An Improved Maximum Torque Per Amp Control Strategy for Induction Machine Drives

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Abstract—In this paper, an improved maximum torque per amp control strategy for induction machine drives is proposed. The proposed control strategy is simple in structure and includes the effects of magnetizing and leakage saturation. It is experimentally shown that the proposed control strategy produces the desired torque with the minimum stator current for the entire torque range and that this control strategy yields superior performance over a control strategy based on the classical qd induction machine model.

I. INTRODUCTION

With improper control, induction machines operate with poor efficiency at light or moderate loads, where some machines are driven for a significant portion of their service life. To improve induction machine efficiency in those operating conditions, there have been a variety of techniques proposed [1]-[7]. These methods aim to find an optimum slip frequency, at which the maximum efficiency of the induction motor is achieved. These methods possess a variety of disadvantages. The loss model based approaches [1]-[2] require substantial look-up tables. On-line search techniques [3]-[5] have the potential to exhibit hunting. Other approaches include [6], wherein Pontriagin’s maximum principle is used to improve efficiency and [7], wherein a particularly elegant approach to minimize the stator current amplitude for a given load torque is set forth. However, since these methods [6]-[7] are based on classical qd model whose deficiencies such as failure to represent leakage saturation, magnetizing saturation, and distributed system effects in the rotor circuits have long been noted in the literature [8]-[13], degradation of their ability to find an optimum slip frequency is inevitable.

In this paper, a new control strategy is proposed based on an alternate qd induction machine model (AQDM) [14], which addresses the deficiencies in classical qd model (CQDM). This AQDM has been experimentally shown to be a good mathematical representation of the machine, and one that is sufficiently accurate for control design [14]-[16].

The proposed maximum torque per amp (MTPA) control strategy is experimentally validated. It is shown that the open loop MTPA control strategy achieves commanded torque with good accuracy and that the maximum torque per amp condition is, in fact, achieved. The control is also experimentally compared to the one set forth in [7] and shown to yield superior performance.

II. NOTATION

In this work, electrical rotor position is designated \( \theta_r \) and electrical rotor speed \( \omega_r \). These quantities are related to the mechanical rotor position \( \omega_{rm} \) and mechanical rotor speed \( \omega_{rm} \) by a factor of \( P/2 \) where \( P \) is the number of poles. This work will make use of a transformation of variables to both the rotor and synchronous reference frames. The transformation of stator or inverter quantities may be expressed

\[
\begin{bmatrix}
    f^*_q \\
    f^*_d \\
    f^*_y
\end{bmatrix} = K^*_q(\theta)\begin{bmatrix}
    f_{qy} \\
    f_{dy} \\
    f_{ey}
\end{bmatrix}
\]

(1)

where \( '^* \) may be \( 's \) for stator, \( 'i \) ' for inverter, \( 'r \) for rotor, or \( 'm \) for magnetizing. \( 'f \) may be \( 'v \) for voltage, \( 'i \) for current, or \( '\lambda \) for flux linkage, and where

\[
K^*_q(\theta) = \frac{2}{3} \begin{bmatrix}
\cos(\theta) & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\
\sin(\theta) & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3)
\end{bmatrix}
\]

(2)

The superscript \( 'x' \) will denote frame of reference. In a break with the usual notation, omission of a superscript will designate the rotor reference frame in which \( \theta = \theta_r \). This is done for notational simplicity since most quantities used herein are expressed in the rotor reference frame. Setting \( 'x' \) to \( 'e' \) will denote the synchronous reference frame. A similar transformation is used for rotor variables with the exception that \( \theta \) is replaced by \( \theta - \theta_r \) in (1)-(2).

A related transformations used herein is that the \( q \)- and \( d \)-axis inverter currents may be expressed as

\[
\begin{bmatrix}
    i^*_q \\
    i^*_d
\end{bmatrix} = K^*_{qd}(\theta)\begin{bmatrix}
    i_s \\
    i_B
\end{bmatrix}
\]

(3)

where

\[
K^*_{qd}(\theta) = \frac{2}{\sqrt{3}} \begin{bmatrix}
\cos(\theta - \pi/6) & \sin(\theta) \\
\sin(\theta - \pi/6) & -\cos(\theta)
\end{bmatrix}
\]

(4)
This variation arises from the assumption that the sum of the inverter currents is zero. Additional discussion of reference frame theory and notation is set forth in [8].

III. REVIEW OF INDUCTION MACHINE DRIVE

The test system for this work utilizes a 4-pole, 460 V, 50 Hp, 60 Hz, delta-connected induction machine whose specifications are listed at Table I. The associated inverter is illustrated in Fig. 1.

Therein, for a given commanded torque $T^c$, an MTPA control strategy determines a corresponding inverter current command in the synchronous reference frame as well as a slip frequency command, which are denoted $i_{qdc}^c = [i_{qdc}^c, i_{dqc}^c]^T$ and $\omega_s^c$, respectively. Later in this work, an MTPA control strategy to determine these quantities will be set forth. Other variables which appear in Fig. 1 but which are not defined by the figure include the measured a- and b-phase inverter currents, $i_{ia}$ and $i_{ib}$, the electrical frequency $\omega_s$ (which is integrated to determine the position of the synchronous reference frame $\theta_s$), the vector of inverter phase current commands $\mathbf{i}_{abc}^* = [i_{a}^*, i_{b}^*, i_{c}^*]^T$, and finally the vector of switch commands $\mathbf{s}_{abc}^* = [s_a^*, s_b^*, s_c^*]$. Setting $s_x^*$ high indicates the upper transistor of $x$-phase should be turned on (and the lower transistor off), where $'x'$ may be 'a', 'b', and 'c'.

The SCR block designates a synchronous current regulator, whose implementation is depicted in Fig. 2 [17]. Therein, it can be seen that the $\tilde{i}_{ia}$, $\tilde{i}_{ib}$ are transformed to the synchronous reference frame by $\mathbf{K}_{x_{ij}}$, and used to generate the error between the actual and commanded q- and d-axis currents. This error is multiplied by the integral gain $(1/\tau_{scr})$, integrated to $\pm i_{fd}$, and added back to $i_{qdc}^*$. The modified q- and d-axis current commands are transferred back to abc variables using $\mathbf{K}_{x_{ij}}$, which denotes the left two columns of $\mathbf{K}_{x_{ij}}$.

A delta modulator is used to generate the switching commands, $\mathbf{s}_{abc}$, for switching devices of the inverter, $T_1$, ..., $T_6$. In this modulator, every $\tau_{sc}$ seconds, the phase current error between $i_{ij}^*$ from SCR and $i_{ij}$ are calculated as $e_{ij} = i_{ij}^* - i_{ij}$

\begin{equation} (5) \end{equation}

where $'x'$ may be 'a', 'b', and 'c'. Based on the sign of error in each phase in (5), switching command $s_x$ is determined by $s_x = \begin{cases} 1 & e_{ij} \geq 0 \\ 0 & e_{ij} < 0 \end{cases}$

(6)

The switching of the three phases is evenly staggered in time. For the study here, $\tau_{sc}$ is set to 10 kHz.

IV. MTPA CONTROL STRATEGY BASED ALTERNATE QD INDUCTION MACHINE MODEL

In order to design an MTPA control strategy, an accurate mathematical induction machine model is essential. Therefore, prior to setting forth the design of an MTPA control strategy,
the AQDM is briefly reviewed.

The steady-state equivalent circuit corresponding to the AQDM is depicted in Fig. 3 [15]. Therein, \( \omega_s \) is defined as \( \omega_m - \omega_e \), \( \lambda_m \) is equal to \( \sqrt{2} \left| \lambda_{qm} \right| \), and slip \( S \) is defined as 
\[
S = \frac{\omega_e}{\omega_m}.
\]

