Exam 2 Review

1) In a DC circuit when steady-state conditions are reached a Capacitor acts like a what?
   
   OPEN

2) In a DC circuit when steady-state conditions are reached an Inductor acts like a what?
   
   SHORT

3) The circuits below have been sitting for a long time; apply the steady-state conditions of the Capacitor and the Inductor to determine the voltages at terminals a-b. (Assume the Capacitors and Inductors are ideal).

   \[ V_{OUT} = 12 \, V \]

   \[ V_{OUT} = 0 \, V \]
4) Initially the switch is at **Position 1** for a long time and then moves to **Position 2**. Determine the initial Voltage \( (V_o) \) across the Capacitor.

\[
V_o = \frac{10 \times 10^3}{(6 \times 10^3) + (10 \times 10^3)} (40) = 25 \, \text{V}
\]

5) Redraw the circuit in Position 2 and determine the equivalent Resistance \( (R_{eq}) \) as seen by the Capacitor. Also determine the time constant \( (\tau) \) and the final Voltage \( (V_f) \) across the Capacitor.

\[ R_{eq} = (7 \times 10^3) + (2 \times 10^3) = 9 \, \text{k}\Omega \]

\[ \tau = R_{eq}C = (9 \times 10^3)(5 \times 10^{-6}) = 45 \, \text{ms} \]

\[ V_f = 0 \, \text{V} \]

6) Develop the transit response equation for the Voltage across the Capacitor and plot the response below. Make sure to include the time (in milliseconds) when steady-state is reached.

\[ v(t) = V_f + (V_o - V_f)e^{-\frac{t}{\tau}} \]

\[ 5\tau = 5(45 \times 10^{-3}) = 225 \, \text{ms} \]

\[ v(t) = 0 + (25 - 0)e^{-\frac{t}{45 \, \text{ms}}} = 25e^{-\frac{t}{45 \, \text{ms}}} \, \text{V} \]
7) Initially the switch is at **Position 2** for a long time and then moves to **Position 1**. Determine the initial Current \( i_o \) through the Inductor.

\[ i_o = 0 \text{ A} \]

8) Redraw the circuit in Position 1 and determine the equivalent Resistance \( R_{eq} \) as seen by the Inductor. Also determine the time constant \( \tau \) and the final Current \( i_f \) through the Inductor.

\[
R_{eq} = \frac{(6 \times 10^3)(10 \times 10^3)}{(6 \times 10^3)+(10 \times 10^3)} + 7 \times 10^3 = 10.75 \text{ k}\Omega
\]

\[
\tau = \frac{L}{R_{eq}} = \frac{3 \times 10^{-3}}{10.75 \times 10^3} = 279 \text{ ns}
\]

\[
i_f = \frac{V}{7 \times 10^3} = \frac{16.28}{7 \times 10^3} = 2.33 \text{ mA}
\]

9) Develop the transit response equation for the Current through the Inductor and plot the response below. Make sure to include the time (in milliseconds) when steady-state is reached.

\[
i(t) = i_f + (i_o - i_f)e^{-\frac{t}{\tau}}
\]

\[
i(t) = 2.33 \text{ mA} + (0 - 2.33 \text{ mA})e^{-\frac{t}{279 \text{ ns}}}
\]

\[
i(t) = 2.33(1 - e^{-\frac{t}{279 \text{ ns}}}) \text{ mA}
\]

\[
5\tau = 5(279 \times 10^{-9}) = 1.395 \mu\text{s}
\]
10) How long will it take for the Current through the Inductor to reach 1.94 mA and what will be the energy stored in the Inductor at this point in time?

\[
1.94 \text{ mA} = 2.33(1 - e^{-\frac{t}{279 \text{ ns}}}) \text{ mA}
\]

\[
\ln(0.167) = -\frac{t}{279 \text{ ns}}
\]

\[
\frac{1.94 \text{ mA}}{2.33 \text{ mA} - 1} = e^{-\frac{t}{279 \text{ ns}}}
\]

\[-0.167 = e^{-\frac{t}{279 \text{ ns}}}
\]

\[
l(0.167) = \ln\left(e^{-\frac{t}{279 \text{ ns}}}\right)
\]

\[t = -\ln(0.167) \times (279 \times 10^{-9}) = 0.5 \mu s
\]

\[W = \frac{1}{2}L_i^2 = \frac{1}{2}(3 \times 10^{-3})(1.94 \times 10^{-3})^2 = 5.65 \text{ nJ}
\]

11) Express the Voltage Sources as a phasor.

\[21.21 \sin(160t - 63^\circ) \text{ V}\]

**Make sure to convert the Amplitude to the RMS value.**

\[
\frac{21.21}{\sqrt{2}} = 15 \text{ V} \quad \therefore \quad 15\angle -63^\circ \text{ V}
\]

12) Find the Impedance values for the given elements.

a. Let \( f = 60 \text{ Hz} \)

\[
\omega = 2\pi f = (2\pi)(60) = 377 \text{ rad/s}
\]

\[
Z_L = j\omega L = j(377)(132.63 \times 10^{-3}) = j50 \Omega
\]

b. Let \( \omega = 235 \text{ rad/s} \)

\[
Z_C = -j\frac{1}{\omega C} = -j\frac{1}{(235)(193.42 \times 10^{-6})}
\]

\[
Z_C = -j22 \Omega
\]

c. Let \( \omega = 1500 \text{ rad/s} \)

\[
Z_R = R = 80.5 \Omega
\]
13) Redraw the circuit below. Express the source in phasor form and the elements as Impedances.

\[ V_s = \frac{19.8}{\sqrt{2}} \angle 430^\circ = 14.430^\circ \text{ V} \]
\[ Z_{10} = 10 \text{ } \Omega \]
\[ Z_{C1} = -j \frac{1}{(377)(221 \times 10^{-6})} = -j12 \text{ } \Omega \]

\[ Z_{21} = 21 \text{ } \Omega \]
\[ Z_{C2} = -j \frac{1}{(377)(530.5 \times 10^{-6})} = -j5 \text{ } \Omega \]
\[ Z_L = j(377)(53.05 \times 10^{-3}) = j20 \text{ } \Omega \]

14) Find the total Impedance as seen by the Source.

\[ Z_T = 10 + \left[ \frac{1}{-j12} + \frac{1}{21} + \frac{1}{(-j5 + j20)} \right]^{-1} = 10 + [j0.083 + 0.048 - j0.067]^{-1} \]
\[ Z_T = 10 + \frac{1}{0.048 + j0.016} = 10 + \frac{1}{0.051418.43^\circ} = 10 + 19.614 - 18.43^\circ \]
\[ Z_T = 10 + 18.6 - j6.2 = 28.6 - j6.2 = 29.264 - 12.23^\circ \text{ } \Omega \]

15) Find \( I_s \), \( I_1 \), \( I_2 \), and \( I_3 \). Use Current Divider Rule when finding \( I_1 \), \( I_2 \), and \( I_3 \).

\[ I_s = \frac{V_s}{Z_T} = \frac{14.430^\circ}{29.264 - 12.23^\circ} = 0.4784 \angle 42.23^\circ \text{ } A \]
\[ Z_{eq} = \left[ \frac{1}{-j12} + \frac{1}{21} + \frac{1}{(-j5 + j20)} \right]^{-1} \]
\[ Z_{eq} = 19.614 - 18.43^\circ \text{ } \Omega \]

\[ I_1 = \frac{Z_{eq}(I_s)}{Z_{C1}} = \frac{19.614 - 18.43^\circ}{124 - 90^\circ} (0.4784 \angle 42.23^\circ) = 0.7814 \angle 113.8^\circ \text{ } A \]
\[ I_2 = \frac{Z_{eq}(I_s)}{Z_{21}} = \frac{19.614 - 18.43^\circ}{2140^\circ} (0.4784 \angle 42.23^\circ) = 0.4464 \angle 23.8^\circ \text{ } A \]
\[ I_3 = \frac{Z_{eq}(I_s)}{Z_{C2} + L} = \frac{19.614 - 18.43^\circ}{15490^\circ} (0.4784 \angle 42.23^\circ) = 0.6254 \angle 66.2^\circ \text{ } A \]
16) Find the voltage across the Inductor ($V_L$).

$$V_L = (I_3)(Z_L) = (0.6254 \angle -66.2^\circ)(20490^\circ) = 12.5423.8^\circ V$$

17) When solving Thevenin Equivalent problems there are logical steps that hold true no matter if the circuit is DC or AC. Answer the following with True or False.

a. True If a Load is shown on the circuit, the first step is to remove the Load.
b. False The Thevenin Impedance is always the impedance as seen by the Source.
c. False When solving the Thevenin Impedance, we must “turn off” the sources by replacing both current and voltage sources as SHORTS.
d. False When solving the Thevenin Impedance, we must “turn off” the sources by replacing the current source a SHORT and the voltage sources as an OPEN.
e. True When solving the Thevenin Impedance, we must “turn off” the sources by replacing the current source an OPEN and the voltage sources as a SHORT.
f. True When finding the Thevenin Voltage we use the open terminals were the Thevenin perspective is.

18) Find the Thevenin Impedance and Voltage external to $Z_{Load}$ for the circuit below. Draw the Thevenin circuit with the Load attached.

First step is to remove the load and find $Z_{TH}$.
Remember the current source is replaced with an OPEN.

$$Z_{TH} = \frac{(1640^\circ)(84-90^\circ)}{16 - j8} = \frac{(1640^\circ)(84-90^\circ)}{17.894-26.57^\circ}$$

$$Z_{TH} = 7.154-63.43^\circ = 3.2 - j6.4 \ \Omega$$
Now “turn on” the source and find $V_{TH}$ at the open terminals.

$V_{TH}$ is the voltage across the 16 Ω and the $-j8$ Ω. We can use Nodal Analysis or combine the 16 Ω and $-j8$ Ω impedances in parallel and find the voltage across them.

$$Z_{eq} = (16)||(-j8) = 7.154-63.43^\circ \ \Omega$$

$$V_{TH} = (I_s)(Z_{eq}) = (8.43^\circ)(7.154-63.43^\circ) = 57.24-33.43^\circ \ \text{V}$$

19) Is Maximum Power being delivered to the Load and why?

$$Z_{TH} = Z_L^*$$

$$3.2 - j6.4 = 3.2 + j6.4$$

$$7.154-63.43^\circ = 7.154-63.43^\circ$$

YES!

20) The acronym used for Capacitors in AC circuits is “ICE”. Explain what is meant with this acronym.

Current LEADS Voltage for a capacitive circuit. If we are just analyzing a Capacitor’s current and voltage relationship we will see the current LEADS the voltage by $90^\circ$.

This implies the Current Angle ($\theta_i$) equals to $\theta_V + 90^\circ$. 
21) The acronym used for Inductors in AC circuits is “ELI”. Explain what is meant with this acronym.

Voltage **LEADS** Current for an inductive circuit. If we are just analyzing an Inductor’s current and voltage relationship we will see the voltage LEADS the current by $90^\circ$.

This implies the Voltage Angle ($\theta_V$) equals to $\theta_i + 90^\circ$.

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**The Graph and circuit below are for questions 22 through 25**

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**Time in milliseconds**

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![](image)
22) For the Graph on the other page answer the following questions.

a. What is the period \((T)\) of the waveforms?

\[ T = 5 \text{ ms} \]

b. What is the frequency of the waveforms \((f)\)?

\[ f = \frac{1}{T} = \frac{1}{5 \times 10^{-3}} = 200 \text{ Hz} \]

c. What is the time difference \((\Delta t)\) between \(V_s\) and \(V_R\)?

The \(\Delta t\) between the peak of \(V_s\) and the peak of \(V_R\) is about 0.5 ms.

\[ \Delta t = 0.5 \text{ ms} \]

d. What is the phase difference \((\Delta \theta)\) between \(V_s\) and \(V_R\)?

\[ \Delta \theta = \frac{\Delta t}{T} (360^\circ) = \frac{0.5 \times 10^{-3}}{5 \times 10^{-3}} (360^\circ) = 36^\circ \]

e. Write the function for \(V_s(t)\).

\[ V_s(t) = 6 \sin[2\pi(200)t] \text{ V} \]

f. Express \(V_s(t)\) in phasor form.

\[ V_s = \frac{6}{\sqrt{2}}40^\circ = 4.2440^\circ \text{ V} \]

g. Write the function for \(V_R(t)\). (Use \(V_s\) as the reference angle)

\[ V_R(t) = 3 \sin[2\pi(200)t - 36^\circ] \text{ V} \]

h. Express \(V_R(t)\) in phasor form.

\[ V_R = \frac{3}{\sqrt{2}}4 - 36^\circ = 2.124 - 36^\circ \text{ V} \]
23) Since we know $V_s$ and $V_R$, find the voltage across the Load ($V_L$). Refer to the circuit at the bottom of the other page. (Hint: use KVL)

$$-V_s + V_L + V_R = 0$$

$$V_L = V_s - V_R = 4.24\angle0^\circ - 2.12\angle-36^\circ = 4.24 - (1.72 - j1.25) = 2.52 + j1.25 \text{ V}$$

$$V_L = 2.81\angle26.38^\circ \text{ V}$$

24) Find $Z_{Load}$; Is the Load Capacitive or Inductive?

$$I_s = I_R = \frac{V_R}{R} = \frac{2.12\angle-36^\circ}{2} = 1.06\angle-36^\circ \text{ A}$$

$$Z_{Load} = \frac{V_L}{I_s} = \frac{2.81\angle26.38^\circ}{1.06\angle-36^\circ} = 2.65\angle62.38^\circ = 1.22 + j2.35 \Omega$$

Since the imaginary component of $Z_{Load}$ is positive then the Load is **Inductive**.

25) Draw the Load equivalent circuit below.
26) The following circuit is operating at a frequency of \( \omega = 500 \text{ rad/s} \).

\[ |\vec{S}| = \sqrt{160^2 + 400^2} = 430.81 \text{ VA} \]

\[ \theta = \tan^{-1}\left(\frac{400}{160}\right) = 68.2^\circ \]

b. Find the source current (\( I_s \)).

\[ \vec{S} = \left(\bar{V}_s\right) \left(\bar{I}_s\right)^* \]

\[ (\bar{I}_s)^* = \frac{430.81}{220} 468.2^\circ = 1.96428^\circ \text{ A} \]

\[ I_s = 1.964 - 28.2^\circ \text{ A} \]

c. Typically we like to reduce the source current. This can be accomplished by connecting a Capacitor in parallel to an Inductive Load. The Capacitor component value (\( C \)) is determined by choosing a Capacitance Reactive Power (\( Q_c \)) equal to the Inductance Reactive Power (\( Q_L \)).

Find the Capacitor component value so all of the Reactive Power at the Load is cancelled. In other words, what component value will correct the power factor to unity?

\[ |Q_L| = |Q_c| = 400 \text{ VAR} \]

\[ Q_c = \frac{V^2}{X_c} \Rightarrow X_c = \frac{(220)^2}{400} = 121 \Omega \]

\[ X_c = \frac{1}{\omega C} \Rightarrow C = \frac{1}{(500)(121)} = 16.53 \mu F \]
d. Find the source current ($I_s$) when a Capacitor is connected in parallel with the Load. Assume the Capacitance Reactive Power is $-400 \text{ VAR}$.

\[
\begin{align*}
I_1 &= 1.96 \angle -28.2^\circ \text{ A} \\
(I_2)^* &= \frac{\vec{S}}{\vec{V_s}} = \frac{400 \angle -90^\circ}{220 \angle 40^\circ} = 1.82 \angle -130^\circ \text{ A} \\
I_2 &= 1.82 \angle 130^\circ \text{ A} \\
I_s &= I_1 + I_2 \\
I_s &= (1.96 \angle -28.2^\circ) + (1.82 \angle 130^\circ) \\
I_s &= 0.728 \angle 40.01^\circ \text{ A}
\end{align*}
\]