A Synchronous Machine Model With Saturation and Arbitrary Rotor Network Representation

Dionyssios C. Aliprantis, Member, IEEE, Scott D. Sudhoff, Senior Member, IEEE, and Brian T. Kuhn, Member, IEEE

Abstract—This paper addresses equivalent circuit and magnetic saturation issues associated with synchronous machine modeling. In the proposed synchronous machine model, the rotor equivalent circuits are replaced by arbitrary linear networks. This allows for elimination of the equivalent circuit parameter identification procedure since the measured frequency response may be directly embedded into the model. Magnetic saturation is also represented in both the q- and d-axis. The model is computationally efficient and suitable for dynamic time-domain power system studies.

Index Terms—Electric machines, modeling, power system stability, realization theory, rotating machine transient analysis, synchronous generator transient analysis, synchronous machines, transfer functions.

\[ A_x, B_x, C_x \] Rotor network state equation matrices \( x = \{q_d, d\} \).

\[ B_{dl}, B_{d2} \] Column vectors of the \( N_d \times 2 \) matrix \( B_d \).

\[ C_{dl}, C_{d2} \] Row vectors of the \( 2 \times N_d \) matrix \( C_d \).

\( E \) Voltage-behind-reactance of an average-value exciter-rectifier system representation (in volts).

\( i_{fd}, i_{md} \) Field winding current (in amperes).

\( i_{qu}, i_{ru} \) qd-axes magnetizing branch current (in amperes).

\( i_{qr}, i_{rd} \) Current flowing into the armature side of the rotor networks (in amperes).

\( i_{qs} \) Stator windings current \( x = \{a, b, c, q_d\} \) (in amperes).

\( J \) Moment of inertia (kg \( \cdot \) m²).

\( K_d \) Park’s transformation matrix.

\( L_{ds} \) Stator winding leakage inductance (in Henries).

\( L_{tr} \) Transient inductance of an average-value exciter-rectifier system representation (in Henries).

\( m_Y \) Minimal polynomial of transfer function matrix \( Y \).

\( N_{af}, N_{aq}, N_{aq} \) Number of qd-axes rotor network states.

\( n_q, n_d \) Order of qd-axes rotor network minimal polynomials.

\( P \) Number of poles.

\( P_e \) Electric power supplied to coupling field (in watts).

\( p_Y \) Characteristic polynomial of transfer function matrix \( Y \).

\( R_e \) Effective resistance of an average-value exciter-rectifier system representation (in ohms).

\( r_{fd}, r_{ms}, r_{md} \) Field winding resistance (in ohms).

\( s \) Stator winding resistance (in ohms).

\( \omega \) Complex frequency (in radians per second).

\( \psi_{fd}, \psi_{md}, \psi_{ms} \) Armature voltage \( x = \{a, b, c, q_d\} \) (in volts).

\( \psi_{qs} \) Coupling field energy (in Joules).

\( \psi_{rs}, \psi_{rd} \) qd-axes rotor network states.

\( \psi_{w_s} \) Prime-mover torque (N-m).

\( \psi_{w_d} \) Field winding voltage (in volts).

\( \psi_{w_z} \) qd-axes magnetizing branch voltages (in volts).

\( \psi_{w_z} \) Armature voltage \( x = \{a, b, c, q_d\} \) (in volts).

\( \psi_{w_s} \) Coupling field energy (in Joules).

\( W_f \) damping rotor network states.

\( \alpha, \beta \) \( d \)-axis rotor two-port network transfer function matrix \( \Omega^{-1} \).

\( \alpha_n, \alpha_{nd} \) q-axis rotor network transfer function \( \Omega^{-1} \).

\( \alpha_{nq}, \alpha_{ndq} \) Constant related to the \( d \)-axis rotor network admittance at dc \( \Omega^{-1} \).

\( \alpha_{ny}, \alpha_{nyd} \) Elements (transfer functions) of \( Y_d \).

\( \alpha_{nq}, \alpha_{ndq} \) Incremental inverse magneto-motive force matrix.

\( \beta_{nq}, \beta_{ndq} \) Inverse magneto-motive force \( \Omega^{-1} \).

\( \beta_{nq}, \beta_{ndq} \) Saliency-dependent magnetizing path characteristic constants.

\( \beta_{nq}, \beta_{ndq} \) Constants of the \( d \)-axis rotor network transfer functions \( j = 1, \ldots, n_q - 1 \).

\( \beta_{nq}, \beta_{ndq} \) Constants of the \( q \)-axis rotor network transfer functions \( j = 1, \ldots, n_q - 1 \).

\( \theta_r \) Electrical rotor position (in radians).

\( \theta_{rm} \) Mechanical rotor position (in radians).

\( \lambda_{es}, \lambda_{esd}, \lambda_{esq} \) Stator windings leakage flux linkage \( x = \{a, b, c, q_d\} \) (V \( \cdot \) s).

\( \lambda_{ms}, \lambda_{msd}, \lambda_{msq} \) Effective magnetizing flux linkage \( V \cdot s \).

\( \lambda_{md}, \lambda_{mdq} \) qd-axes magnetizing flux linkage \( V \cdot s \).

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D. C. Aliprantis is with the Greek Armed Forces (e-mail: aliprantis@alumni.purdue.edu).

S. D. Sudhoff is with the Department of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907-1285 USA (e-mail: sudhoff@ecn.purdue.edu).

B. T. Kuhn is with SmartSpark Energy Systems, Inc., Champaign, IL 61820 USA (e-mail: b.kuhn@smartsparkenergy.com).

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\[ \lambda_{x8} \quad \text{Stator windings flux linkage} \quad x = \{a, b, c, q, d\} [V \cdot s]. \]

\[ \omega_r \quad \text{Electrical rotor speed (in radians per second).} \]

\[ \dot{\omega}_{rm} \quad \text{Mechanical rotor speed (in radians per second).} \]

I. INTRODUCTION

MATHEMATICAL models of physical devices are abstractions of reality. Their purpose is to portray, with sufficient accuracy and simplicity, the important characteristics of the systems under consideration. For synchronous machines, relatively low-order models may be employed to predict their steady-state or dynamic behavior, with the orthogonal \(qd\)-axes theory [1] used in the majority of cases. Over the years, a plethora of models has been proposed to represent phenomena related to the distributed circuit behavior of rotor windings, magnetic saturation, and saliency-related issues. However, the common thread among them is that they were focused on—and, thus, limited by—the specific problem at hand. This work provides an integrated perspective on synchronous machine modeling, realizing the need for a generalized formulation that is compatible with modern computational tools.

Synchronous machines are usually employed as generators connected to a power system. In the current IEEE Standard [2], it is acknowledged that synchronous machines may be accurately modeled by two lumped-parameter equivalent circuits representing the \(q\) and the \(d\)-axis, as shown in Fig. 1. The number of rotor “damper” branches is selected in accordance with the rotor design. In low-order models, these branches correspond to the actual amortisseur windings; higher order models utilize these branches to represent the distributed effects in the rotor iron.

The rotor-related part of the equivalent circuits proposed in the standard differs from other “conventional” equivalent circuits [1], [3] since it contains the “differential leakage” inductance \(L_{f12d}\). Its use was proposed by Canay in his seminal paper of 1969 [4], to signify unequal coupling between the rotor windings and the stator, and between the rotor circuits themselves. A variety of other circuit structures has also been proposed in the literature [5]–[8].

New circuits having \(L_{f12d}\) equal to zero may be obtained with an appropriate transformation, as was shown by Canay in his original paper, and again by Kirtley in [9]. This corresponds to forcing equal magnetic coupling between all rotor circuits and the stator winding—often called the “equal mutuals” base. However, the equal mutuals base is restricted to the case of three coupled circuits; that is, when the rotor has, at most, a field winding and one damper [10]. The most unconstrained form of equivalent circuit is one with differential leakage inductances between all damper circuits. Important theoretical results about equivalent circuits were published in [11] and [12] where it was shown that there is no unique RL ladder network representation of a given rotor two-port network with a prescribed impedance matrix.

A further disadvantage of using a mathematical transformation to ensure a specific network structure (one with no differential leakage) is that it causes the new values of the stator leakage and magnetizing inductance to lose their original physical meaning.

If magnetizing path saturation is to be considered—either by adjusting the magnetizing branch inductance or by replacing it by a nonlinear element—the physical significance of these elements should be preserved [13]. Although some physical significance may be attributed to the parameters of models with up to three “damper” circuits [14], it is gradually lost as the order is further increased, as is recommended for accurate transient stability or subsynchronous resonance studies [15].

The purpose of this work is to address these equivalent circuit issues. In the model that is set forth herein, the equivalent-circuit structure of the rotor is replaced by a completely arbitrary linear circuit, a two-port network for the \(d\)-axis, and a single branch for the \(q\)-axis, as shown in Fig. 2. This approach has been adopted from previous modeling work on induction machines [16], emphasizing the importance of the rotor’s actual input-output behavior rather than the debatable physical meaning of the equivalent circuit parameters. It offers the advantage that once the transfer functions of these linear circuits are determined—using standstill frequency response tests, for example, they may be immediately incorporated in the model [17]. It does not require the tedious and time-consuming process of equivalent circuit parameter identification to which numerous studies have been devoted [18], [19]. Indeed, this problem is highly nonlinear, and the solution algorithms often face convergence issues, accentuated by the fact that there is no unique solution. Furthermore, for modern computer simulation software, such as Matlab/Simulink [20] or ACSL [21], the entry of a specific equivalent circuit
structure is not required. Rather, it is more efficient to directly provide a state-space representation of the system to be simulated. The proposed model possesses such a form.

Apart from equivalent circuit-related problems, synchronous machine research has also focused on the accurate incorporation of magnetic saturation, which has been shown to considerably affect their operating characteristics [22]. Different methods to model this complicated phenomenon were analyzed in a variety of publications, such as [32]–[34]. An alternative approach is to employ artificial neural networks to represent the magnetizing path saturation [32]–[34], or the variation of rotor parameters [35, 36].

The form of the proposed model readily lends itself to the modification of the magnetizing inductances for saturation (and cross-saturation) modeling in both axes. Recent evidence suggests that lumping saturation effects in the magnetizing branch is a reasonable assumption [37, 38] and this hypothesis is further corroborated by our results. Although the leakage flux paths may saturate due to excessive current or by high magnetizing flux levels, synchronous generators are usually operated close to their nominal values of flux, so the leakage inductance may be assumed to remain constant for a wide range of studies. A derivation of the model’s torque equation, and restrictions on the magnetizing inductances arising from the assumption of a lossless (conservative) coupling field will be presented in later sections.

II. PROPOSED SYNCHRONOUS MACHINE MODEL

From a computer simulation point of view, the proposed model is of the voltage-in, current-out type; the stator and field winding voltages are the inputs, while the stator and field winding currents are the outputs. The model is computationally efficient in the sense that it is noniterative at each time step, and uses only a minimum number of states obtained from a minimal realization of the measured input–output behavior. It is especially suitable for time-domain dynamic simulations of power systems, as well as for the design and optimization of control schemes.

A. Notation

In order to assist the reader, the paper’s nomenclature is defined. Throughout this work, matrix and vector quantities appear in bold font. The primed rotor quantities denote referral to the rotor through the turns ratio, which is defined as the ratio of field-to-armature turns \( N_{af} = N_{fr}/N_s \) [39]. The analysis takes place in the rotor reference frame; the often used “\( r \)” superscript [1] is omitted for convenience. The electrical rotor position \( \theta_{r} \) and electrical rotor speed \( \omega_{r} \) are \( P/2 \) times the mechanical rotor position \( \theta_{rm} \) and mechanical speed, \( \omega_{rm} \), where \( P \) is the number of poles. The transformation of stationary \( abc \) to \( dq \) variables in the rotor reference frame is defined by [1]

\[
\mathbf{f}_{g\alpha\beta} = \mathbf{K}_s(\theta_r) \mathbf{f}_{abc}
\]

where

\[
\mathbf{K}_s(\theta_r) = \frac{2}{3} \begin{bmatrix}
\cos \theta_r & \cos \left( \theta_r - \frac{2\pi}{3} \right) & \cos \left( \theta_r + \frac{2\pi}{3} \right) \\
\sin \theta_r & \sin \left( \theta_r - \frac{2\pi}{3} \right) & \sin \left( \theta_r + \frac{2\pi}{3} \right)
\end{bmatrix}.
\]

B. Voltage Equations

The stator voltage equations may be expressed in \( abc \) variables as

\[
\mathbf{v}_{abc} = r_s \mathbf{i}_{abc} + \frac{d}{dt} \mathbf{\lambda}_{abc}
\]

where \( \mathbf{v}_{abc} \), \( \mathbf{i}_{abc} \), and \( \mathbf{\lambda}_{abc} \) denote stator winding (phase-to-neutral) voltages, currents flowing into the machine terminals, and flux linkages, respectively, and \( r_s \) is the stator winding resistance. Transforming (3) to the rotor reference frame yields

\[
\mathbf{v}_{d\alpha\beta} = r_s \mathbf{i}_{d\alpha\beta} + \omega_{r} \mathbf{\lambda}_{d\alpha\beta} + \frac{d}{dt} \mathbf{\lambda}_{d\alpha\beta}
\]

where \( \mathbf{\lambda}_{d\alpha\beta} = [\mathbf{\lambda}_{d\alpha} - \mathbf{\lambda}_{d\beta}]^T \). It will be assumed hereafter that the zero-sequence variables can be neglected.

The state equations of the \( d \)-axis two-port network (Fig. 2) may be expressed by a linear system of order \( N_q \) as

\[
\frac{d}{dt} \mathbf{x}_d = \mathbf{A}_d \mathbf{x}_d + \mathbf{B}_d \begin{bmatrix} v_{md} \\ v_{ld} \end{bmatrix}
\]

\[
= \mathbf{A}_d \mathbf{x}_d + \begin{bmatrix} \mathbf{B}_{dl} & \mathbf{B}_{d2} \end{bmatrix} \begin{bmatrix} v_{md} \\ v_{ld} \end{bmatrix}
\]

\[
\begin{bmatrix}
i_{dr} \\
i_{fdr}
\end{bmatrix} = \mathbf{C}_d \mathbf{x}_d = \begin{bmatrix} \mathbf{C}_{d1} & \mathbf{C}_{d2} \end{bmatrix} \mathbf{x}_d
\]

where \( \mathbf{x}_d \in \mathbb{R}^{N_d \times 1}, \mathbf{A}_d \in \mathbb{R}^{N_d \times N_d}, \mathbf{B}_d \in \mathbb{R}^{N_d \times 2}, \) and \( \mathbf{C}_d \in \mathbb{R}^{2 \times N_d} \). The contents of \( \mathbf{B}_d \) and \( \mathbf{C}_q \) were written as the column vectors \( \mathbf{B}_{dl}, \mathbf{B}_{d2} \), and the row vectors \( \mathbf{C}_{d1}, \mathbf{C}_{d2} \). The magnetizing branch voltages are equal to the derivatives of the corresponding magnetizing flux linkages, which will be computed in the ensuing analysis.

The voltage \( i_{ld} \) may be eliminated from the equations, since it is related to the field voltage and current by \( i_{ld} = i_{fdr} - i_{fdr} / i_{fdr} \). Using this, the following state equation is obtained:

\[
\frac{d}{dt} \mathbf{x}_d = (\mathbf{A}_d - r_{fdr} \mathbf{B}_d \mathbf{C}_d) \mathbf{x}_d + \begin{bmatrix} \mathbf{B}_{d1} & \mathbf{B}_{d2} \end{bmatrix} \begin{bmatrix} v_{md} \\ v_{ld} \end{bmatrix}.
\]

Similarly, the \( q \)-axis state equations are

\[
\frac{d}{dt} \mathbf{x}_q = \mathbf{A}_q \mathbf{x}_q + \mathbf{B}_q v_{mq}
\]

\[
i_{qr} = \mathbf{C}_q \mathbf{x}_q
\]

where \( \mathbf{x}_q \in \mathbb{R}^{N_q \times 1}, \mathbf{A}_q \in \mathbb{R}^{N_q \times N_q}, \mathbf{B}_q \in \mathbb{R}^{N_q \times 1}, \mathbf{C}_q \in \mathbb{R}^{1 \times N_q}, \) and \( N_q \) is the order of the \( q \)-axis system.

It will be useful to derive expressions for the current derivatives. From (6)–(9)

\[
\frac{d}{dt} \begin{bmatrix} i_{dr} \\ i_{fdr} \end{bmatrix} = \mathbf{C}_d (\mathbf{A}_d - r_{fdr} \mathbf{B}_d \mathbf{C}_d) \mathbf{x}_d
\]

\[
+ \mathbf{C}_d \mathbf{B}_d \begin{bmatrix} v_{md} \\ v_{ld} \end{bmatrix}
\]

\[
\frac{d}{dt} i_{qr} = \mathbf{C}_q \mathbf{A}_q \mathbf{x}_q + \mathbf{C}_q \mathbf{B}_q v_{mq}.
\]
C. Rotor Transfer Functions

In the frequency domain, the rotor currents and voltages are related by the transfer functions

\[
\begin{bmatrix}
\vec{y}_{dr} \\
\vec{y}_{fdr}
\end{bmatrix} = \begin{bmatrix} Y_{d}(s) \\ Y_{f}(s) \end{bmatrix} \begin{bmatrix}
\vec{\delta}_{md} \\
\vec{\delta}_{ld}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\vec{y}_{11}(s) \\
\vec{y}_{12}(s) \\
\vec{y}_{22}(s)
\end{bmatrix} = \begin{bmatrix} Y_{11}(s) \\ Y_{12}(s) \\ Y_{22}(s) \end{bmatrix} \begin{bmatrix}
\vec{\delta}_{md} \\
\vec{\delta}_{ld}
\end{bmatrix}
\]

(12)

\[
\gamma_{q} = Y_{q}(s)\vec{v}_{mq}
\]

(13)

where \( s = j\omega \) and the tilde are used to denote phasor quantities. The elements of \( Y_{d}(s) \) and \( Y_{f}(s) \), \( y_{11}(s) \), \( y_{12}(s) \), and \( y_{22}(s) \) are strictly proper rational polynomial functions of the complex frequency \( s \) of the form

\[
y(s) = \frac{a_{0} + a_{1}s + \cdots + a_{n}s^{n-1}}{b_{0} + b_{1}s + \cdots + b_{n}s^{n}}.
\]

(14)

The coefficients \( a_{0} \) and \( b_{0} \) may not be simultaneously equal to zero since that would result in a pole-zero cancellation at the origin. This is a representation of a most general form, but a simplified version may be obtained for the \( d \)-axis if the physics of the rotor are taken into account.

To this end, by applying Faraday’s law to the field winding (i.e., \( v_{fdr} = i_{fdr} - \frac{dN_{fd}}{dt} \)), it can be seen that there is no dc voltage drop besides the ohmic drop of the winding’s resistance (which is external to the two-port network). This implies that there exists a direct path for dc current between the primary and secondary sides of the two-port network.

