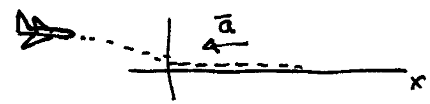
Ave Accel = $\frac{16-10}{1} = 6$ m/s^2

d) Instantaneous Acceleration

$V = 6t - 2$

$a = \frac{dv}{dt} = 6$ m/s^2

23.



$x(t) = x_0 + V_{0x}t + \frac{1}{2}a_x t^2 \qquad V(t) = V_0 + a_x t$

$x_0 = 0$ at touchdown

$V_{0x} = 100$ m/s

$a_x = -5$ m/s^2 because decelleration & the way I have chosen +x axis.

$x = 100t + \frac{1}{2}(-5)t^2 \qquad V = 100 - 5t$

a) Time to stop.

Airplane stopped when $v = 0$

$V = 0 = 100 - 5t$

$t = 20$ s

b) Distance to stop

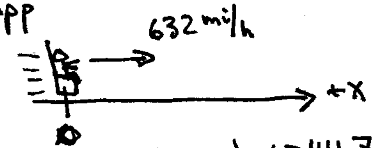
position of plane at 20 s

$x = 100(20) + \frac{1}{2}(-5)(20)^2 = 1000$ m

The plane will therefore run off a 800 m runway

27. Col John Stapp

Came to rest in 1.4 s



100 mph ⇔ 44.7 m/s

$x(t) = x_0 + V_{0x}t + \frac{1}{2}a_x t^2 \qquad V(t) = V_{0x} + a_x t$

$x_0 = 0$

$V_{0x} = 632$ mph $= 282.5$ m/s

$a_x = ?$

a) ~~Examine~~ time to stop

$V = 0 = 282.5 - a_x(1.4)$

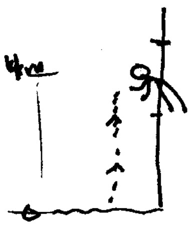
$a_x = -201.8$ m/s^2 ≈ -20 g's

b) Distance travelled

$x = 282.5t + \frac{1}{2}(-201.8)t^2 = 282.5(1.4) + \frac{1}{2}(-201.8)(1.4)^2$

$= 197.7$ m

39. ~~the~~



keys caught at 1.5 s

$y(t) = y_0 + v_{oy}t + \frac{1}{2}a_y t^2 \qquad v_y(t) = v_{oy} + a_y t$

$y_0 = 0$

$v_{oy} = ?$

$a_y = -9.8$

a) Keys caught at $y = 4$ & 1.5 s

$y = v_{oy}t + \frac{1}{2}(-9.8)t^2$

$4 = v_0(1.5) + \frac{1}{2}(-9.8)(1.5)^2$

$= v_0\, 1.5 - 11.0$

$v_0 = 10.0$ m/s

b) Velocity of key when caught

$V(t) = v_{oy} + a_y t$

$= 10 + (-9.8)(1.5)$

$= -4.7$ m/s

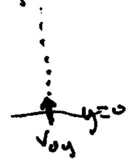
the keys were caught as they were coming back down.

51. Rocket

1) catapulted to ground level
2) engines fire until hgt = 1000 m
   init speed 80 m/s
   accel 4 m/s²
3) engines stop
   projectile, unpowered.

There are three completely separate problems here, but we only need consider part 2) and 3).

Part 2)



$y(t) = y_0 + v_{oy}t + \frac{1}{2}a_y t^2 \qquad v_y(t) = v_{oy} + a_y t$

$y_0 = 0$

$v_{oy} = 80$ m/s

$a_y = +4$ m/s²

$y(t) = 80t + \frac{1}{2}(4)t^2 \qquad v_y(t) = 80 + 4t$

$= 80t + 2t^2$

This continues until hgt = 1000 m

$1000 = 80t + 2t^2$

$0 = 2t^2 + 80t - 1000$

$t = \frac{-80 \pm \sqrt{80^2 + 4(2)1000}}{4} = \frac{-80 \pm 120}{4} = -40, 10$ sec

The height and velocity at 10 sec will be:

$y = 80t + 2t^2$

$= 80(10) + 2(10)^2$

$= 1000$ m

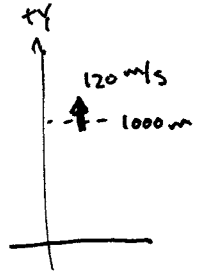
$v_y = 80 + 4t$

$= 80 + 4(10)$

$= 120$ m/s

51 cont

Part 3)



$y(t) = y_0 + v_{oy}t + \frac{1}{2}a_y t^2$ $\quad$ $v_y(t) = v_{oy} + a_y t$

$y_0 = 1000$ m
$v_{oy} = 120$ m/s
$a_y = -9.8$ m/s²

$y(t) = 1000 + 120t + \frac{1}{2}(-9.8)t^2$ $\quad$ $v_y(t) = 120 - 9.8t$

At the maximum height $v_y = 0$

$$0 = 120 - 9.8t$$
$$t = 12.24 \text{ s}$$

$$y(t) = 1000 + 120t + \tfrac{1}{2}(-9.8)t^2$$
$$= 1000 + 120(12.24) + \tfrac{1}{2}(-9.8)(12.24)^2$$
$$= 1735 \text{ m}$$

The rocket strikes the earth $(y=0)$

$$y = 1000 + 120t + \tfrac{1}{2}(-9.8)t^2$$
$$0 = 1000 + 120t - 4.9t^2$$
$$0 = -4.9t^2 + 120t + 1000$$
$$t = \frac{-120 \pm \sqrt{120^2 + 4(4.9)(1000)}}{-9.8}$$
$$= \frac{-120 \pm 184.4}{-9.8} = +31.1,\ -6.57 \text{ sec}$$

The velocity when the rocket strikes is:

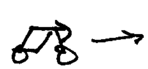
$$v_y(t) = 120 - 9.8t$$
$$= 120 - 9.8(31.1)$$
$$= -185 \text{ m/s}$$

The total flight time is

$$10 \text{ sec} + 31.1 \text{ sec} = 41.1 \text{ sec}$$

33.

 max speed $= 50$ mph accel $= 9$ mi/h/s reaches max speed at $\frac{50}{9} = 5.55$ sec

 max speed $= 20$ mph accel $= 13$ mi/h/s reaches max speed at $\frac{20}{13} = 1.54$ sec

$\longrightarrow +x$

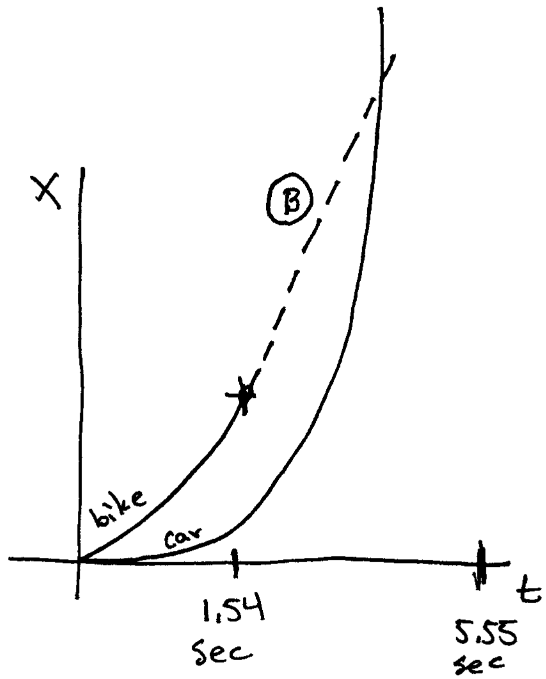
Each vehicle maintains constant speed after reachin, it's max.

TALK ABOUT SOME SCREWY UNITS!

50 mph $\Rightarrow$ 22.35 m/s 20 mph $\Rightarrow$ 8.94 m/s

9 mi/h/s $\Rightarrow$ 4.03 m/s² 13 mi/h/s $\Rightarrow$ 5.82 m/s²

I NEED TO DO SOME PRELIMINARY CALCs AND GRAPHS TO GET A FEELING FOR WHEN THEY MIGHT MEET



Where are the bike & car at $t = 1.54$ sec?

$$X_{CAR} = \cancel{X_0}^{0} + \cancel{V_0 t}^{0} + \frac{1}{2}At^2 \qquad X_{bike} = \cancel{X_0}^{0} + \cancel{V_0 t}^{0} + \frac{1}{2}at^2$$

$$X_{CAR} = \frac{1}{2}(4.03)t^2 \qquad X_{bike} = \frac{1}{2}(5.82)t^2$$

$$= \frac{1}{2}(4.03)(1.54)^2 \qquad = \frac{1}{2}(5.82)(1.54)^2$$

$$= 4.78\,m \qquad = 6.90\,m$$

Where are the car & bike at $t = 5.55$ sec

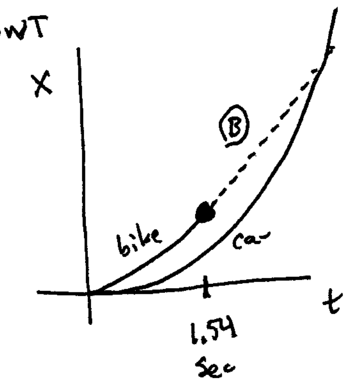
$$X_{CAR} = \frac{1}{2}(4.03)(5.55)^2 = 62.1\ m$$

$X_{bike} =$ above formula doesn't apply ~~past~~ 1.54 s

conclusion is that car catches bike after 1.54 sec but before 5.55 sec

We need to develop an expression for the line B in the figure above

33 CONT



line B is a line, so let's write it

$$x_B = "x_0" + vt$$
$$= "x_0" + 8.94t$$

where "x_0" is a pseudo-intercept. we can find its value because the line has to go through the dot ●

$$6.90 = "x_0" + 8.94(1.54)$$
$$"x_0" = -6.87 \text{ m}$$

so

$$x_B = -6.87 + 8.94t$$

a) NOW, the bike & car will meet when

$$\tfrac{1}{2}(4.03)t^2 = -6.87 + 8.94t$$
$$2.02t^2 - 8.94t + 6.87 = 0$$
$$t = \frac{8.94 \pm \sqrt{(8.94)^2 - 4(2.02)(6.87)}}{4.04}$$
$$= 0.99, \quad 3.44 \text{ sec}$$

reject because too early (<1.54s)

b) Greatest Separation

If the greatest separation occurs at <1.54 sec then it would occur at 1.54 sec

$$6.90 - 4.78 = 2.12 \text{ m}$$

The greatest separation could also occur in the middle region. The separation between bike & car could be written:

$$s = \underbrace{\left[-6.87 + 8.94t\right]}_{\text{bike}} - \underbrace{\left[\tfrac{1}{2}(4.03)t^2\right]}_{\text{car}}$$

The max occurs at

$$\frac{ds}{dt} = 0 = 8.94 - 4.04t$$
$$t = 2.21 \text{ sec}$$

which is

$$s = \left[-6.87 + 8.94(2.21)\right] - \left[2.02(2.21)^2\right]$$
$$= 3.02 \text{ m}$$