A Problem…

MIDN A, in Annapolis, has a 6 MHz analog signal that he wants to send to his friend MIDN B, who is vacationing in Outer Mongolia. The transmission will use a satellite as a relay. But, satellite communications cannot support a baseband 6 MHz signal, since this frequency will be refracted off the ionosphere. The satellite requires MIDN A’s transmission be 5 GHz.
Ok, let’s just go terrestrial…

Suppose two midshipmen are yakking on their phones, and each is generating a PCM signal that has a bit rate of 64 kbps. The midshipmen would like to share a channel that has a bandwidth of 128,000 Hz. They might consider sharing on a frequency division basis, where MIDN A is assigned the lower half of the 128,000 Hz channel and MIDN B is assigned the upper half… But how does MIDN B shift his frequency to the upper half?

Bottom Line…

We often have to shift the frequency range of our signal to a different range of frequencies. This shifting is accomplished by modulation.

**Definition:**

**Modulation:** The process by which some characteristic of a carrier is varied in accordance with the modulating wave.
Carrier Modulation – 3 ways to impart information onto a sinusoidal carrier

I could change the amplitude, increasing it and decreasing it so as to make it represent some data…. This is called AMPLITUDE MODULATION

\[ A \sin(2\pi f_c t + \phi) \]

Changing the amplitude of the sine wave as time passes…

Carrier Modulation – 3 ways to impart information onto a sinusoidal carrier

I could change the frequency, increasing it and decreasing it so as to make it represent some data…. This is called FREQUENCY MODULATION

\[ A \sin(2\pi f_c t + \phi) \]

Changing the frequency of the sine wave as time passes…
Carrier Modulation – 3 ways to impart information onto a sinusoidal carrier

I could change the phase of the carrier, increasing it and decreasing it so as to make it represent some data.…

This is called **PHASE MODULATION**

\[ A \sin(2\pi f_c t + \phi) \]

**DSB-SC Amplitude Modulation**

**Definition:**
Information signal, modulated signal, and carrier frequency are defined as:

- \( m(t) \) Information signal – analog baseband
- \( s(t) \) Modulated Signal – analog bandpass
- \( f_c \) Carrier Frequency – high frequency sinusoid

**Recall:** When multiplying a time function by a pure sinusoid, the result is to shift the original spectrum both up and down in frequency and multiply the amplitude by half.

\[ s(t) = m(t)\cos(2\pi f_c t) \quad \Leftrightarrow \quad S(f) = \frac{1}{2} M(f - f_c) + \frac{1}{2} M(f + f_c) \]

![Frequency Spectrum of a DSB-SC AM Signal](image)
DSB-SC AM Modulation

\[ S(f) = \frac{1}{2} M(f - f_c) + \frac{1}{2} M(f + f_c) \]

Note 1: Suppressed Carrier, nothing appears at \( f_c \).

Note 2: Transmitted bandwidth is given by \( BW = 2f_{\text{max}} \) (AM is bandwidth inefficient)

Note 3: DSB-SC isn’t a useful way to communicate – it requires a synchronous receiver.

Creation and Recovery of DSB-SC AM

To modulate AM signals, we use a device known as a mixer.

\[ s(t) = m(t)s_c(t) \]
\[ s(t) = m(t)\cos(2\pi f_c t) \]

To demodulate AM signals, we use a mixer and LPF (Requires Phase Sync.).

\[ \hat{m}(t) = m(t)\cos(2\pi f_c t) = m(t)\cos(2\pi f_c t)\cos(2\pi f_c t) \]
\[ \hat{m}(t) = m(t)\cos^2(2\pi f_c t) \]

Note: \( \cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A) \)

\[ \hat{m}(t) = \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos(4\pi f_c t) \]

\[ \hat{m}(t) = \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos(4\pi f_c t) \]

Rejected by LPF

\[ \hat{m}(t) = \frac{1}{2}m(t) = m(t) \]
Example Problem – Lathi 4.1 (Handout)

Given: A baseband signal is of the form: \( m(t) = \cos(2\pi f_m t) \).
This signal AM modulates a high-frequency carrier.

Find: Sketch the resulting DSB-SC AM signal in both the time-domain and frequency domain.

Note: This particular example is referred to as tone modulation because the underlying modulating signal is a pure sinusoid (or tone).

Example Problem Solution

The spectrum of the baseband signal \( m(t) \) is given by:

\[
M(f) = \frac{1}{2} \left[ \delta(f - f_m) + \delta(f + f_m) \right]
\]

In the time domain we have:

\[
s(t) = m(t) \cos(2\pi f_c t) \\
s(t) = \cos(2\pi f_c t) \cos(2\pi f_m t) \\
s(t) = \frac{1}{2} \left[ \cos(2\pi (f_c + f_m) t) + \cos(2\pi (f_c - f_m) t) \right]
\]

We can plot this in Matlab, and observe the following:

```
% Setup system parameters
fc = 200;   % Carrier Freq
fm = 10;    % Message Freq
% Setup timebase
fs = 10e3;  % Sampling Freq
Tend = 0.2; % Stop Time
t = 0:1./fs:Tend; % Time
% Generate the AM Signal
m = cos(2.*pi.*fm.*t); u = m.*cos(2.*pi.*fc.*t);
```

```
DSB-SC AM Time Domain
```

\[ -1 \quad 0 \quad 0.5 \quad 1 \quad 0 \quad 0.5 \quad 1 \]

\[ 0 \quad 0.02 \quad 0.04 \quad 0.06 \quad 0.08 \quad 0.1 \quad 0.12 \quad 0.14 \quad 0.16 \quad 0.18 \quad 0.2 \]

\[ -1 \quad -0.5 \quad 0 \quad 0.5 \quad 1 \]

\[ \text{time (sec)} \quad \text{Amplitude (V)} \]

\[ \text{DSB-SC AM Time Domain} \]
Example Problem Solution

In the frequency domain we have:

\[ S(f) = \frac{1}{2} m(f - f_c) + \frac{1}{2} m(f + f_c) \]

\[ S(f) = \frac{1}{4} \left[ \delta(f - f_c - f_m) + \delta(f - f_c + f_m) \right] + \frac{1}{2} \left[ \delta(f + f_c - f_m) + \delta(f + f_c + f_m) \right] \]

A better version of AM: DSB-TC

Transmit a tone carrier along with the AM modulated message signal.

\[ s(t) = [A + m(t)] \cos(2\pi f \cdot t) \]

\[ S(f) = \frac{1}{2} A \delta(f + f_c) + \frac{1}{2} A \delta(f - f_c) + \frac{1}{2} M(f - f_c) + \frac{1}{2} M(f + f_c) \]

\[ BW = 2f_{\text{max}} \]
**AM Modulation Envelope – Two Cases**

\[ s(t) = \left[ A + m(t) \right] \cos(2\pi f_c t) \]

The quantity \( A + m(t) \) forms an envelope that bounds the amplitude of the carrier.

To illustrate:
- Sketch \( A + m(t) \).
- Sketch \(-A + m(t)\).
- Fill in carrier in between.

**DSB-TC AM in the time domain**

To ease analysis, rewrite in terms of amplitude-normalized message signal \( \tilde{m}(t) \) and modulation index \( \mu \).

\[ s(t) = A [1 + \mu \tilde{m}(t)] \cos(2\pi f_c t) \quad m(t) = m_p \cos(2\pi f_c t) \]

Criteria for Envelope Detection: \( 0 \leq \mu \leq 1 \)
AM Envelope Example  (Lathi 4.3)

Given: Suppose \( m(t) = b \cos(2\pi f_a t) \)
Find: Sketch the resulting AM Signal if \( \mu = 0.5 \)

\[
\text{Note: } \mu = \frac{b}{A} \Rightarrow b = \mu A
\]
\[
m(t) = \mu A \cos(2\pi f_a t)
\]
\[
s(t) = A \left[ 1 + \mu \cos(2\pi f_a t) \right] \cos(2\pi f_t)
\]
\[
\mu = 0.5
\]

![Diagram](a)

AM Envelope Example  (Lathi 4.3)

Given: Suppose \( m(t) = b \cos(2\pi f_a t) \)
Find: Sketch the resulting AM Signal if \( \mu = 1.0 \)

\[
\text{Note: } \mu = \frac{b}{A} \Rightarrow b = \mu A
\]
\[
m(t) = \mu A \cos(2\pi f_a t)
\]
\[
s(t) = A \left[ 1 + \mu \cos(2\pi f_a t) \right] \cos(2\pi f_t)
\]
\[
\mu = 1
\]

![Diagram](b)
AM Power and Efficiency

A carrier makes it easier to demodulate the incoming signal, but we pay a price in terms of efficiency. Some of the transmitted power is being used to broadcast a pure sinusoid which does not convey any information.

Define power efficiency as:

$$\eta = \frac{\text{signal power}}{\text{total power}} = \frac{P_s}{P_c + P_i}$$

Assuming tone modulation, expand the AM equation:

$$s(t) = A \cos(2\pi f_c t) + \frac{\mu A}{2} \cos(2\pi (f_c + f_m) t) + \frac{\mu A}{2} \cos(2\pi (f_c - f_m) t)$$

Power in Carrier: \(P_c = \frac{A^2}{2}\)

**Note:**

Power in Sidebands: \(P_{\text{USB}} = P_{\text{LSB}} = \frac{\left(\frac{\mu A}{2}\right)^2}{2} = \frac{m^2}{8}\)

If we rewrite the above expressions in terms of the modulation index:

$$s(t) = A \cos(2\pi f_c t) + \frac{\mu A}{2} \cos(2\pi (f_c + f_m) t) + \frac{\mu A}{2} \cos(2\pi (f_c - f_m) t)$$

$$P_{\text{USB}} = P_{\text{LSB}} = \frac{\left(\frac{\mu A}{2}\right)^2}{2} = \frac{(\mu A)^2}{8}$$

Thus:

$$\eta = \frac{P_s}{P_c + P_i} = \frac{(\frac{\mu A}{2})^2}{\frac{\mu A}{2} + \frac{\mu A}{2}} = \frac{(\mu A)^2}{\frac{\mu A}{2} + \frac{\mu A}{2}} = \frac{\mu^2}{\mu^2 + 2}$$

**Note That:** As the index of modulation decreases, the efficiency will also decrease. In fact, for sinusoidal modulating signals and envelope detection, we find that:

$$\eta \leq 33\% \quad \text{for} \quad 0 \leq \mu \leq 1$$
**AM Example**

Given: AM Transmitter, 1 kW unmodulated output power, 50Ω load.
5V sinusoidal input to the modulator gives the amplitude of each sideband to be 40% of the amplitude of the carrier.

Find: (a) What is the modulation index?
(b) What is the power in the Carrier, USB, and LSB
(c) What is the efficiency of the AM Transmitter.

![AM Transmitter Diagram]

**Recovering the Message: Envelope Detector**

Observe: If we can trace out the Envelope of the AM signal, we can effectively recover the underlying information signal.
**Envelope Detector Mathematically**

**Received Signal:** \[ r(t) = A[1 + \mu m(t)] \cos(2\pi ft) \]

**Absolute Value Operation:** \[ r_{\text{env}}(t) = A[1 + \mu m(t)]|\cos(2\pi ft)| \]

If \( 1 + \mu m(t) \) is constrained to be always positive (i.e., \( 0 \leq \mu \leq 1 \))

\[ r_{\text{env}}(t) = A[1 + \mu m(t)]|\cos(2\pi ft)| \]

**Note:** The full-wave rectified cosine can be expanded in a **Fourier Series**

\[ r_{\text{env}}(t) = A[1 + \mu m(t)]a_0 + a_2 \cos(2\pi f_2 t) + a_4 \cos(2\pi f_4 t) + \ldots \]

**LPF:** Block everything except the \( a_0 \) term.

\[ r_{\text{env}}(t) = a_0 A(1 + \mu m(t)) \]

**DC Block does the rest:** \( \hat{m}(t) = a_0 \mu m(t) \)

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**AM Demodulation in Software**

```matlab
fs_rf = 1e6;  \% sampling frequency of the AM signal
fco = 0.02e6;  \% cutoff frequency for the low pass filter
\% Will need to downsample the recovered signal to an audio-level sampling frequency for output to sound card.
downsample_rate = floor(fs_rf./44.1e3);

\% Perform AM Demod using Envelope Detection.
r_env = abs(am_samples);  \% Envelope Detector
r_lpf = filter_audio(x_env,fco,fs_rf);  \% Low Pass Filter
\% Downsample to audio sampling frequency
fs_audio = fs_rf./downsample_rate;
m_hat = downsample(r_lpf, downsample_rate);

\% Perform AM Demod using Square-Law Detector
r_sq = am_samples.^2;  \% Square-Law Detector
\% Low Pass Filter and Square Root operation
m_hat = sqrt(filter_audio(r_sq,fco,fs_rf));
\% Downsample to audio sampling frequency
fs_audio = fs_rf./downsample_rate;
x_bb = downsample(m_hat, downsample_rate);

\% Output the demodulated signal to the sound card
sound(m_hat, fs_audio, 16);
```
Frequency Division Multiplexing

Consider: MIDN A likes listening to old-fashioned grunge music. MIDN B is more of a modern music fan. How do we satisfy their listening desires?

We could TDM music (0900 Grunge Hour; 1000 Alternative; 1100 Classical), or we could establish multiple stations on different frequencies and multiplex them in the Frequency Domain.

Note: Stations transmit their signals simultaneously in time, but are separated in frequency.

Example: FDM Across the AM Radio Band

\[
\begin{align*}
&f_{m1} = 4 \text{ kHz}, & &f_{m2} = 4 \text{ kHz} \\
&m_{m1}(t) & &\rightarrow & &\sum & &\rightarrow & &s_1(t) \\
&m_{m2}(t) & &\rightarrow & &\sum & &\rightarrow & &s_2(t) \\
&f_{cn} = 500 kHz, & &f_{cs} = 510 kHz, & &\ldots, & &f_{cn} = 1700 kHz
\end{align*}
\]