Under this assumption, the constants \( a_{0} \) of the polynomials will all have the same absolute value, and the system will possess a pole at the origin. To see this, assume that the field side of the admittance block is short circuited (\( \vec{v}_{ld} = 0 \)) so that

\[
\begin{align*}
\gamma_{dr} &= \vec{y}_{11}(s)\vec{\delta}_{md} \\
\gamma_{fdr} &= \vec{y}_{12}(s)\vec{\delta}_{md}.
\end{align*}
\]

(15)

(16)

As the frequency approaches zero (\( s \to 0 \)), the hypothesis that at low frequency the two-port network behaves as an ideal series inductor implies that \( \gamma_{dr} \approx \gamma_{fdr} \approx 0 \).

\[
\lim_{s \to 0} \vec{y}_{11}(s) = -\lim_{s \to 0} \vec{y}_{12}(s).
\]

(17)

Since the denominator of (14) corresponds to the least common denominator of all elements of \( Y_{d}(s) \), the \( a_{0} \) element of \( \vec{y}_{11}(s) \) is equal to \( -a_{0} \) of \( \vec{y}_{12}(s) \). Similarly, setting \( \vec{\delta}_{md} = 0 \) and letting the frequency approach zero yields

\[
\lim_{s \to 0} \vec{y}_{22}(s) = -\lim_{s \to 0} \vec{y}_{22}(s).
\]

(18)

Thus, the \( a_{0} \) element of \( \vec{y}_{22}(s) \) is equal to \( -a_{0} \) of \( \vec{y}_{22}(s) \). This observation, coupled with the fact that these admittances become infinite as \( s \to 0 \) (again because of the assumption that the network acts as an ideal series inductance at sufficiently low frequency), implies that the \( d \)-axis transfer functions must have the following forms:

\[
y_{1}(s) = \frac{1 + \alpha_{1}s + \cdots + \alpha_{n}s^{n-1}}{s(1 + \beta_{1}s + \cdots + \beta_{n}s^{n-1})}
\]

(19)

\[
y_{2}(s) = -\frac{1 + \beta_{1}s + \cdots + \beta_{n}s^{n-1}}{s(1 + \delta_{1}s + \cdots + \delta_{n}s^{n-1})}
\]

(20)

\[
y_{22}(s) = \frac{1 + \gamma_{1}s + \cdots + \gamma_{m}s^{m-1}}{s(1 + \delta_{1}s + \cdots + \delta_{n}s^{n-1})}
\]

(21)

The formulation of the \( q \)-axis transfer function is more straightforward due to the absence of a field winding. In particular, the \( q \)-axis transfer function may be expressed as

\[
y_{q}(s) = \frac{1 + \epsilon_{1}s + \cdots + \epsilon_{m-1}s^{m-2}}{s(1 + \zeta_{1}s + \cdots + \zeta_{n}s^{n-1})}.
\]

(22)

In this case, poles and zeros at the origin are not allowed.

D. Realization Theory

The rotor transfer functions are the starting point for the computation of the time-domain state matrices, as used in (7) and (8). The problem is to determine an internal, space-state description of a linear system, given its external, input–output description. This is the subject of system realization theory [40].

Numerous algorithms exist for deriving a realization. However, for increased computational efficiency, it is desirable to obtain a system of the least possible order, a so-called minimal realization. In contrast to the single-input/single-output case, where it is rather straightforward to obtain a realization, the multiple-input/multiple-output case (like the two-port \( d \)-axis rotor system) is more complicated. The difficulty arises when determining the minimal realization order. Consider a transfer function written in the form \( Y(s) = N(s)/M(s) \), where \( N(s) \) is a matrix of polynomials and \( M(s) \) is the minimal polynomial of \( Y(s) \). The roots of \( M(s) \) constitute a subset of the eigenvalues of the minimal realization; hence, a minimal polynomial with two roots may correspond to a system with a second-, third-, or fourth-order minimal realization.

In the case of the proposed model, the transfer function matrix entries are the functions (19)–(21), the degree of the minimal polynomial is \( n_{rd} \), and the order of the minimal realization is \( N_{d} = 2n_{rd} - 1 \). This is proved using the following theorem:

The order of any minimal realization is equal to the degree of the characteristic polynomial \( p_{y}(s) \) of the transfer function matrix \( Y(s) \) [40, p. 397].

It remains to compute \( p_{y}(s) \), given the functional forms (19)–(21). Let us write the \( d \)-axis transfer function as

\[
Y_{d}(s) = \frac{1}{m_{y}(s)}N(s) = \frac{1}{m_{y}(s)}\begin{bmatrix} n_{11}(s) & n_{12}(s) \\ n_{21}(s) & n_{22}(s) \end{bmatrix}
\]

(23)

where

\[
m_{y}(s) = s^{(m_{1}-1)} + \cdots + \delta_{n_{1}-1}.
\]

(24)

It is assumed that no common factors exist between \( n_{11}(s) \), \( n_{12}(s) \), \( n_{21}(s) \), and \( m_{y}(s) \). To obtain \( p_{y}(s) \), it is necessary to compute all nonzero minors of \( Y_{d}(s) \). The first-order minors are the entries of \( Y_{d}(s) \), and their monic least common denominator is (by definition) the minimal polynomial \( m_{y}(s) \). The second-order minor is equal to the determinant

\[
D(s) = \frac{n_{11}(s)n_{22}(s) - n_{12}(s)n_{21}(s)}{m_{y}^{2}(s)}.
\]

(25)

Taking into account the specified forms of the transfer functions (19)–(21), it is readily shown that the numerator of \( D(s) \) has a

1The minimal polynomial is defined herein as the monic least common denominator of the transfer function entries. A polynomial is monic when the highest order term has a coefficient of one.

2The characteristic polynomial \( p_{y}(s) \) is defined as the monic least common denominator of all nonzero minors of \( Y(s) \).
root at the origin, which cancels out with one of the two zero roots of \(n_f \gamma^2(s)\). Hence
\[
pr(s) = s \left( s^{n_d-1} + \cdots + \delta^{n_d-1}_{n_d} \right)^2.
\]
(26)
The degree of the characteristic polynomial—the order of the minimal realization—is thus \(N_d = 2n_d - 1\). An algorithm to obtain a minimal state-space realization from the transfer function matrix is described in the Appendix.

The order of the \(q\)-axis realization, which is a single-input/single-output system, is \(N_q = n_q\).

E. Leakage and Magnetizing Path Magnetics

The stator flux linkage is separated into leakage and magnetizing flux terms as
\[
\lambda_{dls} = \lambda_{dls}^\ast + \lambda_{mqd} = L_{ds}i_{dls} + \lambda_{mqd}
\]
(27)
where \(\lambda_{dls}^\ast\) is the stator leakage flux, and \(\lambda_{mqd}\) is the magnetizing flux. The same value of leakage inductance \(L_{ds}\) is used for both the \(q\)- and \(d\)-axis.

The functional forms of main path saturation in the proposed model are similar to [30]. Since in the general case of a salient rotor machine, the magnetizing magnetomotive force (mmf) and flux vectors are not aligned, the following relations may be assumed for the magnetizing path:
\[
i_{mq} = \Gamma_{mq}(\lambda_m)\lambda_{mq}
\]
(28)
\[
i_{md} = \Gamma_{md}(\lambda_m)\lambda_{md}
\]
(29)
The magnitude of the effective magnetizing flux vector is
\[
\lambda_m = \sqrt{\lambda_{md}^2 + \alpha^2 \lambda_{mq}^2}
\]
(30)
where \(\alpha\) is a saliency-dependent parameter. Differentiating (28), (29) with respect to time yields
\[
\frac{d}{dt}i_{mqd} = \Gamma_{mq}(\lambda_m)\frac{d}{dt}\lambda_{mqd}
\]
(31)
where
\[
\Gamma_{mq}(\lambda_{mqd}) = \left[ \frac{d\lambda_{mqd}}{d\lambda_m} \alpha \frac{\lambda_{mq}^2}{\lambda_m} + \frac{d\lambda_{mqd}}{d\lambda_m} \lambda_{mq} \frac{d\lambda_{md}}{d\lambda_m} \lambda_{md} \lambda_{mqd} \right].
\]
(32)
The “\(\triangle\)” subscript denotes incremental value.

F. Torque Equation

The proposed model’s electromagnetic (EM) torque equation may be derived by examining the energy balance of the machine’s coupling field [1]. The coupling field is created by the magnetizing flux, which links both stator and rotor circuits.

The electric power supplied to the coupling field is equal to the input power, minus the power lost in the stator and field resistances, minus the power that supplies the stator leakage field, minus the power that is dissipated or stored inside the rotor admittance block. Using the equivalent circuit of Fig. 2 and (27), the electric power supplied to the coupling field may be expressed after manipulation as
\[
P_e = \frac{3}{2} \Omega_r (i_{qs} \lambda_{md} - i_{ds} \lambda_{mq}) + \frac{3}{2} \left( i_{rq} \frac{d}{dt} \lambda_{mq} + i_{md} \frac{d}{dt} \lambda_{md} \right)
\]
(33)
The electrical energy provided to the coupling field is partly stored, and the rest is transferred to the mechanical system. If \(W_f\) denotes the coupling field energy and \(T_e\) the EM torque, then the power balance may be written as
\[
P_e = \frac{d}{dt} W_f + T_e \omega_{rm}
\]
(34)
where the product \(T_e \omega_{rm}\) is positive when energy is supplied to the rotor. Equating (33) and (34), and solving for the rate of change of the field energy yields
\[
\frac{d}{dt} W_f = \left[ \frac{3}{2} (i_{qs} \lambda_{md} - i_{ds} \lambda_{mq}) - T_e \frac{2}{P} \right] \frac{d}{dt} \theta_r
\]
(35)
\[
+ \left( \frac{3}{2} i_{rq} \right) \frac{d}{dt} \lambda_{mq} + \left( \frac{3}{2} i_{md} \right) \frac{d}{dt} \lambda_{md}.
\]
Hence, the differential change of the coupling field energy may be written as
\[
dW_f = \frac{\partial W_f}{\partial \theta_r} d\theta_r + \frac{\partial W_f}{\partial \lambda_{mq}} d\lambda_{mq} + \frac{\partial W_f}{\partial \lambda_{md}} d\lambda_{md}
\]
(36)
where
\[
\frac{\partial W_f}{\partial \theta_r} = \frac{3}{2} (i_{qs} \lambda_{md} - i_{ds} \lambda_{mq}) - T_e \frac{2}{P}
\]
(37)
\[
\frac{\partial W_f}{\partial \lambda_{mq}} = \frac{3}{2} i_{mq}
\]
(38)
\[
\frac{\partial W_f}{\partial \lambda_{md}} = \frac{3}{2} i_{md}.
\]
(39)
The change of coupling field energy from an initial state \(\{\theta_{q0}, \lambda_{mq0}, \lambda_{md0}\}\) to an arbitrary final state \(\{\theta_r, \lambda_{mq}, \lambda_{md}\}\) is obtained by integrating (36)
\[
W_f(\theta_r, \lambda_{mq}, \lambda_{md})
= W_{f0} + \int_{\theta_{q0}}^{\theta_r} \left[ \frac{3}{2} (i_{qs} \lambda_{md} - i_{ds} \lambda_{mq}) - T_e \frac{2}{P} \right] d\theta_r
\]
+ \frac{3}{2} \int_{\lambda_{mq0}}^{\lambda_{mq}} i_{mq} d\lambda_{mq} + \frac{3}{2} \int_{\lambda_{md0}}^{\lambda_{md}} i_{md} d\lambda_{md}.
\]
(40)
Since the field is assumed to be conservative, the integration may be performed over an arbitrary trajectory. Assume that the initial energy is \(W_{f0} = 0\), integrate the first term from \(\theta_{q0}\) to \(\theta_r\), while the fluxes are maintained at zero—which forces \(T_e\) to be zero as well. This transition does not change the field energy. Then, consecutively integrate each flux from zero to an arbitrary final value, while keeping \(\theta_r\) and the other flux constant. Recall that the transformation to the rotor reference frame eliminates the dependence of the magnetizing inductances from the rotor position. The magnetizing currents are independent of \(\theta_r\)—as in (28) and
Therefore, the final value of field energy \( W_f(\theta_r, \lambda_{mq}, \lambda_{md}) \) is independent of the angular position \( \theta_r \), that is
\[
\frac{\partial W_f}{\partial \theta_r} = 0. \tag{41}
\]
This observation coupled with (37) yields the following well-known expression for the EM torque:
\[
T_e = \frac{3}{2} \frac{P}{2} (\lambda_{qs} \lambda_{md} - i_{ds} \lambda_{mq}). \tag{42}
\]
For generator action (and \( \omega_r > 0 \)), the torque will be negative.

G. Restrictions on the Inverse Magnetizing Inductances

In (28) and (29), the inverse magnetizing inductances were defined as any arbitrary function of flux. However, to be consistent with the assumption of a lossless coupling field, certain modeling restrictions must be imposed [41], [42].

Specifically, the coupling field’s energy expression, which in view of (41), has become
\[
W_f(\theta_r, \lambda_{mq}, \lambda_{md}) = W_{0f} + \frac{3}{2} \lambda_{mq} \int_{\lambda_{m0}}^{\lambda_{mq}} d\lambda_{mq} + \frac{3}{2} \lambda_{md} \int_{\lambda_{md0}}^{\lambda_{md}} d\lambda_{md}, \tag{43}
\]
must satisfy the requirements of a conservative field. A necessary and sufficient condition for this is [43]
\[
\frac{\partial i_{mq}}{\partial \lambda_{md}} = \frac{\partial i_{md}}{\partial \lambda_{mq}}. \tag{44}
\]
Substitution of (28) and (29) into (44) yields
\[
\frac{d\lambda_{mq}}{d\lambda_{md}} \frac{\lambda_{mq}}{\lambda_{m}} = \frac{d\lambda_{md}}{d\lambda_{mq}} \frac{\alpha \lambda_{mq}}{\lambda_{m}} \frac{\lambda_{md}}{\lambda_{md0}}. \tag{45}
\]
Canceling common terms and integrating both sides yields
\[
\Gamma_{mq}(\lambda_m) = \alpha \Gamma_{md}(\lambda_m) + \beta, \quad \beta \in \mathbb{R}. \tag{46}
\]
This restriction has to be enforced during the magnetizing characteristics’ curve-fitting procedure. Note that it also renders the incremental inverse inductance matrix \( \Gamma_{mq} \) symmetric. The method for determining the constants \( \alpha \) and \( \beta \) is described in detail in [17].

H. Model Integration

The state variables are selected as \( x = \{\lambda_{mq}, x_q, x_d\} \). The goal of the ensuing analysis is the formulation of equations for the time derivatives of the state variables.

First, note that both the rotor and magnetizing currents have been previously expressed as functions of the states: the rotor currents are given in terms of the rotor admittance states from (6) and (9); the magnetizing currents depend on the magnetizing flux states, as seen from (28)–(30). Hence, the stator currents
\[
i_{qs} = i_{mq} + i_{qr} \tag{47}
i_{ds} = i_{md} + i_{dr} \tag{48}
\]
are also functions of state variables.

Next, the time derivatives of the stator flux linkages are calculated from (4)
\[
\frac{d}{dt}\lambda_{ds} = v_{ds} - r_s i_{ds} + \omega_r \lambda_{qs} \tag{49}
\]
\[
\frac{d}{dt}\lambda_{qs} = v_{qs} - r_s i_{qs} - \omega_r \lambda_{ds} \tag{50}
\]
as functions of state variables and model inputs (the stator voltages). However, they are only evaluated as an intermediate calculation; they are not integrated since \( \lambda_{ns} \) are not states.

The differentiation with respect to time of (27) yields an alternate expression for the derivatives of the stator flux linkages [cf. (49) and (50)]
\[
\frac{d}{dt}\lambda_{ds} = L_{ds} \frac{d}{dt}(i_{md} + i_{dr}) + \frac{d}{dt}\lambda_{md} \tag{51}
\]
\[
\frac{d}{dt}\lambda_{qs} = L_{qs} \frac{d}{dt}(i_{mq} + i_{qr}) + \frac{d}{dt}\lambda_{mq}. \tag{52}
\]
The derivatives of the rotor and magnetizing currents may be evaluated using (10) and (11) and (31), so
\[
\frac{d}{dt}\lambda_{ds} = L_{ds} \left[ \Gamma_{mq}(\lambda_m) \frac{d}{dt}\lambda_{mq} + \Gamma_{md}(\lambda_m) \frac{d}{dt}\lambda_{md} + C_{dl} (A_d - r_{fdr} B_d C_{d2}) x_d + C_{dl} B_{d1} \frac{d}{dt}\lambda_{md} + C_{dl} B_{d2} r_{fdr} x_d \right] + \frac{d}{dt}\lambda_{md} \tag{53}
\]
\[
\frac{d}{dt}\lambda_{qs} = L_{qs} \left[ \Gamma_{mq}(\lambda_m) \frac{d}{dt}\lambda_{mq} + \Gamma_{md}(\lambda_m) \frac{d}{dt}\lambda_{md} + C_q A_q x_q + C_q B_q \frac{d}{dt}\lambda_{mq} \right] + \frac{d}{dt}\lambda_{mq}. \tag{54}
\]
The following linear system of equations may therefore be formulated:
\[
L_{ds} \Gamma_{mi}(\lambda_m) \frac{d}{dt}\lambda_{mq} + \{1 + L_{ds} \Gamma_{mi}(\lambda_m) \{2 + C_{dl} B_{d1}\} \right\} \frac{d}{dt}\lambda_{md} \tag{55}
\]
\[
= \frac{d}{dt}\lambda_{ds} - L_{ds} C_{dl} \times [(A_d - r_{fdr} B_d C_{d2}) x_d + B_{d2} r_{fdr} x_d] \tag{55}
\]
\[
= \frac{d}{dt} \lambda_{ds} - L_{ds} C_{dl} A_d x_q \tag{56}
\]
where the quantities \( (d/dt)\lambda_{ds} \) and \( (d/dt)\lambda_{qs} \) are obtained from (49) and (50). The solution of (55) and (56) yields the time derivatives of two of the state variables (the magnetizing flux linkages \( \lambda_{mq} \) and \( \lambda_{md} \)). Apart from being integrated themselves, these derivatives are also inserted in (7) and (8)—as the magnetizing voltages \( v_{mq} \) and \( v_{md} \)—for calculating the derivatives of the remaining state variables \( x_d \) and \( x_q \).
1. Model Summary

The proposed model is of the voltage-in, current-out type; inputs are the armature and field winding voltages, as well as the rotor angular position and speed, whereas outputs are the armature and field winding currents, and the machine’s developed EM torque. The model’s computational structure may be summarized as follows.

1) Transform the abc stator voltages to the rotor reference frame using (1).
2) Compute the magnetizing currents, using (28) and (29).
3) Compute the rotor currents using (6) and (9).
4) Compute the qd-axis armature currents using (47) and (48).
5) Calculate the abc armature currents by applying the inverse of transformation (1).
6) Calculate the EM torque from (42).
7) Compute the derivatives of the stator flux linkages, using (27), (49), and (50).
8) Compute the incremental inverse inductance matrix (32).
9) Substitute known quantities into (55) and (56), and solve the 2 × 2 linear system for the derivatives of the magnetizing flux linkages.
10) Compute the derivatives of the rotor network states using (7) and (8).
11) Interface the synchronous machine model with the rest of the system that is simulated, numerically integrate the state equations, and repeat step 1 from now on.

The model is not computationally intensive, since it only involves simple numerical computations at each time step. Thus, it can be readily implemented in a dynamic simulation environment, such as Matlab/Simulink [20].

In order to initialize the model from a load-flow study—where the machine is normally connected to a generator (P = V) bus or a swing (V = δ) bus, the model states may be calculated by solving a set of nonlinear equations. Specifically, it can be seen from (7), (8), and the proposed realization (75) that in the steady-state, all rotor network states must be equal to zero; the exception is ẋd, which is directly related to the field winding voltage through ẋd = −v′f,dr/ o( 1 / f,dr V). The mathematical expressions for the prespecified quantities (P, V, o) may be readily manipulated and written in terms of λmq and λmd. The nonlinear system may then be solved for the magnetizing flux states.

J. External Impedance Incorporation

The model is flexible enough to allow the incorporation of an external resistance and inductance, connected in series with the field winding. For example, such would be the case of a brushless excitation system, when the detailed model of the exciter-rectifier system is replaced by a nonlinear average-value model. Essentially, this simplification produces a voltage behind reactance representation of the exciter. As was shown in [44], the effective voltage at the generator field has the form

\[ v_{f,dr}' = E' - R_{f,dr} i_{f,dr} - L_{f,dr} \frac{d}{dt} i_{f,dr} \] (57)

where \( E' \) is the voltage behind the effective resistance \( R_{f,dr} \) and the transient inductance \( L_{f,dr} \); the primes denote that all quantities have been referred to the stator. These voltage drops may be incorporated in the proposed model in a straightforward manner.

In particular, the voltage at the field side of the d-axis admittance block may be written as

\[ v_{d2} = E' - (R'_{e} + R_{f,dr}) i_{f,dr} - L_{f} \frac{d}{dt} i_{f,dr} \] (58)

Using (5) and (6), this equation becomes

\[ v_{d2} = E' - (R_{e} + R_{f,dr}) C_{d2} x_{d} - L_{f} C_{d2} (A_{d} x_{d} + B_{d} v_{md} + B_{d2} v_{d2}) \] (59)

and solving for \( v_{d2} \) yields

\[ v_{d2} = \kappa \left[ E' - (R_{e} + R_{f,dr}) C_{d2} x_{d} - L_{f} C_{d2} (A_{d} x_{d} + B_{d} v_{md}) \right] \] (60)

where \( \kappa = (1 + L_{f} C_{d2} B_{d2})^{-1} \). After the substitution of (60) into (5), the following modified state equation is obtained:

\[ \frac{d}{dt} x_{d} = \left[ (I_{N_{d}} - \kappa L'_{e} B_{d2} C_{d2}) A_{d} - \kappa (r'_{f,dr} + R_{e}) B_{d2} C_{d2} \right] x_{d} + \left[ (I_{N_{d}} - \kappa L'_{f,d} B_{d} C_{d2}) B_{d} v_{md} + \kappa B_{d2} E' \right] \] (61)

where \( I_{N_{d}} \) denotes the identity matrix of dimension \( N_{d} \times N_{d} \). The derivatives of the magnetizing flux linkages are given by the solution of the linear system of equations formed by (56) and

\[ L_{d} \frac{d}{dt} \lambda_{mq} + \left\{ 1 + L_{d} [T_{m}^\prime(\omega)]_{22} + C_{d} \right\} \frac{d}{dt} \lambda_{md} = \frac{d}{dt} \lambda_{ls} - L_{d} C_{d} \]

\[ \times \left\{ [(I_{N_{d}} - \kappa L'_{e} B_{d2} C_{d2}) A_{d} - \kappa (r'_{f,dr} + R_{e})] \times B_{d2} C_{d2} \right\} x_{d} + \kappa B_{d2} E' \] (62)

which was obtained by a procedure analogous to the one described in the previous section.

III. EXPERIMENTAL VALIDATION

The experimental setup is depicted in Fig. 3 and contains a Leroy–Somer brushless synchronous generator, model LSA 432L.7. This is a salient four-pole machine rated for 59 kW,
600 V, at 1800 r/min. The generator’s prime mover is a Dyne Systems 110-kW, 590 N·m, vector-controlled induction motor-based dynamometer that is programmed to maintain constant rated speed. The voltage regulator uses a proportional-integral control strategy to maintain the commanded voltage (560 V line-to-line, fundamental rms) at the generator terminals; the brushless exciter’s field current is controlled with a hysteresis modulator. The rotor’s angular position and speed are measured with an optical position encoder that is fitted on the machine shaft. The generator is loaded with an uncontrolled rectifier that feeds a resistive load through an LC filter.

The complete methodology for characterizing the main synchronous machine is described in detail in [17]. The obtained parameters and functional forms are as follows: \( r_s = 0.108 \, \Omega \), \( r_{fd} = 201 \, \Omega \), \( N_{af} = 11.5 \), \( L_d = 0.97 \, mH \)

\[
\Gamma_{md}(\lambda_m) = 10^3 \cdot \frac{1 - 1.122 \lambda_m + 0.3348 \lambda_m^2}{2020 - 32.48 \lambda_m + 9.261 \lambda_m^2}
\]

\[
\Gamma_{mq}(\lambda_m) = -6.580 + 2.461 \Gamma_{md}(\lambda_m)
\]

\[
y_{11}(s) = \frac{Y_{d0}}{s(1 + 15.7 \cdot 10^{-3} s)}
\]

\[
y_{12}(s) = \frac{1 + 12.87 \cdot 10^{-3} s}{s(1 + 15.7 \cdot 10^{-3} s)}
\]

\[
y_{22}(s) = \frac{1 + 9.241 \cdot 10^{-3} s}{s(1 + 1.57 \cdot 10^{-3} s)}
\]

\[
Y_q(s) = \frac{5.82}{1 + 1.46 \cdot 10^{-3} s}
\]

where \( Y_{d0} = 1229.6 \, \text{H}^{-1} \) and \( n_d = 2 \), \( n_q = 1 \). Note that the expression for \( \Gamma_{md} \) is valid for \( \lambda_m < \lambda_{ml} = 1.6 \, V \cdot s \) (which is above the knee of the saturation curve). For greater values, the rational function diverges, and an appropriate continuation is defined so that the flux linkage increases linearly with current (with the same slope as at \( \lambda_{ml} \)). The load parameters are \( L = 7.5 \, mH \), \( C = 300 \, \mu F \), \( R_1 = R_2 = 32.5 \, \Omega \). With this load, the machine operates at the region of the knee of the magnetizing curve (\( \lambda_m \approx 1.35 \, V \cdot s \) [17]). The brushless exciter is represented by an average-value model that incorporates magnetic hysteresis using Preisach’s theory; this model is discussed in detail in [45] and a forthcoming publication [46]. The prime mover model’s output is the mechanical torque, which is inserted in the well-known equation of motion. The mechanical rotor speed may thus be computed by numerically integrating \( \omega_{rm} = (1/J) \int (T_m + T_e) \, dt \). The electrical rotor position is then calculated by integrating the rotor’s speed (i.e., \( \theta_e = (P/2) \int \omega_{rm} \, dt \)). The prime mover’s model is documented in [45].

For the first experiment, the switch \( S \) is closed so that the total resistive load is \( R = 16.3 \, \Omega \). The generator’s voltage reference is modified according to the profile shown in Fig. 4. This series of commanded voltage steps creates an extended period of significant disturbances, and tests the validity of the model for large-transients simulations. The actual voltage exhibits an overshoot, which is more pronounced for the faster slew rate steps. Moreover, due to the exciter’s magnetically hysteretic behavior, it does not fall to zero. The predictions for mechanical speed (Fig. 5) are in excellent agreement with the experimental results. Finally, detailed voltage and current waveforms are depicted in Fig. 6. Since the proposed model is based on the \( qd \)-axes theory, higher harmonics attributable to the machine’s design are not represented. This reflects on
the voltage waveforms of Fig. 4, wherein the experimental waveform contains more ripple than the simulated waveform. However, the harmonics that are caused by the nonlinearity of the load are predicted accurately.

The second experiment involves sudden load changes. Initially, the switch $S$ is open; at $t = t_2$, it is closed, and at $t = t_2$, it is opened again. In Fig. 7, a lowpass-filtered version of the line-to-line voltage “envelope” is depicted. On average, the simulated and experimental waveforms are similar. As discussed above, the experimental voltage includes higher-order harmonics caused by slot effects. The mechanical speed waveforms are illustrated in Fig. 8.

IV. CONCLUSION

This work presents an integrated perspective on synchronous machine modeling, using arbitrary transfer function representations that replace the rotor’s equivalent circuit structures. This approach offers several advantages, such as the direct incorporation of frequency response results into the model—without further consideration of equivalent circuit parameter identification—and accurate representation of magnetic saturation effects. The model retains the computational efficiency of the $q, d$-axes theoretical framework and is suitable for small- and large-signal time-domain simulations of power systems.

APPENDIX

REALIZATION ALGORITHM

The following algorithm produces a realization with matrix $\mathbf{A}_d$ diagonal [40]. It is valid only for the case where the roots of the minimal polynomial are distinct. It is assumed that the transfer function matrix elements are (19)–(21).

1) Compute the roots of the minimal polynomial

$$ m_Y(s) = s(s - \lambda_1) \cdots (s - \lambda_{n_d-1}). \quad (69) $$

2) Expand $Y_d(s)$ into partial fractions

$$ Y_d(s) = \frac{1}{s} \mathbf{R}_0 + \sum_{k=1}^{n_d-1} \frac{1}{s - \lambda_k} \mathbf{R}_k. \quad (70) $$

The $2 \times 2$ residue matrices may be computed by

$$ \mathbf{R}_k = \lim_{s \to \lambda_k} (s - \lambda_k) Y_d(s), \quad \text{for } k = 1, \ldots, n_d - 1 \quad (71) $$

and are of full rank; however

$$ \mathbf{R}_0 = Y_{d0} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (72) $$

is a matrix of rank 1.

3) Write

$$ \mathbf{R}_k = \mathbf{C}_k \mathbf{B}_k, \quad \text{for } k = 1, \ldots, n_d - 1 \quad (73) $$

for example, by computing the LU decomposition. The matrices $\mathbf{C}_k, \mathbf{B}_k$ are $2 \times 2$. $\mathbf{R}_0$ may be factored as

$$ \mathbf{R}_0 = \mathbf{C}_0 \mathbf{B}_0 = \left( Y_{d0} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right) [1 - 1], \quad (74) $$

4) The realization is given by

$$ \mathbf{A}_d = \begin{bmatrix} 0 & \lambda_1 \mathbf{I}_2 & \cdots & \lambda_{n_d-1} \mathbf{I}_2 \\ \vdots & \ddots & \vdots & \vdots \\ \lambda_{n_d-1} \mathbf{I}_2 & \cdots & 0 & \lambda_1 \mathbf{I}_2 \\ \mathbf{I}_2 & \cdots & \mathbf{I}_2 & \mathbf{I}_2 \end{bmatrix}, \quad \mathbf{B}_d = [\mathbf{C}_0 \mathbf{C}_1 \cdots \mathbf{C}_{n_d-1}], \quad (75) $$
REFERENCES


Scott D. Sudhoff (SM’01) received the B.S. (Hons.), M.S., and Ph.D. degrees in electrical engineering from Purdue University, West Lafayette, IN, in 1988, 1989, and 1991, respectively.

Currently, he is a Full Professor at Purdue University. From 1991 to 1993, he was Part-Time Visiting Faculty with Purdue University and as a Part-Time Consultant with P. C. Krause and Associates, West Lafayette, IN. From 1993 to 1997, he was a Faculty Member at the University of Missouri-Rolla. He has authored many papers. His interests include electric machines, power electronics, and finite-inertia power systems.

Brian T. Kuhn (M’93) received the B.S. and M.S. degrees in electrical engineering from the University of Missouri-Rolla in 1996 and 1997, respectively.

He was a Research Engineer at Purdue University, West Lafayette, IN, from 1998 to 2003. Currently, he is a Senior Engineer with SmartSpark Energy Systems, Inc., Champaign, IL. His research interests include power electronics and electrical machinery.