An Adaptive Maximum Torque Per Amp Control Strategy

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Abstract—Maximum Torque Per Amp (MTPA) control of induction motor drives seeks to achieve a desired torque with the minimal possible stator current, which is favorable in terms of inverter operation, and nearly optimal in terms of machine efficiency. This work demonstrates that existing MTPA controls perform sub-optimally as temperature varies. An adaptive MTPA control strategy is proposed that always achieves optimal performance regardless of rotor temperature and does so without exhibiting the hunting phenomenon. The proposed control is experimentally shown to (i) accurately achieve the desired torque, and (ii) always satisfy the maximum torque per amp condition.

I. INTRODUCTION

MAXIMUM torque per amp control strategy of induction machine drives produces a desired torque with the minimum possible stator current, which is favorable in terms of inverter loss, and nearly optimal in terms of efficiency [1]. Recently, an improved maximum torque per amp (MTPA) control strategy for induction machine drives was proposed in [2]. One interesting aspect of the control strategy proposed in [2] is that it is based on an alternate qd induction machine model (AQDM) [3], which was shown to lead to improved performance over MTPA control strategies based on the classical qd model, such as in [4]. The MTPA control strategy set forth in [2] has been shown to achieve the commanded torque with good accuracy; it has also been shown that the maximum torque per amp condition is, in fact, achieved. However, in this work it is shown that the MTPA control strategy [2] performs sub-optimally as the stator surface temperature (and presumably rotor temperature) varies. In consequence, consideration of rotor resistance variation is required if the maximum torque per amp condition is always to be achieved.

In this work, a new adaptive MTPA control strategy is proposed that always achieves optimal performance. In the proposed approach, the current and slip frequency commands are formulated in terms of desired torque and rotor resistance, which is obtained using an on-line rotor resistance estimator [5]. The proposed adaptive MTPA control strategy is experimentally shown to obtain the desired torque with high accuracy and to satisfy the maximum torque per amp condition as temperature (and hence rotor resistance) varies. Significantly, the proposed approach does not exhibit any hunting behavior, unlike other adaptive schemes [6].

II. NOTATION

In this work, electrical rotor position is designated \( \theta_r \) and electrical rotor speed \( \omega_r \). These quantities are related to the mechanical rotor position \( \theta_m \) and mechanical rotor speed \( \omega_m \) by a factor of \( P/2 \) where \( P \) is the number of poles.

This work will make use of a transformation of variables to both the rotor and synchronous reference frames. The transformation of stator or inverter quantities may be expressed

\[
[f^x_{ay}\ f^x_{by}\ f^x_{cy}]^T = K^x_s(\theta)[f_{ay}\ f_{by}\ f_{cy}]^T
\]

where ‘y’ may be ‘s’ for stator, ‘i’ for inverter, or ‘m’ for magnetizing, ‘f’ may be a ‘v’ for voltage, ‘i’ for current, or ‘\( \lambda \)’ for flux linkage, and

\[
K^x_s(\theta) = \frac{2}{3} \begin{bmatrix}
\cos(\theta) & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\
\sin(\theta) & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3)
\end{bmatrix}
\]

The superscript ‘x’ will denote frame of reference. In a break with the usual notation, omission of a superscript will designate the rotor reference frame in which \( \theta = \theta_r \). This is done for notational simplicity since most quantities used herein are expressed in the rotor reference frame. Setting ‘x’ to ‘e’ will denote the synchronous reference frame. A similar transformation is used for rotor variables with the exception that \( \theta \) is replaced by \( \theta - \theta_r \) in (1)-(2).

A related transformation used herein is the q- and d- axis inverter currents transformation given by

\[
[i^x_{di}\ i^x_{dq}]^T = K^x_{s,i}(\theta)[i_{di}\ i_{dq}]^T
\]

where

\[
K^x_{s,i}(\theta) = \frac{2}{\sqrt{3}} \begin{bmatrix}
\cos(\theta - \pi/6) & \sin(\theta) \\
\sin(\theta - \pi/6) & -\cos(\theta)
\end{bmatrix}
\]
This variation arises from the assumption that the sum of the inverter currents is zero. A more detailed discussion of reference frame theory and notation is set forth in [1].

III. Alternate QD Induction Machine Model

The steady-state equivalent circuit corresponding to the AQDM in [7] is shown in Fig. 1. Therein, the AQDM has been modified slightly in that the rotor impedance corresponds to a 1st order rather than an arbitrary order of network. This is possible because the model is only being used to predict fundamental frequency behavior at low slip values. As can be seen, \( \omega_c \) is defined as \( \omega_c - \omega_s \), and \( \lambda_m \) is equal to \( \sqrt{2 j \lambda q_m} \), slip

\[
S = \frac{\omega_c - \omega_s}{\omega_s},
\]

and the rotor impedance, \( Z_r(j \omega_s) \), is separated into a real and imaginary part, which are denoted \( r_r(j \omega_s) \) and \( j \omega_s L_{fr2}(j \omega_s) \), respectively. In particular,

\[
Z_r(j \omega_s) = r_r(j \omega_s) + j \omega_s L_{fr2}(j \omega_s)
\]

wherein

\[
r_r(j \omega_s) = \text{Re}(Z_r(j \omega_s))
\]

\[
L_{fr2}(j \omega_s) = \frac{1}{\omega_s} \text{Im}(Z_r(j \omega_s))
\]

The functional forms for AQDM parameters, which are the stator and rotor leakage inductances, the absolute inverse magnetizing inductance, and the rotor admittance, are specified as follows:

\[
L_{ls} = l_{s1} \quad \text{(a constant)}
\]

\[
L_p(\lambda_m) = l_{r1} + \frac{l_{r2}}{1 + (l_{r3} \cdot \lambda_m)^{1/4}}
\]

\[
\Gamma_m(\lambda_m) = m_1 - m_2 \cdot \lambda_m + e_m(\lambda_m - m_4) + e_m(\lambda_m - m_5)
\]

\[
Y_e(s) = \frac{1}{Z_e(s)} = \frac{y_{a1}}{y_{m1} s + 1} + \frac{y_{a2}}{y_{m2} s + 1} + \frac{y_{a3}}{y_{m3} s + 1}
\]

The test system for this work utilizes a 4-pole, 460 V, 50 Hz, 60 Hz, delta-connected squirrel cage induction machine. Using the methods set forth in [8], the machine parameters of AQDM for the test induction machine are listed in Table I. Using the procedure set forth in [2], the MTPA control strategy based on this model for the test machine may be expressed as

\[
I_e^* \left( T_r^* \right) = 0.102 \cdot T_r^* - 6.41 \cdot T_r^* e^{-0.0110} + 7.79 \cdot T_r^* e^{-0.152}
\]

\[
\alpha_e^* \left( T_r^* \right) = 1.27 + 0.00443 \cdot T_r^* e^{1.15}
\]

IV. Sensitivity Study of Temperature Variation

In order to justify the need for an adaptive MTPA control strategy, consider the performance of a test induction machine with MTPA control strategy (12)-(13) as temperature is varied. Note that this MTPA control law is derived using machine parameters of the test machine at a specific temperature. Fig. 2 illustrates the effect of temperature on the electromagnetic torque using the control law (12)-(13). Therein, the induction machine is driven at a speed of 900 rpm at a torque command of 150 Nm. This corresponds to the inverter current command \( I_e = 61.58 \) A and the slip frequency command \( \omega_s = 2.6829 \) rad/s. The torque was measured by a torque estimator which was shown to be highly accurate when the induction machine is rotating at moderate to high speeds [9], [10].

Trace (a) illustrates the measured electromagnetic torque versus surface temperature of a point on the stator of the test induction machine which was measured using a Fluke 65 infrared thermometer. The time at which each data point was taken relative to the beginning of the study is designated +X where X is the time in minutes. The torque and the surface temperature of the test induction machine were measured every 5 minutes. It is shown in trace (a) that the variation of the measured electromagnetic torque is significant as stator temperature of the induction machine rises. As can be seen, as the stator surface temperature (and presumably rotor temperature) rises, the torque increases, reaches a maximum, and then decreases at the point where the maximum is reached. The rotor resistance at that temperature (near 43 °C) corresponds to the effective rotor resistance used to design the maximum torque per ampere control law in (12)-(13). At this
Fig. 2. Effect of temperature on torque with \( \tau_e^* = 150 \) (Nm) for the MTPA control strategy based on (12)-(13)

In the Fig. 2 trace (b), the measured electromagnetic torque using the estimated optimal slip frequency command in (13) as well as with two additional sets of torque measurements taken at 110 % and 90 % times the estimated optimal slip frequency command \( \omega_s^* \) defined by (13). At low temperatures where rotor resistance is smaller than the rotor resistance used to design MTPA control strategy, the torque measured at 0.9 times of \( \omega_s^* \) given in (13) is maximum and closer to the commanded torque; as the study proceeds in time, torque is maximized with \( \omega_s^* \) given by (13), and later yet it is maximized with 1.1 times of the value given by (13). These observation leads to the conclusion that the MTPA control strategy can not achieve the commanded torque or satisfy the maximum torque per amp condition unless rotor resistance variation is taken into account in the control strategy design.

V. INFLUENCE OF ROTOR RESISTANCE ON MTPA CONTROL STRATEGY

One challenge in making the AQDM based MTPA control strategy adaptive to rotor resistance variation is that the chief parameter that varies, the rotor resistance, does explicitly appear in the model. This is because in the AQDM, rotor circuits are represented by a transfer function so that they may be accurately represented over a wide frequency range (both fundamental frequency and switching frequency). However, for the purpose of control, the rotor circuits only need to be represented in the low frequency range, which sets the stage for a simplified AQDM in which the rotor circuits are represented by circuit elements. To illustrate this, the rotor resistance \( r_{rz} (j\omega_s) \) in (6) and linear portion of the rotor leakage inductance \( L_{rz} (j\omega_s) \) in (7) were investigated over the ‘practical range’ of slip frequency between one rad/s and four rad/s. This ‘practical range’ is determined by the range of values attained by (13) as the torque command is varied from zero to rated value.

Fig. 3 illustrates the value of the rotor resistance and linear portion of the rotor leakage inductance over this practical range of slip frequencies. Therein, as can be seen, \( r_{rz} (j\omega_s) \) is constant at 0.176 \( \Omega \), over this range. \( L_{rz} (j\omega_s) \) is also constant over this range. The fact that the rotor resistance is constant over the desired range of slip frequency suggests that it can be treated as lumped parameter for the purposes of designing MTPA control laws.

This simplification of the AQDM by replacing the rotor transfer function with lumped circuit elements is very reasonable, provided that the slip frequency is within the normal control range, and that only the fundamental component of the waveforms are being utilized (i.e. the model will no longer accurately predict switching frequency behavior). Note that with this change the AQDM is still not equivalent to the
CQDM, because of the inclusion of main path magnetic saturation and rotor leakage saturation.

Now that it has been established that the rotor resistance can be treated as a constant value (at least at a given temperature) for the range of slip frequency of interest, it is interesting to observe the dependence of the MTPA control law on the rotor resistance. To do this, the rotor resistance \( r_{rz} \) is selected to vary linearly from 0.01 \( \Omega \) to 0.21 \( \Omega \) in five steps. Following the procedure in [2], an MTPA control strategy based on the CQDM with each selected rotor resistance \( r_{rz} \) can be achieved.

The resulting MTPA control laws corresponding to the selected rotor resistances are illustrated in Fig. 4. Therein, an interesting observation was made that the optimal current command is not a function of rotor resistance (it is identical for all values of rotor resistance). Of course this is what would be expected from the CQDM based MTPA control [4]. However, it is interesting that this property also holds for the AQDM based MTPA control strategy. In Fig. 4 trace (b), it can be seen that whereas the current command is not a function of rotor resistance, the optimal slip frequency command is a function of rotor resistance.

VI. ROTOR RESISTANCE DEPENDENT MTPA CONTROL STRATEGY

In the previous two sections, the effect of temperature and rotor resistance variation on the performance of an MTPA control strategy that does not include rotor resistance variation was investigated. It was shown that adherence to the true maximum torque per ampere condition as well as the ability to track a commanded torque deteriorates as rotor resistance varies. It was also found that the rotor resistance is constant over the desired range of slip frequency and so can be treated as a lumped parameter in the practical control range.

The stage is now set to set forth an MTPA control strategy that can accommodate rotor resistance variation. To this end, the actual rotor resistance is assumed to be estimated accurately by an on-line rotor resistance estimator, such as in [5]. With this assumption, it is straightforward to derive an adaptive MTPA control strategy from the AQDM steady-state equivalent circuit in Fig. 1.

The derivation begins by noting that the electromagnetic torque in the synchronous reference frame may be written as

\[
T_e = \frac{3P}{2} \left( \lambda_{qm}^e i_{q}^e - \lambda_{qm}^s i_{q}^s \right) \tag{14}
\]

It is convenient to express (14) in terms of slip frequency, rms magnitude of the applied stator current, and rotor resistance. The relationship between the \( q \)-axis stator current in phasor representation and the \( q \)- and \( d \)-axis currents in the synchronous reference frame is

\[
\lambda_{qm}^e = \sqrt{2} i_{q}^e - j i_{d}^e \tag{15}
\]

Without loss of generality, selecting the phase reference such that all the stator current is in the \( q \)-axis, (15) reduces to

\[
\sqrt{2} I_s = i_{q}^e \tag{16}
\]

where \( I_s \) is the magnitude of \( i_{q}^e \). The magnetizing flux linkage may be expressed as

\[
\sqrt{2} \lambda_{qm} = \lambda_{qm}^e - j \lambda_{qm}^s \tag{17}
\]

After algebraic manipulation of (14)–(17), the electromagnetic torque may be rewritten in terms of stator current and magnetizing flux linkage phasor as

\[
T_e = \frac{3P}{2} \text{Imag} \left( \lambda_{qm} I_s \right) \tag{18}
\]

where the overbar ‘—’ indicates complex conjugate. From the AQDM steady-state equivalent circuit in Fig. 1, \( \lambda_{qm} \) is expressed as

\[
\lambda_{qm} (\omega_s, I_s, r_{rz}) = Z_{ag} (\lambda_{m}, \omega_s, r_{rz}) I_s \tag{19}
\]

where
\[ Z_{ag} (\lambda_m, \omega_s, r_{z}) = \frac{\omega_s}{-j \Gamma_m (\lambda_m) + j \omega_s L_p (\lambda_m) + (r_z + j \omega_s L_{trz})} \]  

(20)

The subscript ‘ag’ in \( Z_{ag} \) indicates the impedance looking into the air-gap of the induction machine.

The electromagnetic torque can now be expressed in terms of \( \omega_s, I_s, \) and \( r_{z} \). Substitution of (19) into (18) yields

\[ T_e (\omega_s, I_s, r_{z}) = \frac{3}{2} p \cdot \text{Im} \left[ \frac{Z_{ag} (\lambda_m, \omega_s, r_{z})}{j \omega_s} \right] I_s \]  

(21)

It should be observed that \( \omega_s \) from the definition of \( Z_{ag} \) in (20) cancels the \( \omega_s \) in the denominator of (21), so (21) is not a function of \( \omega_s \). It remains to show how \( \lambda_m (\omega_s, I_s, r_{z}) \) in (21) is computed. In (21), \( \lambda_m \), which is equal to \( \sqrt{2} |\lambda_m| \), can be calculated by solving the nonlinear algebraic equation

\[ |\omega_s \cdot \lambda_m| = \sqrt{2} \cdot |I_s \cdot Z_{ag} (\lambda_m, \omega_s, r_{z})| \]  

(22)

by the Newton-Raphson method for a given \( \omega_s, I_s, \) and \( r_{z} \).

The procedure to derive an adaptive maximum torque per amp control strategy using (21) will proceed as follows. First, a suitable combination of \( I_s \) and \( r_{z} \) can be obtained by combining a stator current command, \( I_s \), and the potential values of rotor resistance \( r_{z} \). In the studies conducted herein, \( I_s \) is selected to vary linearly from 0 A to 33 A in 30 steps (0 to 110\% of the rated value), and \( r_{z} \) also from 0.01 \( \Omega \) to 0.21 \( \Omega \) in 10 steps. The \( j \)-th point of \( I_s \) and \( k \)-th point of \( r_{z} \) will be denoted \( I_{s,j} \) and \( r_{z,k} \) respectively. The optimum slip frequency for a combination of these stator current and rotor resistance selected is then identified by numerically maximizing (21) with \( I_s = I_{s,j} \) and \( r_{z} = r_{z,k} \). The resulting value of slip frequency will be denoted \( \omega_{s,j,k} \), and the corresponding value of torque \( T_{e,j,k} \). These data points are illustrated in Fig. 5.

Next, these data points are used to construct a stator current and slip frequency control law. As for a stator current control law, observe from Fig. 5 (a) that \( I_s \) is not function of \( r_{z} \), and so the form of the stator current control law in (12) is retained. Unlike the relationship of the data points \( \{I_{s,j}, T_{e,j,k}, r_{z,k}\} \), the slip frequency \( \omega_s \) is a function of rotor resistance as well as the electromagnetic torque. The data points \( \{\omega_{s,j,k}, T_{e,j,k}, r_{z,k}\} \) are fit to the functional form

\[ \omega_{s}^{*} = a_{1}r_{z}^{*3} + a_{2}r_{z}^{*2}T_{e}^{*} + a_{3} \]  

(23)

where \( a_1, a_2, n_1, n_2, \) and \( n_3 \) are parameters to be identified. These parameters are identified by maximizing the objective function defined by (24).

In (24), \( \varepsilon \) is a small number \( \left(10^{-3}\right) \) added to the denominator in order to prevent singularities in the unlikely event of a perfect fit, \( N_j \) is the number of a set of stator current command selected, \( N_k \) is the number of a set of rotor resistances selected, and \( \omega_{s,j,k}^{*} \) is given by (23) with \( T_{e,j}^{*} = T_{e,j,k} \) and \( r_{z,j}^{*} = r_{z,j,k} \). Although any optimization method could be used to solve this problem, a genetic algorithm is employed in this work. The genetic algorithm used herein was part of the Genetic Optimization System Engineering Tool (GOSET 1.02), a Matlab based toolbox. Details are set forth in [11]. The resulting expression for a slip frequency control law is

\[ \omega_{s} (T_{e}^{*}, r_{z}^{*}) = 7.22 \cdot 10^{-3} + 0.025 \cdot r_{z}^{*1.00} \cdot T_{e}^{*1.15} \]  

(25)
The resulting MTPA control laws are depicted in Fig. 5. It can be seen that (25) fit the calculated data points \( \{\omega_s^{*}, T_c, r_r\} \) with reasonable accuracy.

VII. Validation

The performance of the adaptive MTPA control strategy was experimentally validated. To do this, the rotor resistance estimator proposed in [5] is employed to predict the actual rotor resistance due to rotor temperature. The test induction machine is driven at a speed of 900 rpm at a command torque of 150 Nm. The electromagnetic torque measured at the estimated optimal slip frequency command \( \omega_s^{*} \) defined in (25) is compared with three sets of torque measurements taken at 90 %, 100 %, and 110 % of the slip frequency command \( \omega_s^{*} \) for the non-adaptive MTPA control as given by (13). These results are shown in Fig. 6. This illustrates that the torque measured at \( \omega_s^{*} \) in (25) is larger than the torque produced by the non-adaptive MTPA control, indicating the maximum torque per amp condition is in fact achieved at the estimated optimal slip frequency command \( \omega_s^{*} \) in (25) regardless of the stator surface temperature (and presumably rotor temperature). Furthermore, when compared with the torque produced by the non-adaptive MTPA control strategy whose \( \omega_s^{*} \) is defined in (13), the torque produced by the adaptive MTPA control strategy was not degraded by rotor temperature rise as much as the torque produced by the non-adaptive MTPA control strategy. Note that the torque produced by the adaptive MTPA control strategy increased very slowly as the temperature rose, though it is always within 1 % of the commanded value. This may be an artifact of changes in magnetic permeability or core loss with temperature.

VIII. Conclusion

It was experimentally shown that the MTPA control strategy performs sub-optimally unless rotor resistance variation due to rotor temperature change is taken into consideration. To always maintain optimal performance, a new adaptive MTPA control strategy was proposed. The actual rotor resistance for the adaptive MTPA control strategy is assumed to be predicted precisely by an on-line rotor resistance estimator such as the estimator set forth in [5]. The laboratory experiment shows that the proposed adaptive MTPA control strategy not only achieves the commanded torque but also satisfies the maximum torque per amp condition.

REFERENCES


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