Objectives

a. Restate the definition of a node and demonstrate how to measure voltage and current in parallel circuits
b. Solve for total circuit resistance of a parallel circuit
c. State and apply KCL in the analysis of simple parallel circuits
d. Demonstrate how to calculate the total parallel resistance given various resistors connected in parallel
e. Evaluate why homes, businesses and ships are commonly wired in parallel rather than series.
f. Demonstrate how to calculate the total current and branch currents in a parallel circuit using the current divider equation
g. Determine the net effect of parallel combining voltage sources
h. Compute the power dissipated by each element in a parallel circuit, and calculate the total circuit power

Parallel Circuits

Recall that two elements are in **series** if they exclusively share a single node (and thus carry the very same current).

Components that are in **parallel**, on the other hand, share the same two nodes. Remember: nodes are connection points between components.

Components that are in parallel have the same voltage across them.

Homes and ships are usually wired in parallel instead of in series. The reason: The parallel circuit will continue to operate even though one component may fail open. All components can operate at rated voltage independent of other loads when wired in parallel.

Series - Parallel Circuits

Circuits may contain a combination of series and parallel components.
To analyze a particular circuit it is often beneficial to:

- First identify the nodes
- Next, label the nodes with a letter or number
- Then, identify types of connections

**Kirchhoff’s Current Law (KCL)** Kirchhoff’s Current Law states that the algebraic sum of the currents entering and leaving a node is equal to zero.

\[ \sum I = 0 \]

By convention, currents entering the node are positive, and those leaving a node are negative. For the picture at the right:

\[ \sum_{n=1}^{N} I_n = I_1 + (-I_2) + (-I_3) + (-I_4) + I_5 = 0 \]

KCL can also be expressed as “The sum of the currents entering a node is equal to the sum of the currents leaving a node”.

\[ \sum I_{\text{in}} = \sum I_{\text{out}} \]

\[ I_1 + I_5 = I_2 + I_3 + I_4 \]

KCL can be understood by considering a fluid flow analogy. When water flows in a pipe, the amount of water entering a point is equal to the amount leaving that point.

Also, note that more water will flow down the tube that has a lower resistance to flow.

**Direction of Current** When we solve a problem using KCL, we have to consider the direction of current flow (e.g., sum of the currents entering equals the sum of the currents leaving). But, how do we determine the direction of current flow if we are trying to determine the current?

To determine a current flow:

- *Assume* a current direction and draw a current arrow.
- If this assumption is incorrect, calculations will show that the current has a negative sign.
- A negative sign simply indicates that the current flows in the opposite direction to the arrow you drew.
Example: Determine the unknown currents in the circuit shown below.

Solution:

**Resistors in Parallel** Consider a circuit with 3 resistors in parallel (such as the circuit below, if \( N = 3 \)).

\[
I_T = I_1 + I_2 + I_3 \quad \Rightarrow \quad \frac{E}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}
\]

Since the voltages across all the parallel elements in a circuit are the same (\( E = V_1 = V_2 = V_3 \)), we have:

\[
\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} \quad \Rightarrow \quad \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
\]

This result can be generalized to provide the total resistance of any number of resistors in parallel:

\[
R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}
\]

**Special Case: Two Resistors in Parallel** For only two resistors connected in parallel, the equivalent resistance may be found by the product of the two values divided by the sum:

\[
R_T = \frac{R_1 R_2}{R_1 + R_2}
\]

If you want to be cool, you should refer to this as the “product over the sum” formula. Your EE friends will really admire this.

**Special Case: Equal Resistors in Parallel** Total resistance of \( n \) equal resistors in parallel is equal to the resistor value divided by the number of resistors (\( n \)):

\[
R_T = \frac{R}{n}
\]
Important check of your calculations: The total resistance of resistors in parallel will always be less that the resistance of the smallest resistor.

Example: Simplify the circuit shown below.

Solution:

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Example: Simplify the circuit shown below.

Solution:

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Example: You need to simplify the circuit shown below. What is your solution?

Solution:
Example: Determine the magnitude and direction of each current in the circuit below:

Solution:

\[ I_1 = \frac{5 \text{ V}}{2.5 \text{ k}\Omega} = 2 \text{ mA} \]
\[ I_2 = 4 \text{ mA} \]
\[ I_3 = \frac{5 \text{ V}}{5 \text{ k}\Omega} = 1 \text{ mA} \]

Example: For each of the four circuits below, determine which elements are connected in parallel and which are connected in series.

Solution:

(a) Connected in series
(b) Connected in series
(c) Connected in parallel
(d) Connected in series
Example. Determine the unknown currents $I_2$ and $I_3$.

Solution:

![Parallel Circuit Diagram]

Example: Determine the total resistance of the circuit below.

Solution:

![Series vs. Parallel Circuit Diagram]

**Voltage Sources in Series vs. Parallel**

Voltage sources (such as the cells shown below) connected in **series** increases the available **voltage**.
Voltage sources connected in parallel increases the available current.

When two equal sources are connected in parallel, each source supplies half the required current. Voltage sources with different potentials should never be connected in parallel: large currents can occur and cause damage.

Example: A 12V and 6V battery (each with an internal resistance of 0.05V) are placed in parallel as shown below. Determine the current $I$.

Solution:
Current through resistors in parallel. The total current $I$ is shared by the resistors in inverse proportion to their resistances. Stated another way:

"More current follows the path of least resistance."

Extreme cases for current division:

Current Divider Rule. The Current Divider Rule (CDR) allows us to determine how the current flowing into a node is split between the various parallel resistors.

Note that $R_T$ in this formula is found using the formula for parallel resistances!! $R_T \neq R_1 + R_2 + R_3 + \ldots$!

Compare the formulas for the voltage divider rule and the current divider rule. Where does $R_T$ appear in each formula? How do you compute $R_T$ in each formula?

Special Case: Two resistors in parallel. For only two resistors in parallel:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \left( \frac{I_T R_T}{R_1} \right) I_T$$
Special Case: If current enters a parallel network with a number of equal resistors, the current will split equally between the resistors.

Note: In a parallel network, the smallest value resistor will have the largest current.

Analysis of Parallel Circuits To analyze parallel circuits we should use the following guidelines:

1. Voltage across all branches is the same as the source voltage
2. Determine current through each branch using Ohm’s Law
3. Find the total current using Kirchhoff’s Current Law

Example: Determine the currents $I_1$ and $I_2$ in the circuit below.

Solution:

Example:

a. Determine $I_2$ in the circuit shown.
b. Use the CDR to determine $I$.

Solution:

Example: Determine $I_2$ in the circuit below.

Solution:
Example: Determine $R_1$ in the circuit below.

Solution:

Example: Determine the voltage $V$ in the circuit below:

Solution:

Example: Given the circuit below use the current divider rule to determine all unknown currents:

Solution:
Example: Given the circuit shown below:

a. Determine all unknown currents
b. Determine the total resistance.
c. Verify KCL for node a.

Solution:
It is often easy to solve problems involving parallel currents by inspection, just looking at the currents in the parallel branches.

Example. Determine the currents $I_1$, $I_2$, and $I_3$ in the circuit below.
Solution:

Example. Determine the currents $I_1$, $I_3$, and $I_5$ in the circuit below.
Solution:
Power Calculations

1. To calculate the power dissipated by each resistor, use either $VI$, $I^2R$, or $\frac{V^2}{R}$.
2. Total power consumed is the sum of the individual powers.
3. Compare with $I_T^2R_T$.

Example: Given the circuit below:
Determine:
a. The values of all currents.
b. The power dissipated by each resistor
c. Verify that the total power equals the sum of all power dissipated

Solution: