

### **A Problem...**

**MIDN A, in Annapolis, has a 6 MHz analog signal that he wants to send to his friend MIDN B, who is vacationing in Outer Mongolia. The transmission will use a satellite as a relay. But, satellite communications cannot support a baseband 6 MHz signal, since this frequency will be refracted off the ionosphere. The satellite requires MIDN A's transmission be 5 GHz.**



## Ok, let's just go terrestrial...

Suppose two midshipmen are yakking on their phones, and each is generating a PCM signal that has a bit rate of 64 kbps. The midshipmen would like to share a channel that has a bandwidth of 128,000 Hz. They might consider sharing on a *frequency division basis*, where MIDN A is assigned the lower half of the 128,000 Hz channel and MIDN B is assigned the upper half... But how does MIDN B shift his frequency to the upper half?



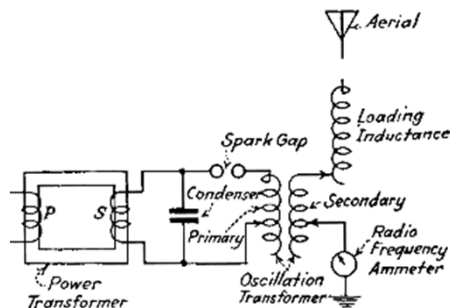
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## Bottom Line...

We often have to shift the frequency range of our signal to a different range of frequencies. This shifting is accomplished by *modulation*.

### Definition:

**Modulation:** The process by which some characteristic of a carrier is varied in accordance with the modulating wave.



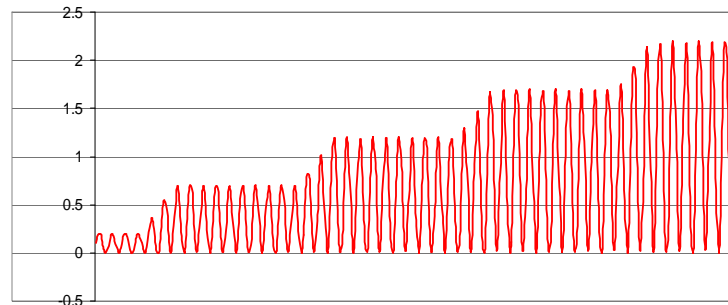
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## Carrier Modulation – 3 ways to impart information onto a sinusoidal carrier

I could change the amplitude, increasing it and decreasing it so as to make it represent some data....

This is called **AMPLITUDE MODULATION**

$$A \sin(2\pi f_c t + \phi)$$



Changing the amplitude of the sine wave as time passes...

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## Carrier Modulation – 3 ways to impart information onto a sinusoidal carrier

I could change the frequency, increasing it and decreasing it so as to make it represent some data....

This is called **FREQUENCY MODULATION**

$$A \sin(2\pi f_c t + \phi)$$



Changing the frequency of the sine wave as time passes...

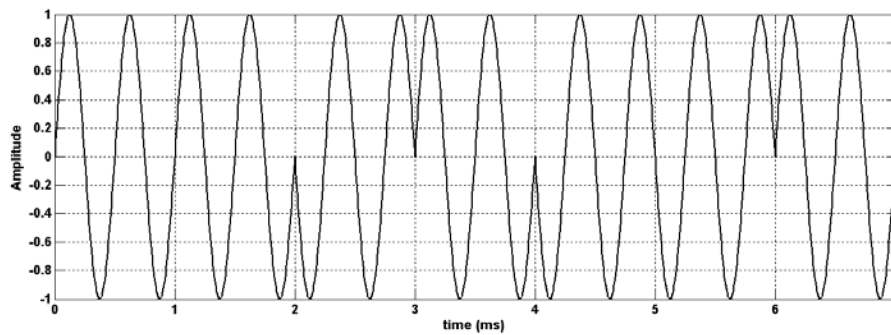
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## Carrier Modulation – 3 ways to impart information onto a sinusoidal carrier

I could change the phase of the carrier, increasing it and decreasing it so as to make it represent some data....

This is called **PHASE MODULATION**

$$A \sin(2\pi f_c t + \phi)$$



Data Pattern: 1 1 0 1 0 0 1

## DSB-SC Amplitude Modulation

### Definition:

Information signal, modulated signal, and carrier frequency are defined as:

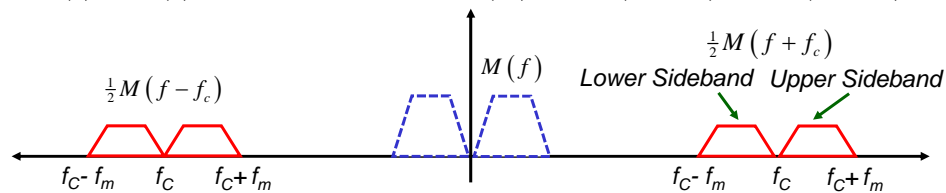
$m(t)$  Information signal – analog baseband

$s(t)$  Modulated Signal – analog bandpass

$f_c$  Carrier Frequency – high frequency sinusoid

**Recall:** When multiplying a time function by a pure sinusoid, the result is to shift the original spectrum both up and down in frequency and multiply the amplitude by half.

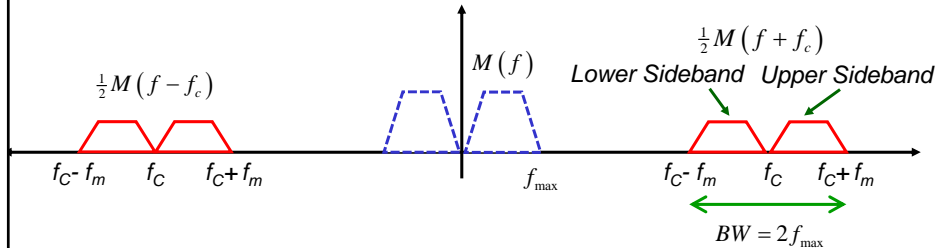
$$s(t) = m(t) \cos(2\pi f_c t) \quad \Leftrightarrow \quad S(f) = \frac{1}{2} M(f - f_c) + \frac{1}{2} M(f + f_c)$$



Frequency Spectrum of a DSB-SC AM Signal

## DSB-SC AM Modulation

$$S(f) = \frac{1}{2}M(f - f_c) + \frac{1}{2}M(f + f_c)$$



**Note 1:** Suppressed Carrier, nothing appears at  $f_c$ .

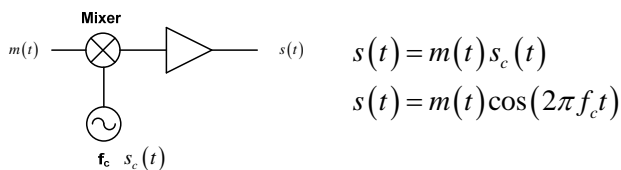
**Note 2:** Transmitted bandwidth is given by  $BW = 2f_{\max}$   
(AM is bandwidth inefficient)

**Note 3:** DSB-SC isn't a useful way to communicate – it requires a synchronous receiver.

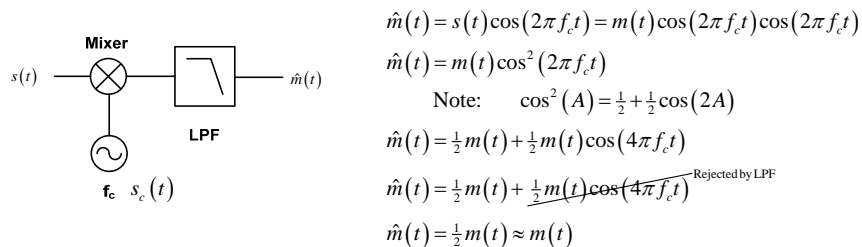
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## Creation and Recovery of DSB-SC AM

To modulate AM signals, we use a device known as a mixer.



To demodulate AM signals, we use a mixer and LPF (Requires Phase Sync.).



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## Example Problem – Lathi 4.1 (Handout)

**Given:** A baseband signal is of the form:  $m(t) = \cos(2\pi f_m t)$ .

This signal AM modulates a high-frequency carrier.

**Find:** Sketch the resulting DSB-SC AM signal in both the time-domain and frequency domain.

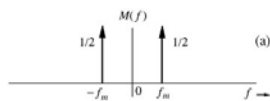
**Note:** This particular example is referred to as tone modulation because the underlying modulating signal is a pure sinusoid (or tone).

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## Example Problem Solution

The spectrum of the baseband signal  $m(t)$  is given by:

$$M(f) = \frac{1}{2} [\delta(f - f_m) + \delta(f + f_m)]$$



In the time domain we have:

$$s(t) = m(t) \cos(2\pi f_c t)$$

$$s(t) = \cos(2\pi f_m t) \cos(2\pi f_c t)$$

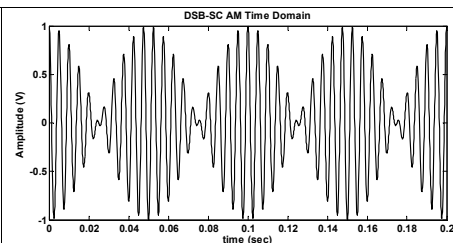
$$s(t) = \frac{1}{2} [\cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)]$$

We can plot this in Matlab, and observe the following:

```
% Setup system parameters
fc = 200; % Carrier Freq
fm = 10; % Message Freq

% Setup timebase
fs = 10e3; % Sampling Freq
Tend = 0.2; % Stop Time
t = 0:1./fs:Tend; % Time

% Generate the AM Signal
m = cos(2.*pi.*fm.*t);
s = m.*cos(2.*pi.*fc.*t);
```



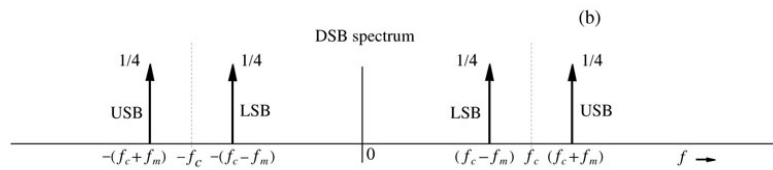
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## Example Problem Solution

In the frequency domain we have:

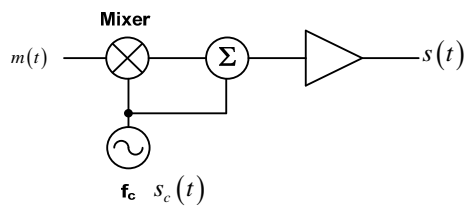
$$S(f) = \frac{1}{2}m(f - f_c) + \frac{1}{2}m(f + f_c)$$

$$S(f) = \frac{1}{4}[\delta(f - f_c - f_m) + \delta(f - f_c + f_m)] + \frac{1}{4}[\delta(f + f_c - f_m) + \delta(f + f_c + f_m)]$$



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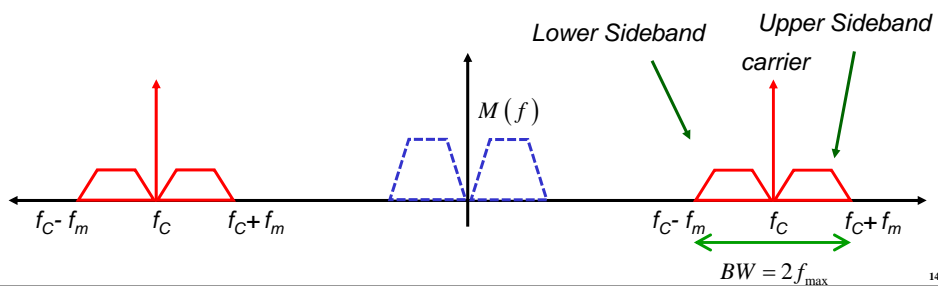
## A better version of AM: DSB-TC



Transmit a tone carrier along with the AM modulated message signal.

$$s(t) = [A + m(t)] \cos(2\pi f_c t) \Leftrightarrow$$

$$S(f) = \frac{1}{2}A\delta(f + f_c) + \frac{1}{2}A\delta(f - f_c) + \frac{1}{2}M(f - f_c) + \frac{1}{2}M(f + f_c)$$



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## AM Modulation Envelope – Two Cases

$$s(t) = [A + m(t)] \cos(2\pi f_c t)$$

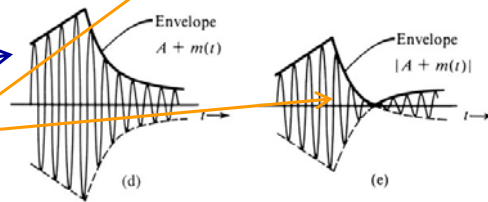
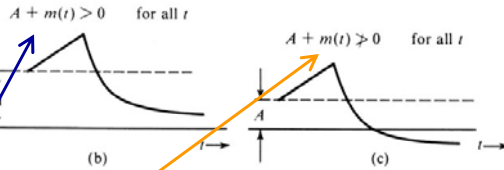
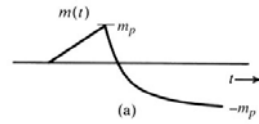
The quantity  $[A + m(t)]$  forms an **envelope** that bounds the amplitude of the carrier.

To illustrate

- Sketch  $[A + m(t)]$ .
- Sketch  $-[A + m(t)]$ .
- Fill in carrier in between.

**Easily recoverable**

**Crossover Distortion**



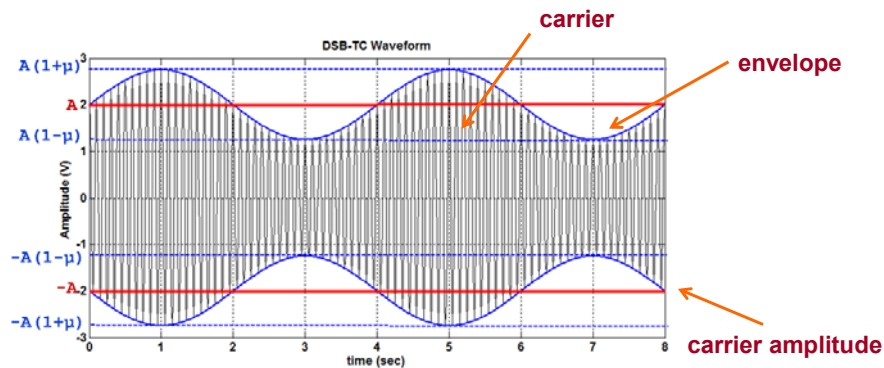
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## DSB-TC AM in the time domain

To ease analysis, rewrite in terms of **amplitude-normalized message signal**  $\tilde{m}(t)$  and modulation index  $\mu$ .

$$s(t) = A[1 + \mu\tilde{m}(t)] \cos(2\pi f_c t) \quad m(t) = m_p \cos(2\pi f_m t) \quad \mu = \frac{m_p}{A}$$

Criteria for Envelope Detection:  $0 \leq \mu \leq 1$



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## AM Envelope Example (Lathi 4.3)

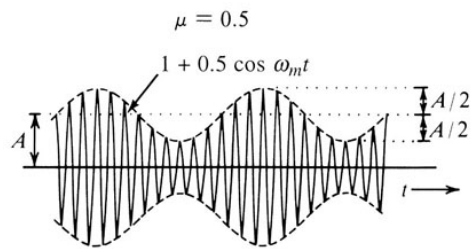
**Given:** Suppose  $m(t) = b \cos(2\pi f_m t)$

**Find:** Sketch the resulting AM Signal if  $\mu = 0.5$

**Note:**  $\mu = \frac{b}{A} \Rightarrow b = \mu A$

$$m(t) = \mu A \cos(2\pi f_m t)$$

$$s(t) = A[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$



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## AM Envelope Example (Lathi 4.3)

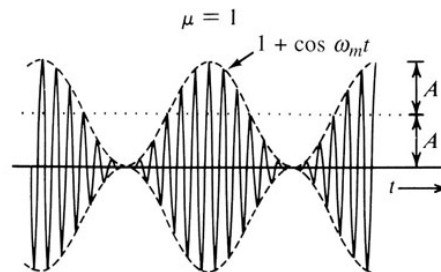
**Given:** Suppose  $m(t) = b \cos(2\pi f_m t)$

**Find:** Sketch the resulting AM Signal if  $\mu = 1.0$

**Note:**  $\mu = \frac{b}{A} \Rightarrow b = \mu A$

$$m(t) = \mu A \cos(2\pi f_m t)$$

$$s(t) = A[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$



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## AM Power and Efficiency

A carrier makes it easier to demodulate the incoming signal, but we pay a price in terms of efficiency.

Some of the transmitted power is being used to broadcast a pure sinusoid which does not convey any information.

Define power efficiency as:  $\eta = \frac{\text{signal power}}{\text{total power}} = \frac{P_s}{P_C + P_s}$

Assuming tone modulation, expand the AM equation:

$$s(t) = \underbrace{A \cos(2\pi f_c t)}_{\text{Carrier}} + \underbrace{\frac{m_p}{2} \cos(2\pi(f_c + f_m)t)}_{\text{USB=UpperSideband}} + \underbrace{\frac{m_p}{2} \cos(2\pi(f_c - f_m)t)}_{\text{LSB=LowerSideband}}$$

Power in Carrier:  $P_C = \frac{A^2}{2}$

**Note:**

Power in Sidebands:  $P_{USB} = P_{LSB} = \frac{\left(\frac{m_p}{2}\right)^2}{2} = \frac{m_p^2}{8}$

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## AM Power and Efficiency

If we rewrite the above expressions in terms of the modulation index:

$$s(t) = \underbrace{A \cos(2\pi f_c t)}_{\text{Carrier}} + \underbrace{\frac{\mu A}{2} \cos(2\pi(f_c + f_m)t)}_{\text{USB=UpperSideband}} + \underbrace{\frac{\mu A}{2} \cos(2\pi(f_c - f_m)t)}_{\text{LSB=LowerSideband}}$$

$$P_{USB} = P_{LSB} = \frac{\left(\frac{\mu A}{2}\right)^2}{2} = \frac{(\mu A)^2}{8}$$

Thus:  $\eta = \frac{P_s}{P_C + P_s} = \frac{\frac{(\mu A)^2}{8}}{\frac{(\mu A)^2}{8} + \frac{A^2}{2}} = \frac{\frac{(\mu X)^2}{8}}{\frac{(\mu X)^2}{8} + \frac{X^2}{2}} = \frac{\mu^2}{\mu^2 + 2}$

**Note That:** As the index of modulation decreases, the efficiency will also decrease. In fact, for sinusoidal modulating signals and envelope detection, we find that:

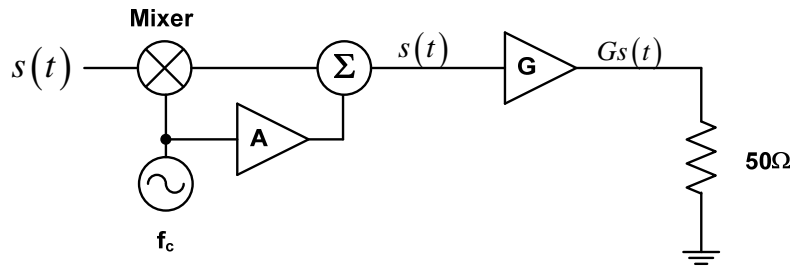
$$\eta \leq 33\% \quad \text{for} \quad 0 \leq \mu \leq 1$$

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## AM Example

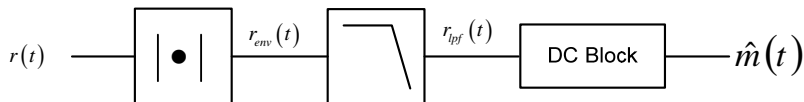
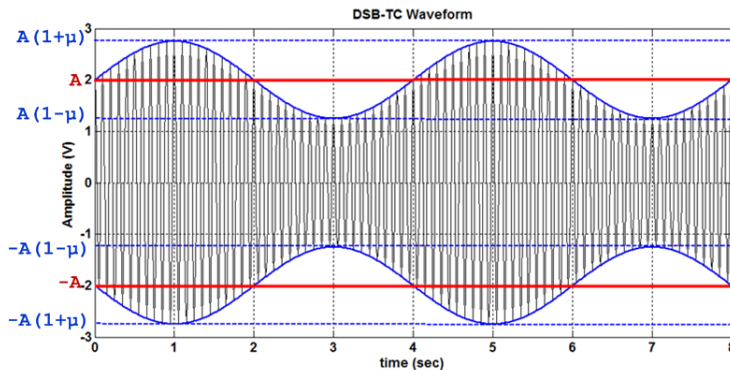
Given: AM Transmitter, 1 kW unmodulated output power,  $50\Omega$  load. 5V sinusoidal input to the modulator gives the amplitude of each sideband to be 40% of the amplitude of the carrier.

Find: (a) What is the modulation index?  
 (b) What is the power in the Carrier, USB, and LSB  
 (c) What is the efficiency of the AM Transmitter.



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## Recovering the Message: Envelope Detector



**Observe:** If we can trace out the Envelope of the AM signal, we can effectively recover the underlying information signal.

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## Envelope Detector Mathematically

**Received Signal:**  $r(t) = A[1 + \mu m(t)] \cos(2\pi f_c t)$

**Absolute Value Operation:**  $r_{env}(t) = A|[1 + \mu m(t)] \cos(2\pi f_c t)|$

If  $1 + \mu m(t)$  is constrained to be always positive (i.e.,  $0 \leq \mu \leq 1$ )

$$r_{env}(t) = A[1 + \mu m(t)]|\cos(2\pi f_c t)|$$

**Note:** The full-wave rectified cosine can be expanded in a Fourier Series

$$r_{env}(t) = A[1 + \mu m(t)][a_0 + a_1 \cos(2\pi(2f_c)t) + a_2 \cos(2\pi(4f_c)t) + \dots]$$

**LPF:** Block everything except the  $a_0$  term.

$$r_{env}(t) = a_0 A(1 + \mu m(t))$$

**DC Block does the rest:**  $\hat{m}(t) = a_0 \mu m(t)$

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## AM Demodulation in Software

```
fs_rf = 1e6; % sampling frequency of the AM signal
fco = 0.02e6; % cutoff frequency for the low pass filter
% Will need to downsample the recovered signal to an audio-
% level sampling frequency for output to sound card.
downsample_rate = floor(fs_rf./44.1e3);

% Perform AM Demod using Envelope Detection.
r_env = abs(am_samples); % Envelope Detector
r_lpf = filter_audio(x_env,fco,fs_rf); % Low Pass Filter
% Downsample to audio sampling frequency
fs_audio = fs_rf./downsample_rate;
m_hat = downsample(r_lpf, downsample_rate);

% Perform AM Demod using Square-Law Detector
r_sq = am_samples.^2; % Square-Law Detector
% Low Pass Filter and Square Root operation
m_hat = sqrt(filter_audio(r_sq,fco,fs_rf));
% Downsample to audio sampling frequency
fs_audio = fs_rf./downsample_rate;
x_bb = downsample(m_hat, downsample_rate);

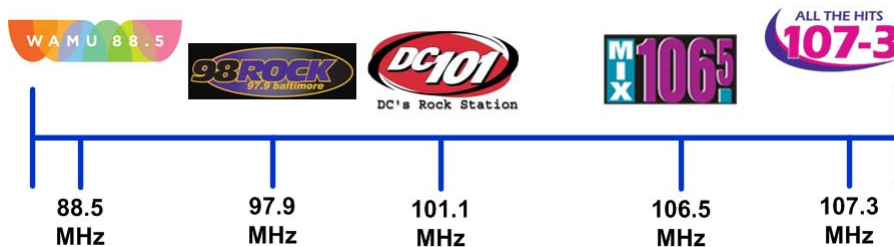
% Output the demodulated signal to the sound card
sound(m_hat, fs_audio, 16);
```

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## Frequency Division Multiplexing

Consider: MIDN A likes listening to old-fashioned grunge music. MIDN B is more of a modern music fan. How do we satisfy their listening desires?

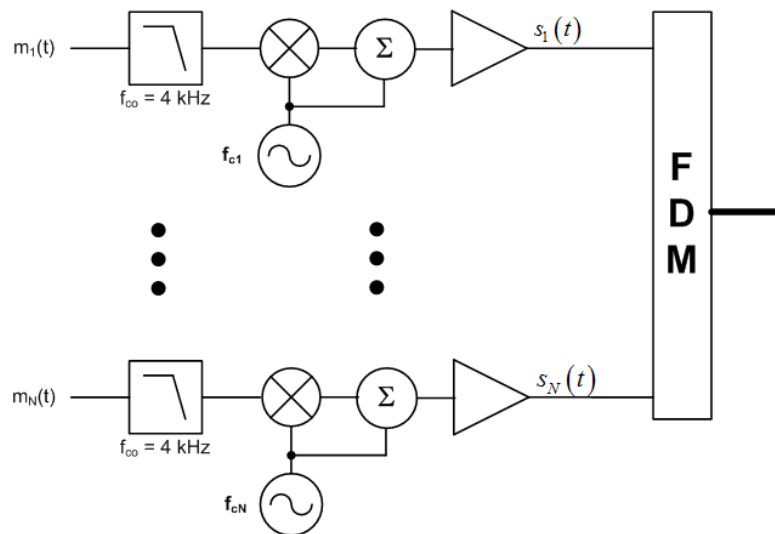
We could TDM music (0900 Grunge Hour; 1000 Alternative; 1100 Classical), or we could establish **multiple stations on different frequencies** and multiplex them in the **Frequency Domain**.



**Note:** Stations transmit their signals simultaneously in time, but are separated in **frequency**.

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## Example: FDM Across the AM Radio Band



$$f_{c_1} = 500 \text{ kHz}, f_{c_2} = 510 \text{ kHz}, \dots, f_{c_N} = 1700 \text{ kHz}$$

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