Appendix B

SIGNIFICANT FIGURES

A. INTRODUCTION

There are two categories of numbers: exact and inexact.

Exact numbers are precise and have no error. Counted numbers, for example, are exact numbers. If there are two pipettes or four test tubes, the numbers two and four are exact numbers. Definitions are also exact numbers. For example, 1 foot = 12 inches and 1 liter = 1000 milliliters. Each number in the foot-inches and liter-milliliter definitions is an exact number. *Inexact* numbers are imprecise and have some error. All measurements are inexact numbers. A 100 mL beaker does not have a volume of exactly 100 mL but 100 mL $\pm$ some small quantity, for example, 5 mL.

The magnitude of the uncertainty of a measurement depends both on the skill of the person making the measurement and on the sensitivity of the instrument being used. For example, an analytical balance is precise to $\pm$ 0.0001 g. The mass determined on such a balance might be reported as 2.6518 $\pm$ 0.0001 g. When a measurement is reported omitting the $\pm$, it is normally understood that there is an uncertainty of one unit in the last digit, e.g.: 2.63 g implies 2.63 $\pm$ 0.01 g.

The precision of a measurement is expressed using significant figures. In this method, all the digits are certain except the last digit, which is estimated. Thus, the measurement 3.16 g has three significant figures; the first two (3 and 1) are certain and the third (6) is estimated.

B. DETERMINING SIGNIFICANT FIGURES

The number of significant figures is independent of the magnitude of the number. The numbers 4502, 45.02 and 4.502 all have four significant figures.

Determining the number of significant figures is straightforward for nonzero digits. All nonzero digits are significant. But zeros must be considered on a case by case basis. There are three types of zeros. The type of zero depends on its position in the number. In the number 0.0034, the zeros are called *leading zeros*. The only function of leading zeros is to fix the decimal point. Leading zeros are not counted as significant figures. Therefore, the number 0.0034 has two significant figures. In the number 5800, the zeros are called *trailing zeros*. The number of significant figures in a number containing trailing zeros before the decimal point is ambiguous. In the number 5800, if the measurement was made to the nearest unit and is 5800 rather than 5801 or 5799, both zeros are significant. If the measurement was made to the nearest ten and is closer to 5800 than to 5810 or 5790, then the first zero is significant and the second zero is not significant. If the number was measured to the nearest hundred and is 5800 rather than 5700 or 5900, then neither zero is significant. The ambiguity in the precision of this measurement can be removed by expressing the number in exponential notation. For example, 5.8×10^3 has two significant figures, 5.80×10^3 has three significant figures, and 5.800×10^3 has four significant figures, signifying the precision of the measurement to the nearest hundred, ten, or unit, respectively. Trailing zeros that follow the decimal point are significant. In the number 5.80×10^3, the trailing zero follows the decimal point and therefore is significant. The third type of zero is a *captive zero*. A captive zero is a zero between two nonzero digits and is significant. For example, the zero in the number 73.04 is a captive zero and is significant.

In summary, zeros before the first nonzero digit are not significant. Zeros following the decimal point (3.50) and zeros between integers (405) are significant. Zeros at the end of a whole number (400) may or may not be significant.

C. CALCULATIONS WITH SIGNIFICANT FIGURES

Exact numbers are considered to have an infinite number of significant figures. When an exact number is used in a calculation, it is not the limiting factor in determining how many significant figures to use to represent the answer.

1. Addition and Subtraction

A sum or difference cannot have an absolute precision greater than that of the least precise number used in the calculation.

EXAMPLE: Sodium chloride (1.32 g) and potassium chloride (0.006 g) are added to 28 g of water. What is the mass of the resulting solution? Remember that the implied precision is ±1 for the last digit of each measurement.

Sodium Chloride	1.32 ± 0.01 g
Potassium Chloride	0.006 ± 0.001 g
Water	28 ± 1 g
Total Mass	29 ± 1 g

Notice that the answer is reported to the nearest gram and has the same precision as that of the least precise measurement (the water). It is always the absolute precision (± 1 g in the above example, ± 0.01 g in the following example) that determines the number of significant figures in addition a and subtraction. If the absolute precision is not given, the last digit of each measurement is considered to be uncertain by ±1 digit.

EXAMPLE: In weighing a sample of sodium chloride by difference, the following measurements are obtained.

Container + Sodium Chloride	32.241 g
Container (empty)	32.13 g
Mass of Sodium Chloride	0.11 g

The mass is expressed to the nearest 0.01 g because the mass of the empty container is the least precise measurement and is measured to the nearest 0.01 g.

In addition and subtraction, the number of significant figures is NOT determined by the number of significant figures in the measurements.

2. Multiplication and Division

A product or quotient contains the same the number of significant figures as the measurement with the least number of significant figures.

EXAMPLES:
a. $(6.10 \times 10^3) (2.08 \times 10^{-4}) = 1.27$, reported to three significant figures

b. $5.92 \times 3.0 = 18$, reported to two significant figures

c. $8.2 / 3.194 = 2.6$, reported to two significant figures

In multiplication and division the number of significant figures in the answer is determined by the measurement with the least number of significant figures.

3. Rounding Numbers

It is often necessary to drop one or more digits to obtain an answer with the appropriate number of significant figures. The rules are:

a. If the first digit to be dropped is less than 5, leave the preceding digit unchanged. (e.g., 3.12467 rounds to 3.12 with three significant figures because the first number to be dropped is 4.)

b. If the first digit to be dropped is greater than or equal to 5, increase the preceding digit by 1. (e.g., 4.26713 rounds to 4.27 with three significant figures because the first number to be dropped is 7.)

4. Logarithms

All logarithms have two parts. The decimal part is called the *mantissa*. The digits in the mantissa are the only significant figures in a logarithm. The integer, or *characteristic*, is the power to which the base (in the following examples, 10) is raised when writing the number in scientific notation. It is an exponent and therefore not a significant figure.

EXAMPLES:

a. $24.3 = 2.43 \times 10^1$ $\log (2.43 \times 10^1) = 1.386$

b. $0.068 = 6.8 \times 10^{-2}$ $\log (6.8 \times 10^{-2}) = -1.17$

The mantissa in the above examples has the same number of digits as the number of significant figures as the number from which it was derived. When calculating natural logarithms (base e), the same rules apply. The above examples yield:

a. $\ln (24.3) = 3.190$

b. $\ln (0.068) = -2.69$

Exponentiation or conversion of logarithms back into their corresponding numbers (called antilog) uses the same rules. The number of digits in the mantissa gives the number of significant figures in the resulting number.

EXAMPLES:

a. antilog $23.32 = 10^{23.32} = 2.1 \times 10^{23}$ c. $e^{23.32} = 1.30 \times 10^{10}$

b. antilog $(-1.873) = 10^{-1.873} = 1.34 \times 10^{-2}$ d. $e^{-1.873} = 1.54 \times 10^{-1}$

Virtually every question asked on homework assignments, quizzes, or examinations describes the results of one or more measurements. Therefore, the numbers used are inexact and your answer must be expressed using the conventions for significant figures.