

Linear DC Motors

The purpose of this supplement is to present the basic material needed to understand the operation of simple DC motors. This is intended to be used as the reference material for the linear and DC motors lectures in EE301, as this material is not covered in the text.

Before introducing electric motors, it is helpful to define electric motors. In simplest terms, *electric motors are machines which convert electrical energy into mechanical energy*. A few examples of familiar electric motors include motors that raise and lower car windows, rotate ceiling fans, and spin CDs and DVDs.

A linear motor is a basic easy-to-understand electrical motor. Although the linear design is not useful for most practical electrical motors, the concepts that are learned here are useful in understanding rotating DC motors. This section will introduce or review: some basic concepts of magnets and magnetic flux, the Lorentz force law, Faraday's law, and then the simple DC linear motor.

15.1 Magnetic Flux

Before electrical machinery can be adequately discussed, it is essential that we first introduce magnets in a manner which explains how they are employed in electrical machinery. It is helpful to recap a few basic conclusions from physics.

- Magnets may exist as permanently magnetized materials, logically called *permanent magnets*, or as a temporary magnet, which exists only when electric current is flowing, also logically called *electromagnets*.
- All magnets have two poles, north and south. These poles give rise to the familiar action between two magnets where opposite poles attract while similar poles repel.
- Lines of magnetic flux can be imagined which flow from the north pole to the south pole. These lines of flux, while they do not physically exist, help us visualize the direction and strength of the magnetic field between magnetic poles.

15.1.1 Permanent Bar Magnets

Figure 1(a) below shows how the lines of magnetic flux are arranged for a permanent bar-shaped magnet. There are two important features of this diagram. First, lines of magnetic flux are continuous. That is, the lines of flux exist as unbroken lines flowing *inside* the magnet from south pole to north pole, and, *externally* to the magnet, flow from the north pole and re-enter the south pole. The same number of lines leave the north pole as enter the south pole. Second, the lines are more tightly spaced close to the magnet than far away. A reasonable – and correct – conclusion is that, since the lines of

magnetic flux are denser close to the magnet, the magnet's magnetic field must be stronger close to the magnet. These two results will apply to all magnets.

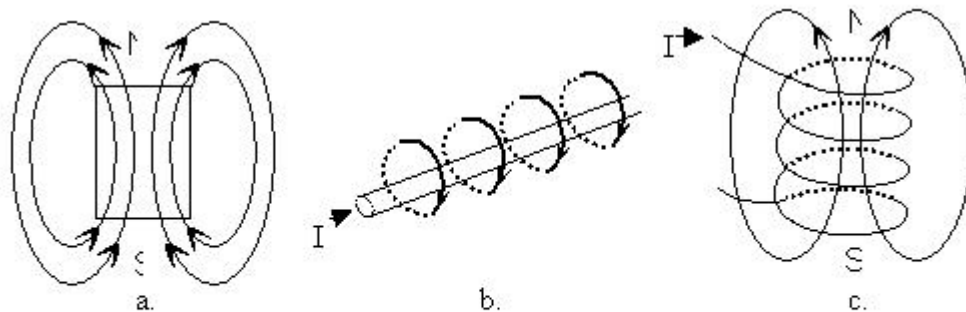


Figure 1: Lines of Magnetic Flux (Φ) for (a) A Permanent Magnet, (b) Current-Carrying Conductor, (c) Current-Carry Coil

15.1.2 Current Carrying Wire

As mentioned previously, a flowing current also produces a magnetic field, and therefore lines of magnetic flux. Figure 1(b) shows how lines of magnetic flux are arranged around a straight section of a current-carrying wire. Notice that the lines of flux all form closed loops around the wire. Since they have no beginning or end we cannot indicate where a north or south pole would exist, but the lines do have a defined direction. This direction is determined using the *right-hand rule* as follows:

- point your right thumb in the direction of the current flow in the wire
- wrap your fingers around the wire
- your fingers indicate the direction of the lines of magnetic flux

It should be apparent that reversing the direction of current flow will also reverse the direction of the lines of magnetic flux. Not as obvious is that the density of lines of flux is dependent on the current: higher current produces a higher density of lines of magnetic flux.

15.1.3 Current Carrying Coil

Next we will consider the lines of magnetic flux that result when a length of current-carrying wire is formed into a tightly wound coil. We will assume that the wire is insulated so that current must flow through the entire length of the wire, not short circuit between adjacent windings.

Figure 1(c) shows how the lines of magnetic flux are arranged around a coil. Notice first that a north and south pole are defined in this diagram even though they were not for a straight wire. Why this is so is easy to demonstrate graphically. Figure 2 shows two

closely spaced wires in cross section which can be imagined to be any two adjacent wires in the coil. Assuming the wire is wrapped around the coil in the same direction for both, the current direction will be the same in both wires (shown here as into the page). Therefore the lines of magnetic flux around each wire will be in the same direction. If the wires are closely spaced, then the lines of flux will *cancel* each other *between the wires* and *reinforce* each other *around the outside* of the wires, forming lines of flux which will now encompass the pair of wires.

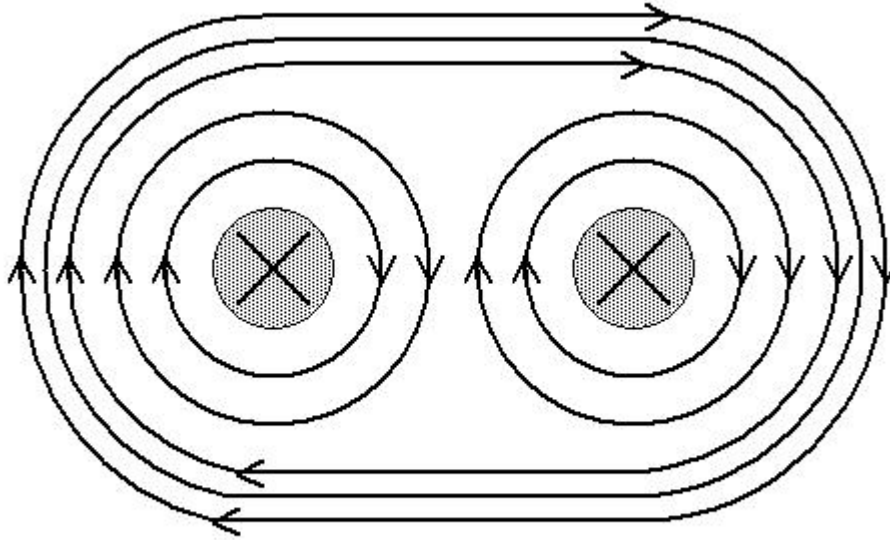


Figure2. Lines of Magnetic Flux for Two Current Carrying Wires

Now extend this simple example to a coil formed by many loops of wire carrying the same current. The net result, shown in Figure 1(c), is then similar to the bar magnet shown in Figure 1(a). The coil has a defined north pole where the lines of flux leave, and a south pole where they re-enter. It should be apparent that more loops of wire, or more *turns*, will produce more lines of magnetic flux.

It is useful to summarize the results regarding the lines of magnetic flux surrounding the electromagnet formed by a coil of current-carrying wire.

- the same number of lines leaves the north pole of the magnet as re-enter the south pole.
- these lines are denser close to the magnet, especially near the poles.
- the direction of the lines depends on the direction of the current through the coil.
- changing current direction changes the poles of the magnet.
- the number of lines depends on the strength of the current: higher current produces more lines of flux.

15.2 Magnetic Flux, Magnetic Flux Density, and Magnetic Field Intensity

It was alluded to in the previous section that lines of magnetic flux are only imagined. Nonetheless, they are useful in understanding how magnets interact. A slightly more rigorous treatment can be achieved by trying to count these lines of flux. Since these lines do not actually exist, we have to construct some model which allows us to count or measure them. In a very simple view, the amount of flux is described by the number of lines. *Magnetic flux* is given the symbol Φ . We measure magnetic flux in units of *Webers* (Wb), which have equivalent units of volt-seconds.

Now, we arrive at the concept of the magnetic field. One failing of using lines to depict magnetic flux is that this seems to indicate that the magnetic field only exists where these lines occur. But numerous other undefined lines also exist. A better method would be to understand that the actual magnetic field from any magnet is not confined to a set of lines which occur along specific paths, but rather that the magnetic field permeates all space. The direction of the magnetic field outside the magnet is taken to be from the north pole of the magnet to the south pole, and the lines of flux are drawn accordingly. The *magnitude of the magnetic field* is the flux **density**.

Thus, we are typically not concerned simply with the magnetic flux, but rather the magnetic flux density, \vec{B} , which is measured in *Teslas* (T), or Wb per square meter. The magnetic field is generally quantified by the magnetic flux density, or simply B-field. Note that magnetic flux density is a vector quantity since the magnetic flux density has a direction and a magnitude.

The final magnetic field quantity to discuss is *magnetic field intensity*, which is given the symbol \vec{H} . The units of \vec{H} are Amperes per meter, or A/m. Magnetic field intensity is related to the magnetic flux density by:

$$\vec{H} = \frac{1}{\mu} \vec{B} \quad \text{or} \quad \vec{B} = \mu \vec{H} \quad [\text{Eq. 1}]$$

where μ is the permeability of a material. *Permeability is a measure of how well a given material is able to conduct magnetic flux lines.* Most materials have a permeability roughly equal to that of free space (air). However, some materials, such as iron and iron-based compounds, have a much higher permeability, up to several thousand times higher, than free space. This explains why objects containing iron respond to magnetic fields even when not magnetized themselves.

We can summarize the key facts about magnetic fields as follows:

- A magnetic field surrounds every magnet.
- Lines of magnetic flux leave the north pole of the magnet and re-enter at the south pole of the magnet.
- The unit of flux, Φ , is the Weber (Wb)
- The magnetic field is generally quantified by the magnetic flux density, \vec{B} or B-field, and is measured in Teslas (T), or (Wb/m²).
- Permeability, μ , is a measure of how well a given material is able to conduct magnetic flux lines, with most materials having the same permeability as free space. Iron and other ferrous materials have a high permeability.
- Magnetic field intensity, $\vec{H} = \vec{B} / \mu$, varies by material. Iron and related materials have a much lower \vec{H} for a given value of \vec{B} than other materials. The units of \vec{H} are A/m.

15.3 Magnetic Force

In section 15.1.2 we learned that a wire carrying a current develops a magnetic field that surrounds the wire. As with any magnetic field, the magnetic field surrounding the wire will interact with any other magnetic fields present. This interaction can result in a force being applied to the current-carrying wire, which can be used to move the wire. This force is described by the Lorentz Force Law.

Consider a straight length of current-carrying wire. If the wire, of length L , is placed in a uniform magnetic field (\vec{B}) as shown in Figure 3, the wire will be subjected to a magnetic force given by:

$$\vec{F}_d = I\vec{L} \times \vec{B} \quad [\text{Eq. 2}]$$

The above equation treats the length as a vector with its direction given by the direction of the current. The equation states that the force developed is equal to the cross product of the current multiplied by the length vector and the magnetic field vector. The cross product implies that the force is proportional to: the current “vector”, the magnitude of the magnetic field vector, and the angle between these vectors. The force is a maximum when the angle between the current-carrying length of wire and the magnetic field is 90 degrees (as in Fig. 3), and the force goes to zero when the current-carrying wire and the magnetic field are parallel.

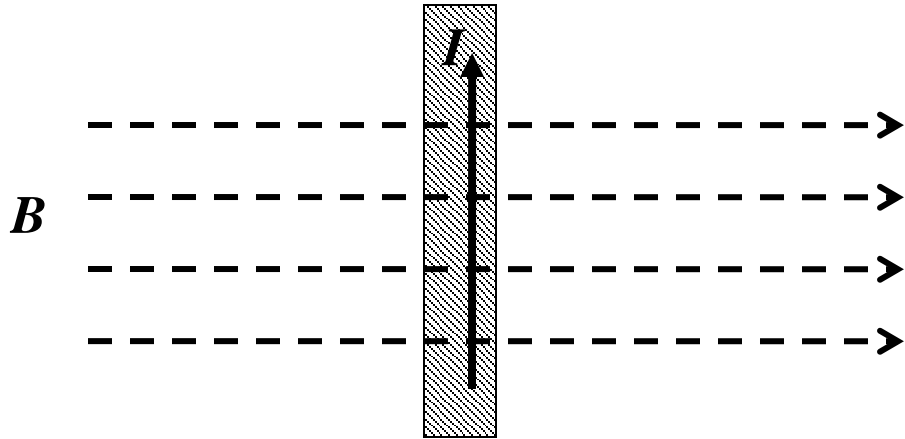


Figure 3. Current Carrying Wire in a Uniform Magnetic Field

The result of the Lorentz Force Law is that we can impose a force on a wire in a magnetic field by pushing current through the wire. That wire can then be used to perform mechanical work. This is the fundamental principle behind linear motors, as well as any other electric motors, and it allows us to convert electrical energy into mechanical energy which can then be used for mechanical work.

15.4 Linear Motors

We now have sufficient understanding to introduce the linear motor. As shown in Figure 4, a simple linear motor consists of a current source, a magnetic field, and a wire that is free to move along a set of rails. Assuming that the current is flowing around the loop as indicated, and the magnetic field is going into the page, the effect of the force on the moveable wire will be to move the wire to the right (as determined by the cross product). This simple concept is how a linear motor operates – a current is moved through a wire where the direction of current flow is perpendicular to the magnetic field. The result is a force applied to the wire that makes it move. The force is dependent on the direction and magnitude of the current, the direction and magnitude of the magnetic field, and the angle between the current and magnetic field vectors.

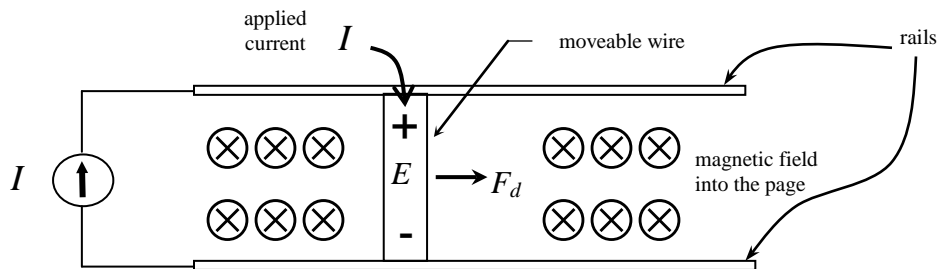


Figure 4. Linear Motor

15.4.1 Faraday's Law

From the description of a simple linear motor given above, we see how a force can be generated on a current-carrying wire in a magnetic field. Equation 2 in section 15.3 indicates that a constant current, perpendicular to a uniform magnetic field, will result in constant force on that wire. If the wire is free to move, the constant force would result in constant acceleration. Constant acceleration would eventually result in physically impossible speeds. It is clear, then, that another equation is needed to describe some opposition to this force.

It turns out that changing the magnetic flux flowing through the area defined by a closed loop of wire produces a current in the closed loop of wire. Faraday's law describes this result. It is the concept behind electric generators, which will be discussed later in the course. For now, we will simply accept that *current is generated by changing the area of a closed loop of wire in a magnetic field*. We needed to introduce Faraday's law in order to understand how this current works to counter the force of the Lorentz law.

For the linear motor in Fig. 4, it should be clear that, as the moveable wire slides to the right, more and more lines of magnetic flux appear within the circuit loop formed by the current source, the rails, and the moveable wire. Faraday's law states that changing the area of a conductor loop in a magnetic field results in a current. This current is driven by an induced voltage which is given by:

$$E_{induced} = (\vec{u} \times \vec{B}) L \quad [\text{Eq. 3}]$$

where, \vec{L} represents the length of, and direction of the current in, the moveable wire
 \vec{u} represents the velocity of the moveable wire
 \vec{B} represents the magnetic field

The above equation indicates that the induced voltage is dependent on the velocity of the wire, the magnitude and direction of the magnetic field, and the current direction and length of the wire in the magnetic field. It is important to understand that the voltage induced by moving the wire generates a current which *opposes* the flow of current from the current source shown in Fig. 4. This opposition is described by Lenz' Law.

15.4.2 Linear Motor Operation

In order to describe a variation of the operation of a linear motor, let's apply a fixed voltage to the rails instead of the DC current. This would be in line with a linear motor used on a roller coaster (not quite how it is done, but it gives the general idea). Figure 5a shows the physical setup of the linear motor, including a voltage source to push the current (rather than a current source as shown in Figure 4), the rails along which a bar can slide, the sliding bar, and a magnetic field. The direction of the initial current flow, I , the force developed, F_d , from the Lorentz law, the resulting velocity, v , and the induced voltage, E , are all indicated on the diagram. Figure 5b shows a circuit diagram which

models the physical setup, including a resistor, R_{rail} , which represents the small but non-zero resistance of the sliding bar contacting the rails. .

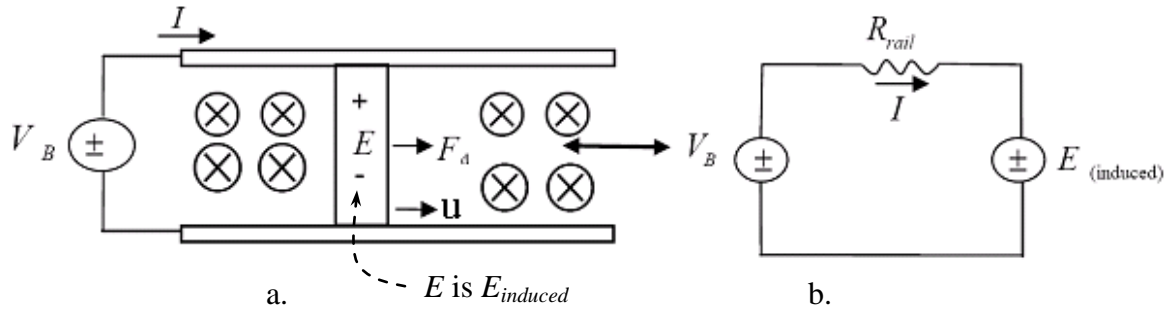


Figure 5. Linear Machine with Voltage Source Applied

Assuming the applied voltage is initially zero, the force on the moveable bar will initially be zero. Similarly, if the bar in the magnetic field is initially at rest, the induced voltage across the bar is zero. Now assume at some time the voltage V_{DC} is turned on and current begins to flow through the bar. The initial value for the current is $I = V_{DC} / R_{rail}$, since $E_{induced}$ is 0. This initial current results in a Lorentz force (F_d) being applied to the bar. Up until the time that the bar begins to move, the induced voltage, $E_{induced}$, given by equation 3 in the previous section, remains zero. However, once the bar moves, the velocity vector \vec{v} is no longer zero, so the induced voltage is no longer zero. The KVL equation for the circuit in Figure 5b would be:

$$V_{DC} - I R_{rail} - E_{induced} = 0 \quad [\text{Eq. 4}]$$

which can be rewritten to solve for the current:

$$I = \frac{V_{DC} - E_{induced}}{R_{rail}} \quad [\text{Eq. 5}]$$

Equation 5 should make it obvious that as $E_{induced}$ increases, the current I through the bar decreases. Equation 2 tells us that, as I decreases, the Lorentz force, F_d , decreases. Eventually, $E_{induced}$ will be equal and opposite to V_{DC} , which means that no current will flow through the bar (from Equation 5). Then the Lorentz force will go to zero and the bar will have no acceleration. In other words, it will move at a constant speed.

To use this linear machine as a motor, it would be required to move some mechanical load such as a roller coaster cart with some friction that opposes movement. This friction force, or load force, would be treated as a force that opposes the force developed from the Lorentz law force, F_d . The net force on the bar would then be the difference between the developed force and the load force:

$$F_{net} = F_d - F_{load}$$

Analysis of the operation of the linear machine can be conducted using these equations. For example, assume that a roller coaster is operating with a fixed applied load. Once steady state operation is achieved, the roller coaster is moving at a constant speed, v , and the net force on the roller coaster would be zero. Hence, a steady state load force would imply that:

$$F_d = BLI = F_{load} \quad [\text{Eq. 6a}]$$

[Note that $F_d = BLI$ is the maximum magnitude from the cross product in Eq. 2]

Rearranging this equation implies a steady state current of:

$$I = \frac{F_{load}}{BL} \quad [\text{Eq. 6b}]$$

Let us rewrite the KVL equation above (Eq. 4) for convenience:

$$V_{DC} - IR_{rail} - E_{induced} = 0 \quad [\text{Eq. 4}]$$

Into this equation we will substitute:

$$E_{induced} = BLu \quad \text{from Eq. 3 [Note that this is the maximum value for the cross product in Eq. 3]}$$

and,

$$I = \frac{F_{load}}{BL} \quad \text{from Eq. 6b above}$$

This gives:

$$V_{DC} - \frac{F_{load}}{BL} R_{rail} - BLu = 0 \quad [\text{Eq. 7a}]$$

Rewriting:

$$BLu = V_{DC} - R_{rail} \frac{F_{load}}{BL} \quad [\text{Eq. 7b}]$$

Dividing both sides by BL gives an expression for the steady state velocity:

$$u = \frac{V_{DC} - R_{rail} \frac{F_{load}}{BL}}{BL} \quad [\text{Eq. 8}]$$

Certainly this analysis does not consider the acceleration of the roller coaster cart, and the algebra gets a bit harried if we make the load force dependent on speed. However, this result is useful in that it illustrates some key concepts that we will see again when we address DC machines. First, in a good design, the voltage drop across the resistance modeling the rails will be small so that the induced voltage will nearly balance the applied DC voltage. Therefore, if we want a high-speed coaster, we would want the induced voltage large and thus V_{DC} as large as practical. Further, we would want the frictional forces to be as small as practical. Second, the current that flows will be sufficient to create an electromagnetic force that balances the load force. The top speed and maximum force required will give us some sense of the power rating of the linear motor.

Here is a summary of the linear motor equations:

Linear Motor Equations

$$P_{in} = V_{DC} I \quad [V_{DC} = \text{supplied voltage}]$$

$$P_{out} = E_{induced} I$$

$$\eta = P_{out} / P_{in}$$

$$E_{induced} = B L u \quad [L = \text{effective length} = n \times \text{length of one wire}]$$

$$V_{DC} = I R_{rail} + E_{induced}$$

$$F_d = B L I \quad [F_d = \text{developed Lorentz force} = \text{Load}_{\text{steady state}}]$$

Example 1: Assume we wish to design a 20 kW (output power) roller coaster that will reach speeds of 60 mph. The maximum B-field that we can achieve is 0.8 T. We have a 240 VDC source available. We desire a full-speed efficiency of 92%.

- a) what is the required rail resistance
- b) what is the source current
- c) what is the effective length of the “bar” or wire

Solution. The desired output power is telling us how much power is being delivered to the induced voltage source shown in Figure 5. Thus,

$$P_{OUT} = E_{induced} I = 20kW$$

The input power is simply the product of the applied voltage and the current:

$$P_{IN} = V_{DC} I$$

Since we desire an efficiency of 92%, this implies that:

$$\eta = \frac{P_{OUT}}{P_{IN}} = 0.92$$

$$P_{IN} = \frac{P_{OUT}}{0.92} = \frac{20kW}{0.92} = 21.74 kW$$

Since the applied voltage is 240 V, we can solve for the current, I:

$$P_{IN} = V_{DC} I$$

$$I = \frac{P_{IN}}{V_{DC}} = \frac{21.74kW}{240 V} = 90.58 A$$

We can also find $E_{induced}$ now since:

$$P_{OUT} = E_{induced} I$$

$$E_{induced} = \frac{P_{OUT}}{I} = \frac{20 kW}{90.58 A} = 220.8 V$$

Next, we can use the KVL equation (Eq. 4) to find the rail resistance from our equivalent circuit.

$$V_{DC} = I R_{rail} + E$$

$$240 V = 90.58 R_{rail} + 220.8 V$$

$$R_{rail} = 0.212 \Omega$$

We now have to convert the given velocity from “miles per hour” to “meters per second”:

$$u = 60 \frac{\text{miles}}{\text{hr}} \times \frac{5280 \text{ ft}}{\text{mile}} \times \frac{12 \text{ in}}{\text{ft}} \times \frac{1 \text{ m}}{39.37 \text{ in}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 26.82 \text{ m/s}$$

To find the effective length of the “bar”, we will use the maximum value of the cross product of Equation 3:

$$E_{induced} = BLu$$
$$L = \frac{E_{induced}}{Bu} = \frac{220.8 V}{(0.8 T)(26.82 m / s)} = 10.29 \text{ meters}$$

(We would now have to get practical in how we would implement this, since for a mile-long track, it would not make much sense to have to drive current that far to develop the required forces. But this at least gives us a bit of a “feel” for designing a linear motor with a constant voltage source applied.)

Example 2: A 12 V linear motor operates with a B-field of 0.4 T. The effective length of the moveable “bar” is 0.2 meters. Assume the steady state mechanical load on the motor is 0.5 N. Find the current flowing through the motor and the velocity of the bar if the rail resistance is 0.025Ω .

Solution

$$F_d = F_{load} \quad \text{since} \quad F_{net} = 0 \quad \text{at steady state.}$$

Hence:

$$F_d = 0.5 N$$

and

$$F_d = BLI$$
$$0.5 = (0.4)(0.2)I$$
$$I = 6.25 A$$

Next, using KVL from our circuit model:

$$V_{DC} = IR_{rail} + E_{induced}$$
$$12 V = (6.25 A)(0.025 \Omega) + E_{induced}$$
$$E_{induced} = 11.84 V$$

and

$$E_{induced} = BLu$$
$$u = \frac{E_{induced}}{BL} = \frac{11.84 V}{(0.4 T)(0.2 m)} = 148 m / sec$$

OK, what have we learned?

- First, we need a magnetic field and a current-carrying conductor to create a force on the conductor. The force is maximum if the field and current are orthogonal.
- Second, a loop (or coil) experiencing a change of flux will exhibit an induced voltage. The induced voltage will attempt to push current in a direction which *opposes the applied current*, thus trying to keep the flux constant.

If the induced voltage (in bullet 2) is created by a conductor moving in a magnetic field, then the factors that determine the magnitude of this induced voltage are:

- the magnitude of the magnetic field
- the velocity of the conductor, and
- the length of conductor in the magnetic field.

These factors (in some form) are common to all electric machinery and will be key to our understanding the operation of a rotating DC motor next!