A Prototype Hardware for Random Demodulation Based Compressive Analog-to-Digital Conversion

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***Abstract*— In this paper, we utilize recent advances in compressive sensing theory to enable signal acquisition beyond Nyquist sampling constraints. We successfully recover signals sampled at sub-Nyquist sampling rates by exploiting additional structure other than bandlimitedness. We present a working prototype of compressive analog-to-digital converter (CADC) based on a random demodulation architecture. The architecture is particularly suitable for wideband signals that are sparse in the time-frequency plane. CADC has the advantage of enhancing the performance of communication and multimedia systems by increasing the transmission rate for the same bandwidth. We report successful reconstruction of AM modulated signals at sampling rates down to $1/8$ of the Nyquist-rate, which represents an up to 87.5% savings in the bandwidth and the storage memory.**

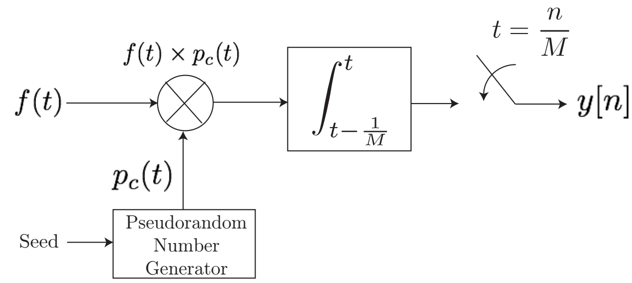

Fig. 1. Pseudo-random demodulation scheme for CADC.

I. Introduction

Advances in computation power have enabled digital signal processing to become the primary modality in many applications, such as, communications, multimedia, and radar detection systems. Converting signals to the digital domain for processing avoids the complicated design considerations for analog processing, such as, feed through, linearity, noise figure, distortion harmonics, and device inherent non-ideal performance. However, the physical limitations of traditional analog-to-digital converters (ADCs) is the main obstacle towards pushing their performance to the GHz-regime. The problem originates from the fact that traditional ADCs, such as, flash ADCs [1], pipelined ADCs [2], and sigma-delta ADCs [3], are based on the Nyquist sampling theorem, which guarantees the reconstruction of a band-limited signal when it is uniformly sampled with a rate of at least twice its bandwidth. However, many signals of interest have additional structure, which can be called *sparsity* or *compressibility*. Consequently, sampling these sparse signals at Nyquist-rate disregards this additional information. Thus, uniform sampling is not a very efficient technique in extracting the information out of sparse signals.

Over the past several years, a new theory of *compressive sensing* (CS) has emerged to enable signal acquisition beyond Nyquist constraints. The main idea of compressive sensing is to recover signals using fewer measurements than the number prescribed by the Nyquist theorem for certain classes of signals. In particular, CS allows reconstruction of signals that are *compressible* by some transform (such as Fourier, wavelets, etc.). Leveraging the CS theory, a compressive analog-to-digital converter (CADC) can be designed to acquire samples at a lower rate while successfully recover the compressible signal of interest. Consequently, the CS theory gives us an opportunity to sample at sub-Nyquist rates, thus alleviating many of the problems in traditional ADCs. Moreover, sending the same information using fewer samples, saves the bandwidth or effectively increases the information transmission rate. Additionally, sending fewer samples saves storage memory space, power consumption, area, and computational power.

In this paper, we introduce a brief background of the CS theory and extend the mathematical framework to random demodulation architecture in Section II. In Section III, we present a working prototype of compressive analog-to-digital converter (CADC) that is based on the random demodulation architecture [4]. We discuss the different implementation concerns in that system and the importance of synchronization. In Section IV, our measurement results show the success of the signal information reconstruction from sampling rates down to $1/8$ of the Nyquist-rate, which represents significant savings in memory and transmission bandwidth.

II. Compressive Sensing for CADC Systems

A. Compressive sensing background

The CS framework [5], [6], demonstrates that a signal that is compressible in one basis $\mathbf{\Psi}$ can be recovered to a quality similar to that of a K-term approximation from $M = O(K \log \frac{N}{K})$ *nonadaptive* linear projections onto a second basis $\mathbf{\Phi}$ that is *incoherent* with the first. By incoherent, we mean that the rows ϕ_j of the matrix $\mathbf{\Phi}$ cannot sparsely represent the elements of the sparsity-inducing basis ψ_i, and vice versa. Thus, rather than measuring the N-point signal $\mathbf{x}$ directly, we acquire the $M \ll N$ linear projections which are then quantized such that $\mathbf{y} = Q(\mathbf{\Phi x} + \mathbf{n})$. The effect of quantization may be modeled as additive noise and thus we

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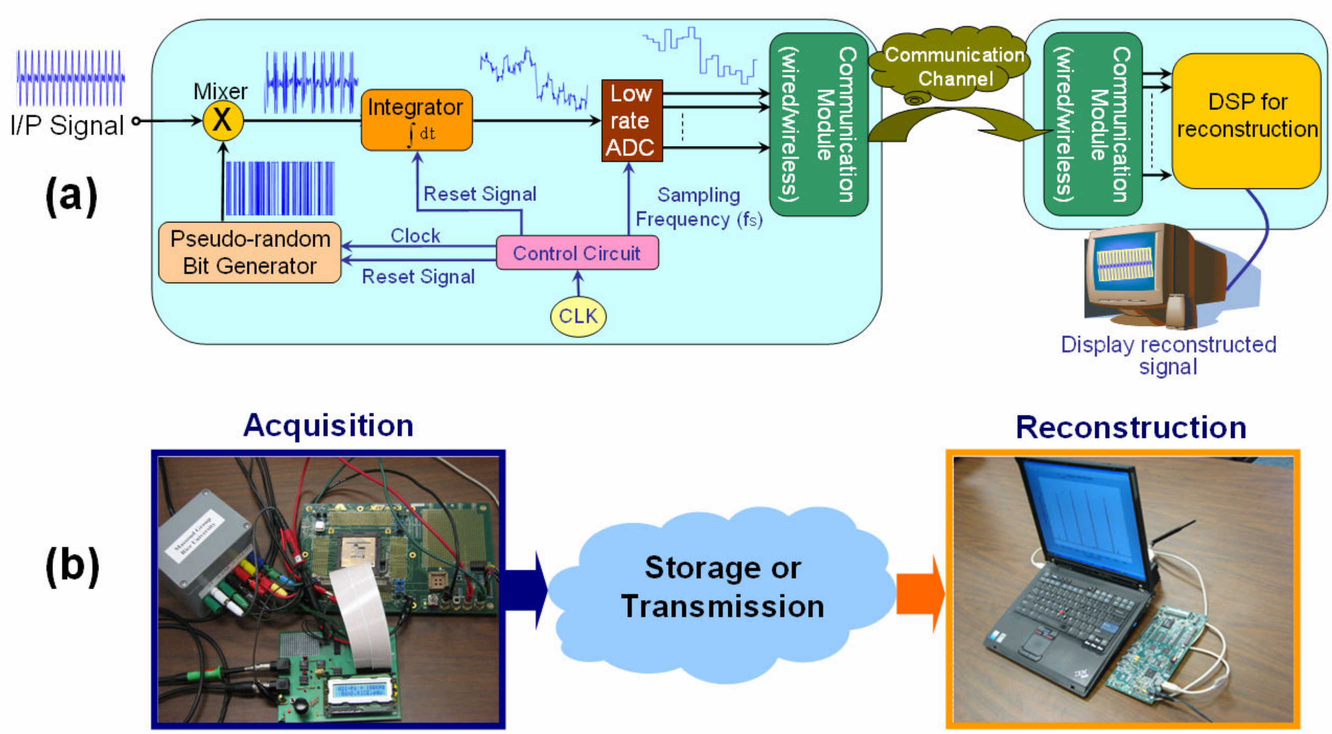

Fig. 2. (a) The hardware block diagram for our CADC prototype and (b) The actual implementation of our CADC prototype.

view the measurements as $\mathbf{y} = \mathbf{\Phi x} + \mathbf{n} = \mathbf{\Phi \Psi} \boldsymbol{\alpha} + \mathbf{n}$, where $\mathbf{n}$ represents the combination of the quantization effect and the noise inherent to the measurement process. For brevity, we define the $M \times N$ matrix $\mathbf{\Theta} = \mathbf{\Phi \Psi}$.

Since $M < N$, recovery of the signal $\mathbf{x}$ from the measurements $\mathbf{y}$ is ill-posed in general; however, the additional assumption of signal *compressibility* in the basis $\mathbf{\Psi}$ makes recovery both feasible and practical. The recovery of the set of transform coefficients $\boldsymbol{\alpha}$ can be achieved through *optimization* [7] by searching for the $\boldsymbol{\alpha}$ with the smallest ℓ_1 norm that agrees with the M observed measurements in $\mathbf{y}$. Other variations of the recovery methods such as basis pursuit denoising (BPDN) [8], allow for measurements with added noise.

B. The random demodulation CADC

As first introduced in [4], the random demodulation CADC consists of three main components; demodulation, filtering, and uniform sampling. As seen in Figure 1, the signal is modulated by a square pulse, with pseudo-random values of $\{\pm 1\}$, generated by a PN sequence. We call this the *chipping sequence* $p_c(t)$, and it must alternate between values at or faster than the Nyquist frequency of the input signal. The purpose of the demodulation is to spread the frequency content of the signal so that it is not destroyed by the second stage of the system, a low-pass filter implemented as an integrator. Finally, the signal is sampled at rate M using a traditional ADC. In its ideal form, this system can be modeled as a discrete vector $\mathbf{x}$ operated on by a banded matrix Φ containing N/M pseudo-random ± 1s per row. For example, with $N = 9$ and $M = 3$, over a period of 1 second such a Φ might look like

$$\Phi = \begin{bmatrix} -1 & 1 & -1 & & & & & & \\ & & & -1 & -1 & 1 & & & \\ & & & & & & 1 & 1 & -1 \end{bmatrix}.$$

Although our system involves the sampling of continuous-time signals, the discrete measurement vector y can be characterized as a linear transformation of the discrete coefficient vector $\boldsymbol{\alpha}$. We assume that an analog signal $f(t)$ is composed of a *discrete, finite* number of weighted continuous basis or dictionary components ψ_n. In cases where there are a small number of nonzero entries in $\boldsymbol{\alpha}$, we may say that the signal f is sparse. Although each of the dictionary elements may have high bandwidth, the signal itself has few degrees of freedom. In this paper, our signals are the superposition of K sinusoids of varying frequency. Thus, the sparsity inducing basis is the Fourier basis and our signals are sparse in the frequency domain.

To improve the performance of the CADC hardware system, we calibrate $\mathbf{\Theta}$. To calibrate, each continuous vector in $\mathbf{\Psi}$ (e.g., sinusoids at different frequencies) is generated with a signal generator and measured with the random demodulator. $\mathbf{\Theta}$ is composed of each of the measurement vectors, and once generated, it is stored to a computer and reused for each reconstruction.

III. CADC Prototype Implementation

In order to demonstrate the effectiveness of the random demodulation based CADC design and evaluate its perfor-

