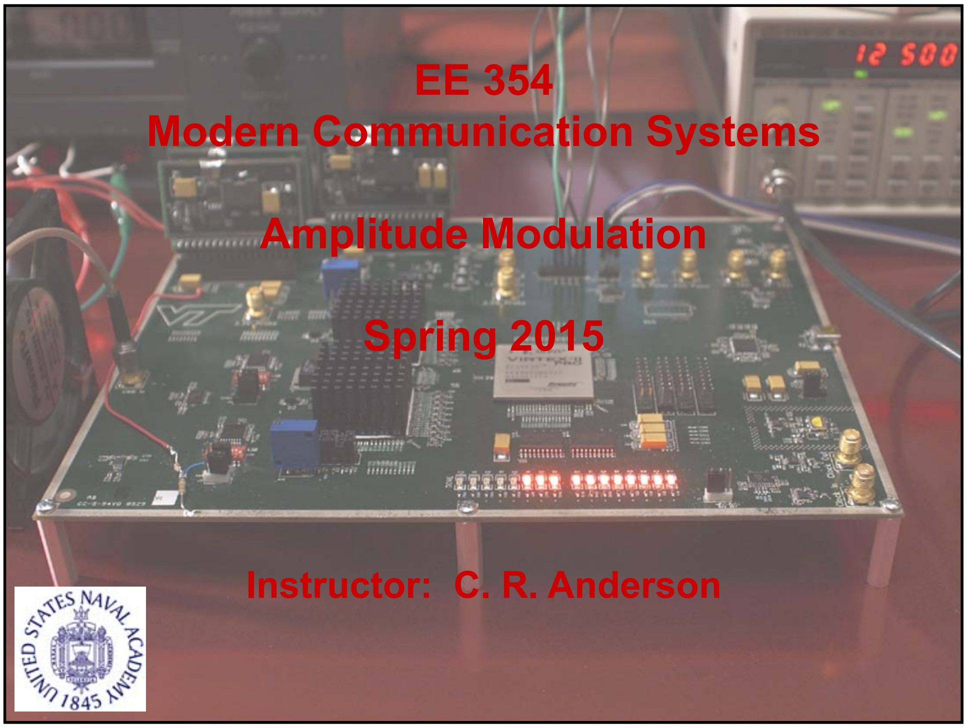

EE 354
Modern Communication Systems

Amplitude Modulation

Spring 2015

Instructor: C. R. Anderson

A Problem…

MIDN A, in Annapolis, has a 6 MHz analog signal that he wants to send to his friend MIDN B, who is vacationing in Outer Mongolia. The transmission will use a satellite as a relay. But, satellite communications cannot support a baseband 6 MHz signal, since this frequency will be refracted off the ionosphere. The satellite requires MIDN A's transmission be 5 GHz.

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Ok, let's just go terrestrial…

Suppose two midshipmen are yakking on their phones, and each is generating a PCM signal that has a bit rate of 64 kbps. The midshipmen would like to share a channel that has a bandwidth of 128,000 Hz. They might consider sharing on a *frequency division basis*, where MIDN A is assigned the lower half of the 128,000 Hz channel and MIDN B is assigned the upper half… But how does MIDN B shift his frequency to the upper half?

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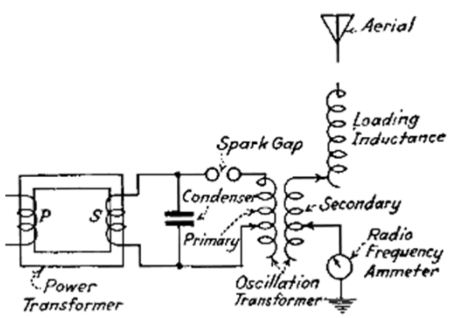
Bottom Line…

We often have to shift the frequency range of our signal to a different range of frequencies. This shifting is accomplished by *modulation*. .

<u>Definition</u>:

Modulation: The process by which some characteristic of a carrier is varied in accordance with the modulating wave.



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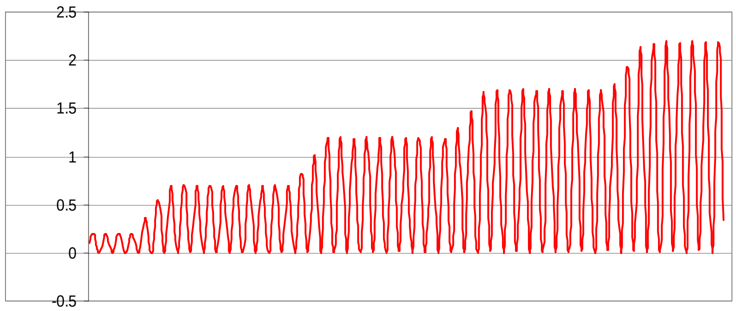
Carrier Modulation – 3 ways to impart information onto a sinusoidal carrier

I could change the amplitude, increasing it and decreasing it so as to make it represent some data….

This is called **AMPLITUDE MODULATION**

$$A\sin(2\pi\ f_c\ t\ +\ \phi)$$



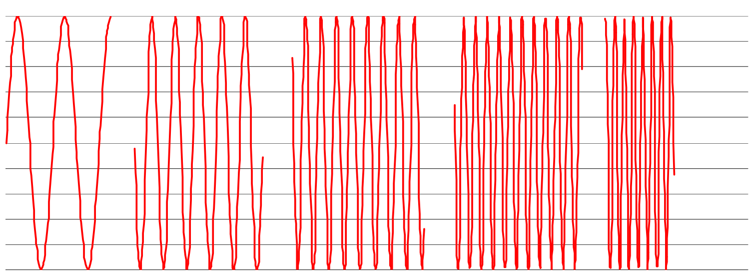
Changing the amplitude of the sine wave as time passes…

Carrier Modulation – 3 ways to impart information onto a sinusoidal carrier

I could change the frequency, increasing it and decreasing it so as to make it represent some data….

This is called **FREQUENCY MODULATION**

$$A\sin(2\pi\ f_c\ t\ +\ \phi)$$



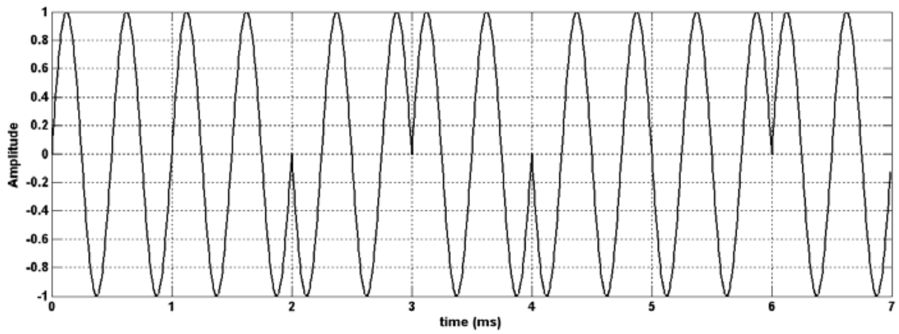
Changing the frequency of the sine wave as time passes…

Carrier Modulation – 3 ways to impart information onto a sinusoidal carrier

I could change the phase of the carrier, increasing it and decreasing it so as to make it represent some data….