The functional forms of machine parameters in this equivalent circuit are taken to be
\[
L_{qr} = L_{qs} \text{ (const)} \tag{7}
\]
\[
L_{qr} \left( \lambda_m \right) = L_1 + \frac{L_2}{1 + \left( L_1 \cdot L_m \right)^{1/2}} \tag{8}
\]
\[
\Gamma_m \left( \lambda_m \right) = m_1 - m_2 \cdot \lambda_m + e^{\pi i (\lambda_m - m_3)} + e^{\pi i (\lambda_m - m_4)} \tag{9}
\]
\[
Y_e (s) = \frac{y_{sl} + y_{sq} + y_{s2}}{y_{sr} + y_{sq} + y_{s2} + 1} \tag{10}
\]

For this work, the AQDM parameters of the test induction machine were characterized using the methods set forth in [16]. The resultant machine parameters of AQDM for this machine are listed in Table II.

The stage is now set to set forth a control strategy based on the AQDM model and its parameters listed in Table II. The objective of this control strategy is to produce a desired torque with the minimum current, which is favorable in terms of inverter losses, and nearly optimal in terms of efficiency [8]. The structure of this control strategy is such that root-mean-square magnitude of the stator current \( I_s \), and slip frequency \( \omega_s \), are expressed as functions of the commanded torque, \( T_e \).

The derivation begins by noting that the electromagnetic torque in the synchronous reference frame may be written as
\[
\tilde{T}_e = I_s \omega_e L_m \tag{11}
\]

It is convenient to express (11) in terms of slip frequency and rms magnitude of the applied stator current. The relationship between the \( q \)-axis stator current in phasor representation and the \( q \)- and \( d \)-axis currents in the synchronous reference frame is
\[
\sqrt{2} i_{qs} = i_d - j i_q \tag{12}
\]

Without loss of generality, selecting the phase reference such that all the current is in the \( q \)-axis, (12) reduces to
\[
\sqrt{2} I_s = I_d \tag{13}
\]

After algebraic manipulation of (11)–(14), the electromagnetic torque may be rewritten in terms of stator current and magnetizing flux linkage phasors as
\[
T_e = \frac{3}{2} P \cdot \text{Imag}(\tilde{T}_{qm}) \tag{15}
\]

where \( T_e \) is the magnitude of \( \tilde{T}_{qm} \), which has only a real component. Likewise in (12), the relationship of magnetizing flux linkages may be expressed
\[
\sqrt{2} \lambda_{qm} = \lambda_{d} - j \lambda_{q} \tag{14}
\]

After algebraic manipulation of (11)–(14), the electromagnetic torque may be rewritten in terms of stator current and magnetizing flux linkage phasors as
\[
T_e = \frac{3}{2} P \cdot \text{Imag}(\tilde{T}_{qm}) \tag{15}
\]

where the overbar ‘ – ’ indicates complex conjugate. From the AQDM steady-state equivalent circuit in Fig. 3, \( \tilde{\lambda}_{qm} \) is expressed as
\[
\tilde{\lambda}_{qm} (\omega_e, I_s) = \frac{Z_a (\lambda_m, \omega_e)}{j \omega_s} I_s \tag{16}
\]

where
\[
Z_a (\lambda_m, \omega_e) = \frac{\omega_e}{j \omega_s L_m (\lambda_m) + 1} \tag{17}
\]

The subscript ‘ag’ in \( Z_a \) indicates the impedance looking into the air-gap of the induction machine.

The electromagnetic torque now can be expressed in terms of only \( \omega_e \) and \( I_s \). Substitution of (16) into (15) yields
\[
T_e (\omega_e, I_s) = \frac{3}{2} P \cdot \text{Imag} \left( \frac{Z_a (\lambda_m, \omega_e)}{j \omega_s} I_s \right) \tag{18}
\]

It should be observed that the \( \omega_e \) from the definition of \( Z_a \) in (18) cancels the \( \omega_e \) in the denominator at (18), so (18) is not applicable.
actually a function of \( \omega_s \). It remains to show how \( \lambda_m(\omega_s, I_s) \) is computed in (18). In (18), \( \lambda_m \), which is equal to \( \sqrt{2 |\lambda_m|} \), can be calculated by solving the nonlinear algebraic equation
\[
|\omega_c \cdot \lambda_m| = \sqrt{2 |I_s \cdot Z_{eq}(\lambda_m, \omega_s)|}
\]
by the Newton-Raphson method for a given \( \omega_s \) and \( I_s \).

The procedure to derive an MTPA control strategy using (18) will proceed as follows. First, a set of stator current commands for \( I_s \) will be assumed (from nearly 0 A to somewhat over rated current). The \( j \)-th point will be denoted \( I_{s,j} \). The optimum slip frequency for this current is then identified by numerically maximizing (18) with \( I_e = I_{s,j} \). The resulting value of slip frequency will be denoted \( \omega_{s,j} \), and the corresponding value of torque \( T_{e,j} \). These data points are illustrated in Fig. 4.

Next, these data points are used to construct a stator current and slip frequency control law. As for a stator current control law, these data points \( \{ I_{s,j}, T_{e,j} \} \) are used to formulate a stator current control law of the form
\[
I_s = a_1 T_e + a_2 T_e^2 + a_3 T_e^3 + a_4 T_e^4
\]
where \( a_1, a_2, a_3, \text{and} a_4 \) are selected by maximizing the objective fitness function \( f_{MTPA} \) defined as
\[
f_{MTPA} = \frac{1}{\epsilon + \frac{1}{N_j} \sum_{j=1}^{N_j} |I_{s,j} - I_{s,j}^*|}
\]
where \( \epsilon \) is a small number \( (10^{-3}) \) and added to the denominator in order to prevent singularities in the unlikely event of a perfect fit. \( N_j \) is the size of a set of stator current commands, and \( I_{s,j}^* \) is given by (20) with \( T_{e,j}^* = T_{e,j} \).

Similarly, the data points \( \{ \omega_{s,j}, T_{e,j} \} \) are used to formulate a slip frequency control law of the form
\[
\omega_s = c_0 + c_1 T_e + c_2 T_e^2 + c_3 T_e^3 + c_4 T_e^4
\]
where \( c_0, c_1, c_2, c_3, \text{and} c_4 \) are also chosen by maximizing (21) with \( \omega_s \) replaced with \( \omega_{s,j} \). \( \omega_{s,j}^* \) is given by (22) with \( T_{e,j}^* = T_{e,j} \). Together, (20) and (22) form the MTPA control strategy. The resultant MTPA control law based on AQDM whose parameters are listed in Table II, by the aforementioned procedure, is given by
\[
I_s(T_e^*) = 0.109 T_e + 17.0 T_e^{0.0177} + 18.5 T_e^{0.079} \quad (23)
\]
\[
\omega_s(T_e^*) = 1.4 - 2.4 \times 10^{-3} T_e + 1.76 \times 10^{-6} T_e^2 - 933 \times 10^{-9} T_e^3 + 1.74 \times 10^{-7} T_e^4 \quad (24)
\]
The final analytical forms of this resulting MTPA control law based on the AQDM are also depicted in Fig. 4.

V. VALIDATION

The performance of the AQDM based MTPA control strategy derived in the previous section is now investigated. For comparison purposes, the performance of a CQDM based MTPA control strategy is also presented. In [7], a derivation of the MTPA control strategy based on CQDM is set forth. However, while [7] demonstrated the ability to achieve a desired torque, it was not demonstrated to be optimal. In fact, the performance of the MTPA control strategy set forth in [7] is sub-optimal.

To show this, consider the 50 Hp machine characterized herein. The parameters of the CQDM were selected to maximize the function, \( f_{CQDM} \)
\[
f_{CQDM} = \frac{1}{\epsilon + \frac{1}{N_{eq} \sum_{l=1}^{N_{eq}} |\tilde{Z}_{	ext{meas}} - \tilde{Z}_{	ext{ref}}|}}
\]

Fig. 4. The MTPA control law based on AQDM.
where \( \varepsilon \) is a small number \( (10^{-6} \text{ in } (25)) \) and \( N_{eq} \) is total number of measured test points, \( \tilde{Z}_{\text{meas}} \) is the measured per-phase fundamental frequency impedance at the \( l \)-th test point, and \( \tilde{Z}_{\text{eq}} \) is given by

\[
\tilde{Z}_{\text{eq}} = r_s + j\omega_L L_s + \frac{\alpha_s}{1 + \frac{j\omega_L}{r_s + j\omega_L L_s}} \tag{26}
\]

The experimentally measured input data set \( \{\alpha_s, \omega_L\} \) and \( \tilde{Z}_{\text{meas}} \) were obtained for the following conditions: the tested induction machine was driven with the rated line-to-line voltage, \( V_{l-l,\text{ rms}} \), of 480 V, rated electrical frequency \( \omega_0 \) of 377 rad/s, and the rotor speed was selected to vary linearly from 1782 to 1800 rpm in 10 steps. The optimization of (25) yields the parameters: \( L_s = 4.16 \text{ mH}, L_p = 4.16 \text{ mH}, L_m = 91.5 \text{ mH}, r_s = 0.159 \Omega \). Using these parameters and the control strategy set forth in [7] yields a CQDM based MTPA control law, which consists of the stator current command, \( i_s^{*}_{\text{CQDM}} \) and slip frequency command, \( \omega_{s}^{*}_{\text{CQDM}} \). Since the machine is delta-connected, the inverter current command in the synchronous frame shown in Fig. 1 is obtained by scaling the stator current command by \( \sqrt{3} \). Fig. 5 depicts the MTPA control law developed using the control strategy set forth in [7].

To investigate whether the CQDM based MTPA control strategy achieves the maximum torque per ampere condition, the measured torque was recorded as the commanded torque was varied from 10 to 200 Nm. In addition, two additional sets of torque measurements were taken at 110 and 90% times the estimated optimal slip frequency command, \( \omega_{s}^{*}_{\text{CQDM}} \). The torque was measured by a torque estimator which was shown to be highly accurate when the induction machine is rotating at moderate to high speeds [18], [19]. Fig. 6 illustrates \( T_e/T_e^{*} \) versus \( T_e^{*} \) for three different slip frequency commands. For a given \( T_e^{*} \), the solid line, dotted line, and dashed line represents the measured torque when estimated optimal slip frequency command \( \omega_{s}^{*}_{\text{CQDM}} \), 90% of \( \omega_{s}^{*}_{\text{CQDM}} \), and 110% of \( \omega_{s}^{*}_{\text{CQDM}} \) are used, respectively. While overall the MTPA control strategy set forth in [7] performs well, the error in achieving the commanded torque is up to 25%. Further, it is apparent that by varying the slip frequency for some torque commands approximately 3% more torque could have been obtained. This is a modest potential improvement, but can nevertheless be significant given the maturity of the field.

To show that maximum torque per ampere condition is achieved by the AQDM based MTPA control strategy, the electromagnetic torque is also measured at three different slip
frequency commands, \( \omega^* \), 90\% of \( \omega^* \), and 110\% of \( \omega^* \), respectively, as it was in AQDM based MTPA control strategy in [7]. Fig. 7 depicts measured torque divided by commanded torque, \( T_c/T^*_c \), versus commanded torque \( T^*_c \). The solid line, dotted line, and dashed line represents \( T_c/T^*_c \) at 90 \% of \( \omega^* \), and \( T_c/T^*_c \) at 110 \% of \( \omega^* \), respectively.

As can be seen, \( T_c/T^*_c \) is maximized with a slip frequency command of \( \omega^* \) over the vast majority of the commanded torque range, indicating that the maximum torque per amp condition is achieved. It is also observed that \( T_c/T^*_c \) is close to unity, indicating that the torque produced closely tracks the commanded torque. In fact, the maximum open loop torque error is less than 3\% (as opposed to 25\% for the CQDM based MTPA control strategy).

VI. CONCLUSION

In this work, an improved maximum torque per amp control strategy for induction machine drives was proposed. The proposed control strategy is simple in structure and produces the desired torque with the minimum stator currents and shown to yield superior performance when compared to a CQDM based MTPA control.

REFERENCES


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