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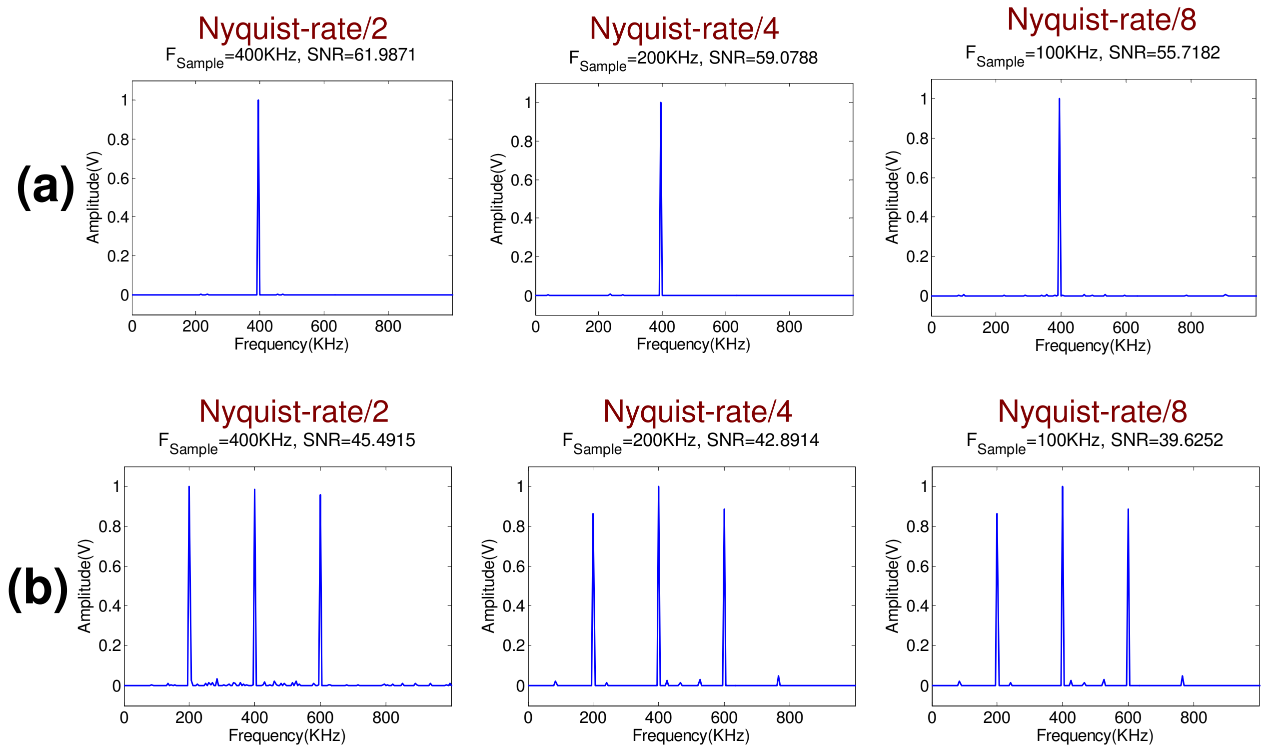

Fig. 3. Measurements from our implemented CADC prototype, (a) Single-tone signal reconstructed from different sub-Nyquist rates, (b) Three-tone (AM modulated) signal reconstructed from different sub-Nyquist rates.

mance, we developed an end-to-end prototype hardware for a compressive signal based acquisition system. The hardware block diagram in Figure 2(a) illustrates an overview of the main building blocks and components. The input signal is mixed with a pseudo-random bipolar wave sequence using an analog multiplier in order to randomly modulate the input signal in the time domain. This is followed by an integrator, which is implemented as Gm-C based differential integrator. Each measurement is digitized using a low-rate ADC and is sent to the reconstruction end. The reconstruction end consists of a communication module that receives the transmitted compressed signal in addition to a digital signal processor (DSP). We used a commercial DSP board that has a 160MHz Fixed-Point Digital Signal Processor to run the reconstruction algorithms on the compressive measurements and retrieve the original signal.

We divide the signal into time-frames, where we reset the integrator after each frame. We use a master clock-driven control circuit to synchronize the process and reset the integrator. Reseting the integrator prevents the cross-coupling of the information in adjacent frames. In addition, keeping the integrator working for long time may cause saturation if the signal amplitude does not follow a normal distribution around the zero DC value.

The implementation of our CADC prototype is shown in Figure 2(b). One advantage of our presented system design is that it relies on simple analog processing components, thus enabling for hardware implementation that is compact in size as well as power efficient. In addition, our system utilizes an analog multiplier in the high frequency regime, which is easy to design, in order to reduce the performance design constraints of the low-rate ADC. This makes our design suitable for sensor networks and portable devices.

The reconstruction algorithm, which is based on orthogonal matching pursuit, is implemented on a programmable DSP board. The reconstruction end needs the most computational power and complexity of the CADC acquisition system. Therefore, our system is suitable for applications that need simplicity on the acquisition side. We use Bluetooth wireless communication modules to send the random measurements from the acquisition end to the reconstruction end. Using actual communication standard in our system enables us to evaluate the performance of our system under the effects of environmental noise and device non-idealities.

IV. Measurements and Results

We conducted a series of experiments on the implemented prototype hardware in order to evaluate the performance of the reconstruction from our CADC prototype hardware. We performed the reconstruction experiments for different AM modulated signals with different frequencies, different sparsity, and different sampling frequencies. We varied the sampling

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frequency from Nyquist-rate/2 to Nyquist-rate/8 to evaluate the effect of deep sub-Nyquist sampling rates on the reconstruction performance measured by signal-to-noise ratio. Figure 3(a) shows the success of the reconstruction of single-tone signal with SNR about 62 dB, 59 dB, and 56 dB for sampling frequencies $1/2$, $1/4$, and $1/8$ from Nyquist-rate, respectively. Figure 3(b) shows the success of the reconstruction of three-tones signal with SNR about $46\ dB$, $43\ dB$, and $40\ dB$ for sampling frequencies $1/2$, $1/4$, and $1/8$ from Nyquist-rate, respectively. These sub-Nyquist sampling rates can be translated into savings in memory and transmission bandwidth by 50%, 75%, and 87.5%, respectively.

The previous results show that SNR values are sensitive to the sparsity of the input signal and to the sampling frequency used in signal acquisition. Therefore, we investigated this relation and tested the reconstruction success for different AM modulated signals under different sampling frequencies. Figure 4 shows the degradation of SNR values (reconstruction success) when the signal becomes more complex (less sparse) or when we decrease the sampling frequency for signal acquisition far from the Nyquist-rate. These results are consistent with CS theory as the performance of the algorithm will degrade for less sparse signals. However, the curves become closer to maintain a constant SNR for more complex signals. When the sampling frequency goes much lower than Nyquist-rate, the reconstruction algorithm does not have enough information to be able to reconstruct the actual signal, and thus degrading the signal-to-noise ratio.

In addition, the SNR values of the reconstructed signals are sensitive to the non-ideal behaviors in the analog devices utilized in our CADC implementation. The most significant sources of non-idealities are the clock jitter of the random number generator, the linearity and intermodulation distortion of the mixer, and the quantization error of the back-end ADC. The sensitivity of the reconstruction is partially due to the fact that these behaviors introduce noise, but also because that we can not produce a matrix that is exactly tuned to the non-idealities for reconstruction process. The difficulty of matrix calibration is due to the stochastic nature of some of these non-linearities.

In order to enhance our reconstructed signals, we used calibration techniques to generate a more accurate reconstruction matrix. Our first calibration technique depends on applying time-shifted impulses to the system and recording the output as a column in the reconstruction matrix, which represents the impulse response of the system. The advantage of this technique is that it can detect the phase differences (sensitive to the phase information). Our second calibration technique depends on applying different input tones to our system, a tone per frame, and build the matrix using the generated measurement outputs. The advantage of this technique is that it builds a dictionary of possible tones and enables the system to reconstruct the signal by extrapolating the information inherent in the matrix. Our reported results and measurements are generated using the second calibration technique. The previous results demonstrate a successful reconstruction of signals at

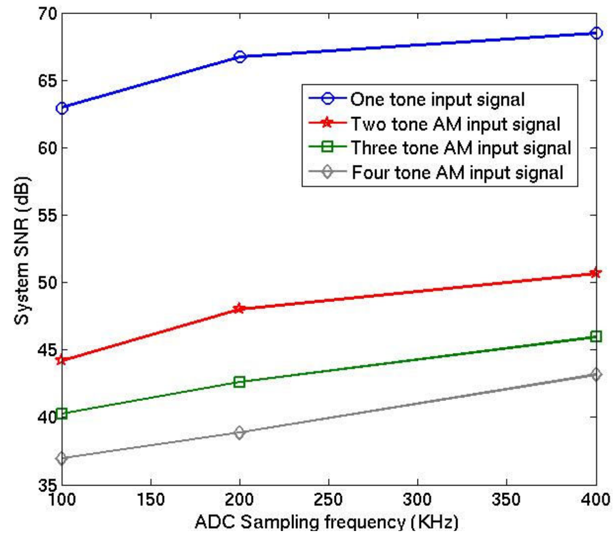

Fig. 4. The effect of signal sparsity and sampling frequency on reconstruction performance measured by SNR.

sampling rates down to $1/8$ of the Nyquist-rate, and therefore, enabling up to an order of magnitude improvement in the performance of analog-to-digital converters.

V. Conclusions

In this paper, we presented a working prototype of compressive analog-to-digital converter (CADC) based on a random demodulation architecture. The compressive analog-to-digital converter has the advantage of enhancing the performance of communication and multimedia systems by increasing the transmission rate for the same bandwidth. We reported successful reconstruction of AM modulated signals at sampling rates down to $1/8$ of the Nyquist-rate, thus providing an up to 87.5% savings in the bandwidth or the storage memory.

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