This is called **PHASE MODULATION**

$$A\sin(2\pi\ f_c\ t\ + \phi)$$



Data Pattern: 1 1 0 1 0 0 1

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DSB-SC Amplitude Modulation

Definition:
Information signal, modulated signal, and carrier frequency are defined as:

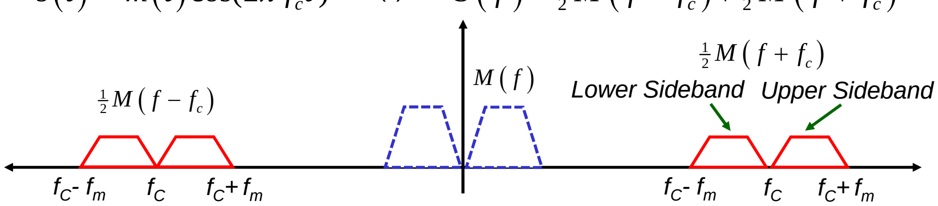
$m(t)$ **Information signal – analog baseband**

$s(t)$ **Modulated Signal – analog bandpass**

f_c **Carrier Frequency – high frequency sinusoid**

Recall: When multiplying a time function by a pure sinusoid, the result is to shift the original spectrum both up and down in frequency and multiply the amplitude by half.

$$s(t) = m(t)\cos(2\pi f_c t) \quad \Leftrightarrow \quad S(f) = \tfrac{1}{2}M(f - f_c) + \tfrac{1}{2}M(f + f_c)$$



Frequency Spectrum of a DSB-SC AM Signal

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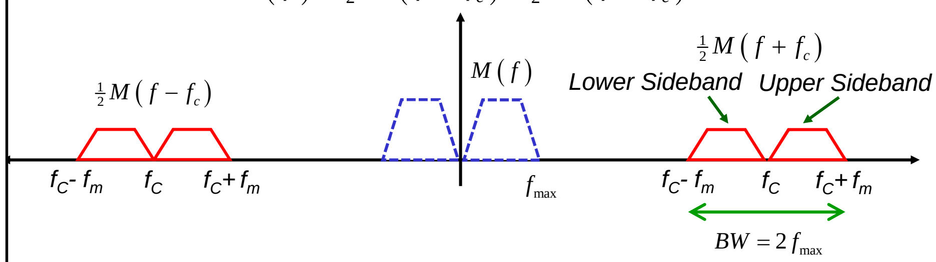
DSB-SC AM Modulation

$$S(f) = \tfrac{1}{2}M(f - f_c) + \tfrac{1}{2}M(f + f_c)$$



Note 1: Suppressed Carrier, nothing appears at f_c.

Note 2: Transmitted bandwith is given by $BW = 2f_{max}$ (AM is bandwidth inefficient)

Note 3: DSB-SC isn't a useful way to communicate – it requires a synchronous receiver.

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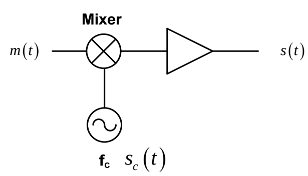
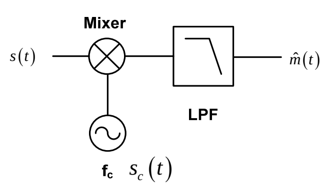
Creation and Recovery of DSB-SC AM

To modulate AM signals, we use a device known as a mixer.



$$s(t) = m(t)s_c(t)$$
$$s(t) = m(t)\cos(2\pi f_c t)$$

To demodulate AM signals, we use a mixer and LPF (Requires Phase Sync.).



$$\hat{m}(t) = s(t)\cos(2\pi f_c t) = m(t)\cos(2\pi f_c t)\cos(2\pi f_c t)$$
$$\hat{m}(t) = m(t)\cos^2(2\pi f_c t)$$
Note: $\cos^2(A) = \tfrac{1}{2} + \tfrac{1}{2}\cos(2A)$
$$\hat{m}(t) = \tfrac{1}{2}m(t) + \tfrac{1}{2}m(t)\cos(4\pi f_c t)$$
$$\hat{m}(t) = \tfrac{1}{2}m(t) + \underbrace{\cancel{\tfrac{1}{2}m(t)\cos(4\pi f_c t)}}_{\text{Rejected by LPF}}$$
$$\hat{m}(t) = \tfrac{1}{2}m(t) \approx m(t)$$

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Example Problem – Lathi 4.1 (Handout)

Given: A baseband signal is of the form: $m(t)=\cos(2\pi f_m t)$ **.**

This signal AM modulates a high-frequency carrier.

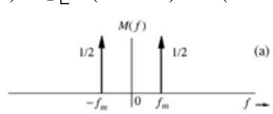
Find: Sketch the resulting DSB-SC AM signal in both the time-domain and frequency domain.

<u>Note:</u> This particular example is referred to as tone modulation because the underlying modulating signal is a pure sinusoid (or tone).

Example Problem Solution

The spectrum of the baseband signal $m(t)$ **is given by:**

$$M(f)=\tfrac{1}{2}\left[\delta(f-f_m)+\delta(f+f_m)\right]$$

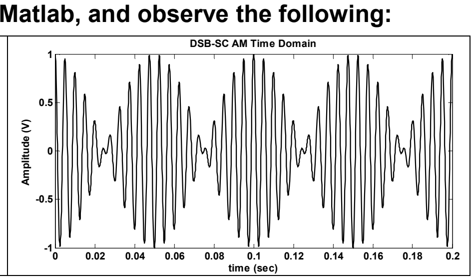


In the time domain we have:

$$s(t)=m(t)\cos(2\pi f_c t)$$
$$s(t)=\cos(2\pi f_m t)\cos(2\pi f_c t)$$
$$s(t)=\tfrac{1}{2}\left[\cos(2\pi(f_c+f_m)t)+\cos(2\pi(f_c-f_m)t)\right]$$

We can plot this in Matlab, and observe the following:

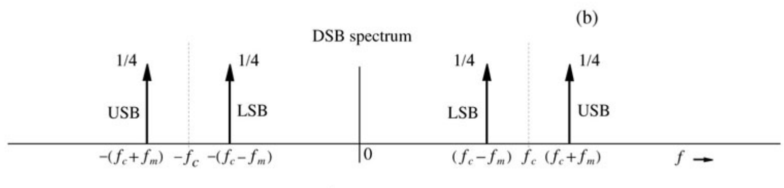
```
% Setup system parameters
fc = 200;    % Carrier Freq
fm = 10;     % Message Freq

% Setup timebase
fs = 10e3;   % Sampling Freq
Tend = 0.2;    % Stop Time
t = 0:1./fs:Tend;   % Time

% Generate the AM Signal
m = cos(2.*pi.*fm.*t);
s = m.*cos(2.*pi.*fc.*t);
```



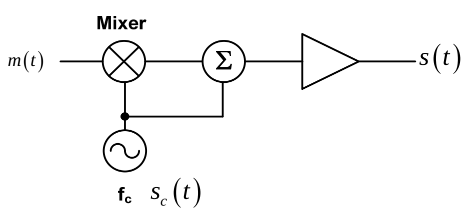
Example Problem Solution

In the frequency domain we have:

$$S(f)=\tfrac{1}{2}m(f-f_c)+\tfrac{1}{2}m(f+f_c)$$
$$S(f)=\tfrac{1}{4}\left[\delta(f-f_c-f_m)+\delta(f-f_c+f_m)\right]+\tfrac{1}{4}\left[\delta(f+f_c-f_m)+\delta(f+f_c+f_m)\right]$$

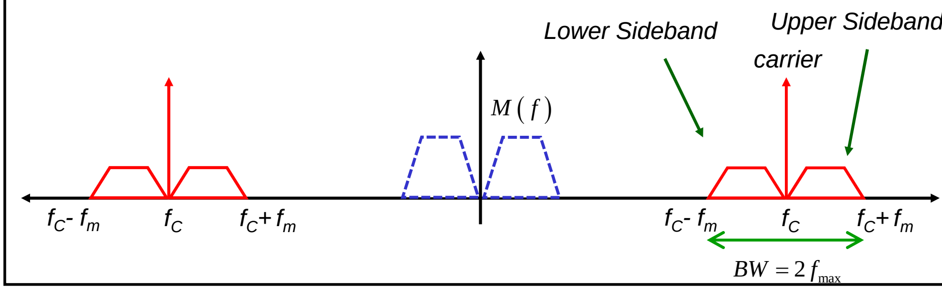


A better version of AM: DSB-TC



Transmit a tone carrier along with the AM modulated message signal.

$$s(t)=\left[A+m(t)\right]\cos(2\pi f_c t) \quad \Leftrightarrow$$
$$S(f)=\tfrac{1}{2}A\delta(f+f_c)+\tfrac{1}{2}A\delta(f-f_c)+\tfrac{1}{2}M(f-f_c)+\tfrac{1}{2}M(f+f_c)$$



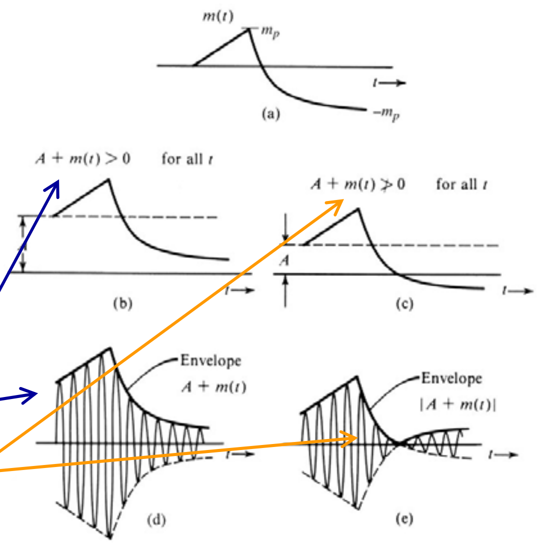
AM Modulation Envelope – Two Cases

$$s(t) = \left[A + m(t)\right]\cos\left(2\pi f_c t\right)$$

The quantity $\left[A+m(t)\right]$ forms an **envelope** that bounds the amplitude of the carrier.

To illustrate

- Sketch $\left[A+m(t)\right]$.
- Sketch $-\left[A+m(t)\right]$.
- Fill in carrier in between.



Easily recoverable

Crossover Distortion

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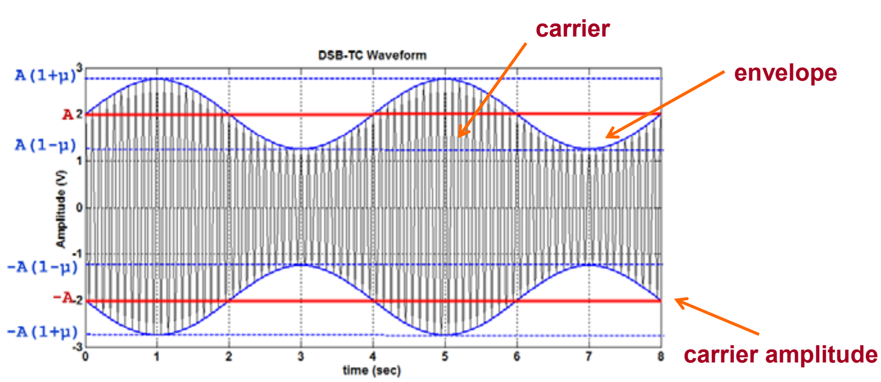
DSB-TC AM in the time domain

To ease analysis, rewrite in terms of **amplitude-normalized message signal** $\tilde{m}(t)$ and modulation index μ .

$$s(t) = A\left[1+\mu\tilde{m}(t)\right]\cos\left(2\pi f_c t\right) \qquad m(t) = m_p\cos\left(2\pi f_m t\right) \qquad \boxed{\mu = \frac{m_p}{A}}$$

Criteria for Envelope Detection: $0 \le \mu \le 1$



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AM Envelope Example (Lathi 4.3)

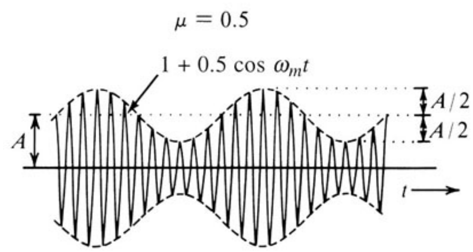
Given: **Suppose** $m(t) = b\cos(2\pi f_m t)$

Find: **Sketch the resulting AM Signal if** $\mu = 0.5$

Note: $\mu = \frac{b}{A} \quad \Rightarrow \quad b = \mu A$

$$m(t) = \mu A\cos(2\pi f_m t)$$

$$s(t) = A\left[1+\mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$



(a)

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AM Envelope Example (Lathi 4.3)

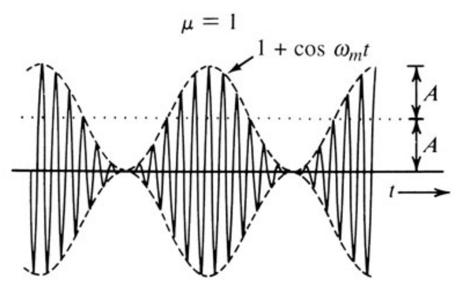
Given: **Suppose** $m(t) = b\cos(2\pi f_m t)$

Find: **Sketch the resulting AM Signal if** $\mu = 1.0$

Note: $\mu = \frac{b}{A} \quad \Rightarrow \quad b = \mu A$

$$m(t) = \mu A\cos(2\pi f_m t)$$

$$s(t) = A\left[1+\mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$



(b)

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AM Power and Efficiency

A carrier makes it easier to demodulate the incoming signal, but we pay a price in terms of efficiency.

Some of the transmitted power is being used to broadcast a pure sinusoid which <u>does not</u> convey any information.

Define power efficiency as: $\eta = \frac{signal\,power}{total\ power} = \frac{P_s}{P_C + P_s}$

Assuming tone modulation, expand the AM equation:

$$s(t) = \underbrace{A\cos(2\pi f_c t)}_{\text{Carrier}} + \underbrace{\tfrac{m_p}{2}\cos\left(2\pi\left(f_c + f_m\right)t\right)}_{\text{USB=UpperSideband}} + \underbrace{\tfrac{m_p}{2}\cos\left(2\pi\left(f_c - f_m\right)t\right)}_{\text{LSB=LowerSideband}}$$

Note:

Power in Carrier: $P_C = \frac{A^2}{2}$

Power in Sidebands: $P_{USB} = P_{LSB} = \frac{\left(\frac{m_p}{2}\right)^2}{2} = \frac{m_p^2}{8}$

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AM Power and Efficiency

If we rewrite the above expressions in terms of the modulation index:

$$s(t) = \underbrace{A\cos(2\pi f_c t)}_{\text{Carrier}} + \underbrace{\tfrac{\mu A}{2}\cos\left(2\pi\left(f_c + f_m\right)t\right)}_{\text{USB=UpperSideband}} + \underbrace{\tfrac{\mu A}{2}\cos\left(2\pi\left(f_c - f_m\right)t\right)}_{\text{LSB=LowerSideband}}$$

$$P_{USB} = P_{LSB} = \frac{\left(\frac{\mu A}{2}\right)^2}{2} = \frac{(\mu A)^2}{8}$$

Thus: $$\eta = \frac{P_s}{P_C + P_s} = \frac{\frac{(\mu A)^2}{8}}{\frac{(\mu A)^2}{8} + \frac{A^2}{2}} = \frac{\frac{(\mu \cancel{A})^2}{8}}{\frac{(\mu \cancel{A})^2}{8} + \frac{\cancel{A}^2}{2}} = \frac{\mu^2}{\mu^2 + 2}$$

Note That: **As the index of modulation decreases, the efficiency will also decrease. In fact, for sinusoidal modulating signals and evelope detection, we find that:**

$$\eta \le 33\% \quad for \quad 0 \le \mu \le 1$$

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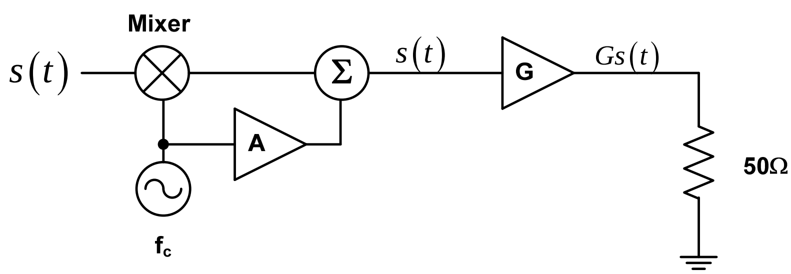
AM Example

Given: AM Transmitter, 1 kW unmodulated output power, 50Ω load. 5V sinusoidal input to the modulator gives the amplitude of each sideband to be 40% of the amplitude of the carrier.

Find: (a) What is the modulation index?
(b) What is the power in the Carrier, USB, and LSB
(c) What is the efficiency of the AM Transmitter.



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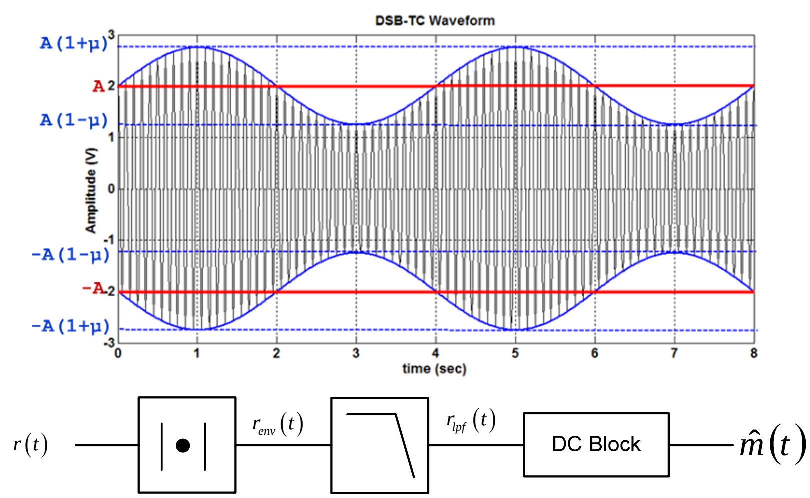
Recovering the Message: Envelope Detector



Observe: If we can trace out the Envelope of the AM signal, we can effectively recover the underlying information signal.

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Envelope Detector Mathematically

Received Signal: $r(t) = A\left[1+\mu m(t)\right]\cos(2\pi f_c t)$

Absolute Value Operation: $r_{env}(t) = A\left|\left[1+\mu m(t)\right]\cos(2\pi f_c t)\right|$

If $1+\mu m(t)$ is constrained to be always positive (i.e., $0 \le \mu \le 1$)

$$r_{env}(t) = A\left[1+\mu m(t)\right]\left|\cos(2\pi f_c t)\right|$$

Note: The full-wave rectified cosine can be expanded in a Fourier Series

$$r_{env}(t) = A\left[1+\mu m(t)\right]\left[a_0 + a_1\cos\left(2\pi(2f_c)t\right) + a_2\cos\left(2\pi(4f_c)t\right) + \ldots\right]$$

LPF: Block everything except the a_0 term.

$$r_{env}(t) = a_0 A\left(1+\mu m(t)\right)$$

DC Block does the rest: $\hat{m}(t) = a_0 \mu m(t)$

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AM Demodulation in Software

```
fs_rf = 1e6;   % sampling frequency of the AM signal
fco = 0.02e6;  % cutoff frequency for the low pass filter
% Will need to downsample the recovered signal to an audio-
% level sampling frequency for output to sound card.
downsample_rate = floor(fs_rf./44.1e3);

% Perform AM Demod using Envelope Detection.
r_env = abs(am_samples); % Envelope Detector
r_lpf = filter_audio(x_env,fco,fs_rf);  % Low Pass Filter
% Downsample to audio sampling frequency
fs_audio = fs_rf./downsample_rate;
m_hat = downsample(r_lpf, downsample_rate);

% Perform AM Demod using Square-Law Detector
r_sq = am_samples.^2; % Square-Law Detector
% Low Pass Filter and Square Root operation
m_hat = sqrt(filter_audio(r_sq,fco,fs_rf));
% Downsample to audio sampling frequency
fs_audio = fs_rf./downsample_rate;
x_bb = downsample(m_hat, downsample_rate);

% Output the demodulated signal to the sound card
sound(m_hat, fs_audio, 16);
```

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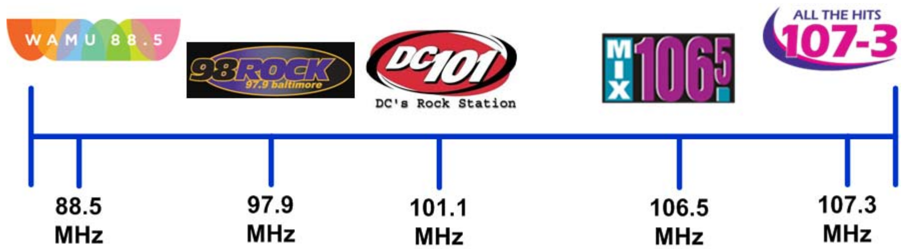
Frequency Division Multiplexing

Consider: MIDN A likes listening to old-fashioned grunge music. MIDN B is more of a modern music fan. How do we satisfy their listening desires?

We could TDM music (0900 Grunge Hour; 1000 Alternative; 1100 Classical), or we could establish multiple stations on different frequencies and multiplex them in the Frequency Domain.



Note: Stations transmit their signals simultaneously in time, but are separated in frequency.

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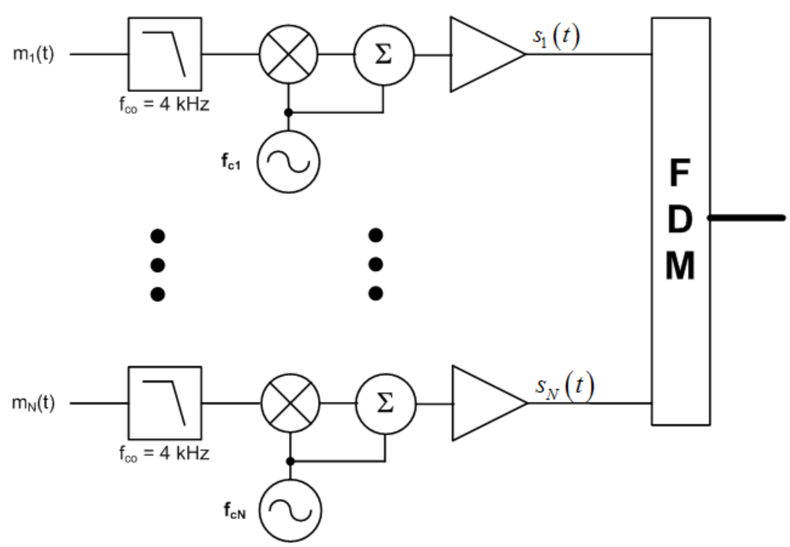
Example: FDM Across the AM Radio Band



$$f_{c_1} = 500\,kHz,\ f_{c_2} = 510\,kHz, \ldots,\ f_{c_N} = 1700\,kHz$$

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