EE301 – AC Thévenin and Max Power

Learning Objectives
1. Apply Thévenin’s Theorem to simplify an AC circuit for analysis
2. Explain under what conditions a source transfers maximum power to a load
3. Determine the value of load impedance for which maximum power is transferred from the circuit

Thévenin’s theorem for AC  Analogous to DC:
- \( E_{Th} \) is the open circuit voltage at the terminals,
- \( Z_{Th} \) is the input or equivalent resistance at the terminals when the independent sources are turned off.

Determining \( E_{Th} \)  Remove the load (open-circuit) and measure the resulting voltage.

Determining \( Z_{Th} \)  With the load disconnected, turn off all independent sources.
- **Voltage sources** – 0 V is equivalent to a short-circuit.
- **Current sources** – 0 A is equivalent to an open-circuit.
\( Z_{Th} \) is the equivalent resistance looking into the “dead” circuit through terminals \( a-b \).
Applying Thévenin equivalent  Once $E_{th}$ and $Z_{th}$ have been found, the original circuit is replaced by its equivalent and solving for $I_{LD}$ and $V_{LD}$ becomes trivial.

$$I_{LD} = \frac{E_{th}}{Z_{th} + Z_{LD}}$$

$$V_{LD} = \frac{Z_{LD}}{Z_{th} + Z_{LD}} E_{th}$$

Much like Nodal Analysis, Thèvenin Analysis is a procedure:

Step 1. Identify the load, disconnect it (label the terminals $a$ and $b$) then find the voltage where the load was. (remember that Thèvenin is a model for a source and has nothing to do with the load!)

Step 2. Zeroize all the sources and find the complex impedance looking from the load back into the source. Voltages become short circuits (0 Volts) and current sources become open circuits (0 Amps).

Step 3. Redraw the new source with the value calculated in Step 1 as $E_{th}$ and the value calculated in Step 2 as $Z_{th}$.

There should be a minimum of three circuits drawn for each Thèvenin Analysis. The circuit in Step 1 is different than the original circuit! So is the circuit in Step 2!

Example: Convert the source below into a Thévenin equivalent and determine the power dissipated by the load. Note that the load is purely resistive.
Solution:
First identify the load and remove it from the circuit, label the nodes \( a \) and \( b \) then find the voltage between the terminals where the load was – this is the open circuit voltage \( E_{th} \). (Notice we have a NEW CIRCUIT so we need a NEW SCHEMATIC!)

You can combine the 8Ω resistor and 16Ω capacitor into a single impedance \( Z' \):

\[
Z' = \left( \frac{1}{-j16\Omega} + \frac{1}{8\Omega} \right)^{-1} = 6.4 - j3.2\Omega
\]

Now use the Voltage Divider Rule to find \( E_{th} \).

\[
E_{th} = 5\, \text{V} \cdot \frac{6.4 - j3.2\Omega}{+j24\Omega + (6.4 - j3.2\Omega)} = 1.64V \angle -99^\circ
\]

Step 1 complete.

Now we must \textit{Zeroize} all the voltage and current sources. Since we will be changing the circuit we \textbf{can} \textbf{not} analyze the old one!!! We need a new schematic again!!

\[
Z_{th} = +j24\Omega/(6.4 - j3.2\Omega) + 12\Omega = (19.8 - j1.3)\Omega
\]

Step 2 Complete.
Now draw the Thévenin Circuit using the values we’ve just calculated adding the old load back. Again this is a brand new circuit unlike the previous three so we must draw it and label all the parts!

Notice that each of the values we calculated are part of the Thévenin circuit and are labeled accordingly. This is an important part of the Analysis and frequently neglected by MIDSHIPMEN!

Now that the Thévenin circuit is complete we can easily find the power dissipated in the load resistor. Although this is a simple example and we could have found it without first finding the Thévenin equivalent circuit, imagine if we had to find the power dissipated for ten different values of the load resistor! Suddenly Thévenin becomes extremely useful.

The load current $i_L$ is the same as total current (since all components are in series) so we need to find total current:

$$i_L = i_T = \frac{E_T}{Z_T} = \frac{E_{TH}}{Z_{TH} + Z_L}$$

$$i_L = \frac{1.64V \angle -99^\circ}{(19.8 - j1.3)\Omega + 10\Omega}$$

$$i_L = 55mA \angle -97^\circ$$

$$P_L = i_L^2 \cdot R_L = (55mA)^2 \cdot 10\Omega = 30.3mW$$

Remember that we’re looking for the real power in the load (which is a purely resistive component). The answer is a real number - not a complex number. We only use the magnitude of the current in the above calculation.
Complex Conjugates  The conjugate of C, denoted \( C^\ast \), has the same real value but the opposite imaginary part:
\[
C = a + jb = C\angle \theta \\
C^\ast = a - jb = C\angle -\theta
\]

Max Power Transfer in AC Circuits  In AC circuits, the max power transfer occurs when the load impedance (\( Z_L \)) is the complex conjugate of the Thévenin equivalent impedance (\( Z_{TH} \)).

So, if \( Z_{TH} = R_{TH} + jX_{TH} \) then \( Z_{LD} \) for max power = \( R_{TH} - jX_{TH} \)

This means that if the Thévenin impedance includes an inductor (as shown on the right), then the load that achieves maximum power transfer will have a capacitor; \( X_{LD} \) cancels out \( X_{TH} \)!

\[
Z_{LD} = R_{LD} - jX_{LD} \\
Z_{TH} = R_{TH} + jX_{TH}
\]

\[
I_L = \frac{E_{TH}}{(Z_{TH} + Z_{LD})} = \frac{E_{TH}}{(R_{TH} + R_{LD}) + j(X_{TH} - X_{LD})}
\]

\[
= \frac{E_{TH}}{R_{TH} + R_{LD}} \quad \text{for max power transfer, } X_{TH} = X_{LD}
\]

\[
P_L = I_L^2 R_{LD} = \left( \frac{E_{TH}}{2R_{TH}} \right)^2 R_{LD} = \frac{E_{TH}^2}{4R_{TH}^2} R_{TH} = \frac{E_{TH}^2}{4R_{TH}} \quad \text{for max power transfer, } R_{TH} = R_{LD}
\]

In summary, if the load is set equal to the complex conjugate of \( Z_{TH} \), the maximum power will be transferred to the load and the value of this maximum power will be

\[
P_{\text{MAX}} = \frac{E_{TH}^2}{4R_{TH}}
\]

Note that since \( R_{LD} = R_{TH} \) and the reactances cancel out, the resulting equation for \( P_{\text{MAX}} \) is the same as the equation for the DC case. The biggest mistake with this equation is not using the real value of resistance when calculating \( P_{\text{MAX}} \). You must remember to transform \( Z_{TH} \) into rectangular form to determine \( R_{TH} \)

\[
Z_{TH} = Z_{TH} \angle \theta = R_{TH} \pm jX_{TH}
\]

\[
P_{\text{MAX}} = \frac{E_{TH}^2}{4R_{TH}^2} \quad \text{when } Z_{LD} = Z_{TH}^\ast
\]
Example: The circuit shown below is operated at a frequency of 191.15 Hz.

a. Determine the load $Z_L$ that will allow maximum power to be delivered to the load the circuit.
b. Find the maximum power delivered to the load.
c. What will happen to power if the frequency is changed to 50% of the original value?

Solution:
Example: Determine the load \( Z_{LD} \) that will allow maximum power to be delivered to the load the circuit. Then find the power dissipated by this load.

Solution: