Learning Objectives
a. Describe the relationship between battery capacity, current drain and battery’s useful life
b. Calculate the total cost given a rate of energy consumption
c. Calculate power supplied/dissipated in a circuit
d. Calculate the power efficiency of a circuit

Batteries
Batteries are the most common direct current (dc) source. The approximate capacity of a battery is the product of the drain current and the length of time it can provide that current (i.e. 110 Ah). Battery capacity is specified in ampere hours (Ah).

If the drain current is known, the life of battery can be calculated:

\[
\text{life} = \frac{\text{capacity}}{\text{current drain}}
\]

Although most of our examples will consider battery capacity to be a fixed quantity, battery capacity is actually a function of discharge rate, operating schedules, temperature and other factors.

Example: You have a battery rated for 200 Ah. How long can you expect the battery to last if you draw a continuous current of 5A?

Solution:

Example: A car battery has a Cold Cranking Amps (CCA) rating of 750 amps, which means it can supply 750 amps for 30 seconds. What is the battery’s capacity?

Solution:

Example: A car battery has a Reserve Capacity of 135 minutes (it can sustain 25 amps discharge for 135 minutes). What is the battery’s capacity?

Solution:
Power

Power is defined as the rate of doing work, or as the rate of transfer of energy.

\[ P = \frac{W}{t} \quad [\text{watts, W}] \]

Power is measured in units of Watts.

Please do not be confused by the two “W’s” in the formula above; the “\(W\)” in the numerator is “work” (i.e., energy), measured in Joules. The second “W” (in the square brackets) represents the SI unit of power, which is the watt (W). One watt equals one joules per second.

The English unit of power is horsepower (hp), which is related to the watt by the conversion factor:

\[ 1 \text{ hp} = 746 \text{ watts} \]

Power in electrical systems

We need to express power in terms of voltage and current. Recall that:

- **voltage** \( V = \frac{W}{Q} \quad [\text{joules/coulomb, J/C}] \)
- **current** \( I = \frac{Q}{t} \quad [\text{coulomb/sec, C/sec}] \)

Combining them we have

\[ P = \frac{W}{t} = \frac{W}{Q} \cdot \frac{Q}{t} = VI \quad [\text{watts, W}] \]

Applying Ohm’s law \((V = IR \text{ and } I = V/R)\) we can also express power as

\[ P = VI = (IR)I = I^2R \quad [\text{watts, W}] \]

or

\[ P = VI = V \left( \frac{V}{R} \right) = \frac{V^2}{R} \quad [\text{watts, W}] \]

Example: A resistor draws 3 amps from a 12V battery. How much power does the battery deliver to the resistor?

Solution:
Example: You have a 5 kΩ resistor. You must ensure that the power rating of 20 mW is never exceeded. What is the maximum current that should be permitted to flow through the resistor?

Solution:

Example: A 10Ω and 100Ω resistor. The current through each resistor is \( I = 1 \) A.

a. Determine the power dissipated by each resistor using the formula:

\[ P = I^2 R \]

b. Determine the power dissipated by each resistor using the formula:

\[ P = \frac{V^2}{R} \]

c. If the total power dissipated by the resistors must be the total power supplied by the battery, what is the value of the voltage source \( E \)?

Solution:

Energy We can rearrange our formula for power to solve for energy:

\[
\text{work (or energy)} = \text{power} \times \text{time}
\]

From the above formula, energy can be measured in watt-seconds. (In fact, 1 Joule = 1 watt-second). The quantity “watt-seconds” usually leads to small values, so energy is more frequently measured in watt-hours (Wh) or, more commonly, kilowatt-hours (kWh).
Example: You realize that you fell asleep in your dorm room studying EE301 at your desk. Your 150 W light-bulb was on for 4 hours. How much energy did you waste in kWh?

Solution:

Midshipmen don’t have to worry about paying electric bills since they live in a lap of luxury where all their needs are provided for free, but the rest of us mere mortals are charged for electricity by the kilowatt-hour.

\[
\text{Cost} = \text{Energy} \times \text{cost per unit} \\
\text{Cost} = \text{Power} \times \text{time} \times \text{cost per unit}
\]

Example: An indicator lamp on a control panel operates continuously, drawing 20 mA from a 120V supply. At $0.09 per kWh, how much does it cost per year to operate this lamp?

Solution:

Example: How long would you have to leave your 150 W light bulb on unnecessarily if you had the goal of wasting 3 kWh of energy?

Solution:
**Efficiency** In the process of converting energy, losses inevitably occur. For example, we might have a motor that requires electrical power as an input and provides mechanical power as an output. Some losses inevitably occur in the motor (e.g., as thermal energy or heat).

The measure of output energy (or power) to input energy (or power) is called **efficiency**. Efficiency is usually expressed in percent and denoted by the symbol $\eta$.

\[
\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% \\
\eta = \frac{W_{\text{out}}}{W_{\text{in}}} \times 100\%
\]

Since $P_{\text{in}} = P_{\text{out}} + P_{\text{losses}}$, efficiency can also be expressed as

\[
\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{losses}}} \times 100\%
\]

**Cascaded equipment**  The overall efficiency of a cascaded system is equal to the product of individual efficiencies of all subsystems:

\[
\eta_{\text{Total}} = \eta_1 \times \eta_2 \times \eta_3 \times \ldots
\]
Example: Your friend wants to check his EE301 homework answers against yours. You look at his answer to the first question, which asked for an efficiency calculation. Your friend calculated a value of $\eta = 115\%$. What do you say to your friend?

Solution:

Example: 120 V, 1.5 HP motor has an efficiency of 60%. What is the input current?

Solution:

Example: A system consists of three components connected in cascade. The first has an efficiency of 99%, the second has an efficiency of 98% and the third has an efficiency of 20%. What is the overall efficiency of the system?

Solution:
Example: A 120 V dc motor connects to a pump via a gearbox as shown in the picture.

The power output to the pump is 1100 W.

The gearbox efficiency is 75%.

Power input to the motor is 1600W.

a. What is the HP output of the motor?

b. What is the efficiency of the motor?

c. What is the overall efficiency of the cascaded system?

Solution: