UNITED STATES NAVAL ACADEMY
DEPARTMENT OF ECONOMICS
WORKING PAPER 2014-46

BUSINESS CYCLE ACCOUNTING IN A SMALL OPEN ECONOMY

by

Jacek Rothert
United States Naval Academy

&

Mohammad Rahmati
Sharif University of Technology
Abstract

Building on Chari et al. (2007), we develop a method to assess theories of business cycles in small open economies. We build a diagnostic economy with time-varying distortions (wedges), which measure the gap between model generated aggregates and the data. We introduce two new wedges, which allow us to fully account for the movements in the trade balance and the current account: (i) the trend-shock wedge and (ii) the debt price wedge. We show how various detailed models with frictions map to economy with new wedges. Finally, we empirically evaluate different theories of fluctuations in emerging economies.

JEL classification: E32, F41, F44

Keywords: emerging markets, business cycle accounting, trend shocks, country risk, terms of trade

1 Introduction

Sources of business cycles is one of the central topics in macroeconomic research. Recently, Chari et al. (2007) (henceforth, CKM) suggested an empirical method to guide the development of theories...
of business cycle fluctuations. The method—*Business Cycle Accounting* (BCA herafater)—starts from the observation that many detailed, micro-founded models with frictions generate the same time paths of major macroeconomic aggregates, as does the neoclassical real business cycle model with time varying exogenous distortions (CKM call these distortions “wedges”). The method then exploits the behavior of macroeconomic aggregates (GDP, investment, employment and consumption) to inform researchers about relative importance of different wedges and—through theoretical equivalence results—importance of different frictions. The methodology has attracted much attention, and been applied to study numerous economies\(^1\). In this paper we extend the CKM method, in order to better understand the sources of fluctuations in emerging economies.

We make three contributions. *First*, we exploit information contained in the behavior of the trade balance, and the current account. This cannot be done in the original CKM method, because the trade balance is in itself part of the government spending wedge (a “distortion” that ensures the GDP identity holds). We believe this to be an important limitation in some applications of the method. In open economies trade balance is one of the central variables of interest. This is particularly true in emerging markets. In order to use the information contained in this variable, we extend the original methodology by adding two additional wedges: (i) the non-stationary efficiency (trend shock) wedge and (ii) the debt price wedge.

We chose those two wedges\(^2\), because they allow us to address important questions in the literature on emerging markets business cycles. We view it as our *second* contribution: we use the business cycle accounting to better understand the sources of fluctuations in emerging economies. The two most widely discusses theories in the literature are (i) permanent productivity shocks (*Aguiar and Gopinath* (2007)) and (ii) interest rate shocks coupled with financial frictions (*Neumeyer and Perri* (2005) and *Uribe and Yue* (2006)). We use BCA to quantitatively assess the contribution of

---

\(^1\) *Lama* (2010) applies BCA to a large sample of Latin American economies; *Simonovska and Soderling* (2008) to Chile; *Cociuba and Ueberfeldt* (2008) to Canada; *Hevia* (2009) to Canada and Mexico.

\(^2\) It is plausible that other distortions, which we do not consider here, can also account for movements in trade balance and current account.
non-stationary productivity shocks and of interest rate shocks to fluctuations in Mexico during the period 1986-2009.

Finally, our third contribution is to provide equivalence results. This is important, because we added additional wedges. If these wedges appear to be quantitatively important, the empirical results are only helpful if we have some idea of what underlying frictions those wedges may represent. First, we show that NIPA-measured allocations in a detailed economy with time varying policies which enhance or reduce incentives to innovate, are identical to NIPA-measured allocations in the diagnostic economy with time varying non-stationary efficiency wedge (combined with an investment / capital income wedge). Second, we show that allocations in an economy suffering from original sin\(^3\) and exposed to terms of trade shocks, are identical to the allocations in the diagnostic economy with movements in the debt price wedge.

**Related literature**

There has been a growing interest in applying BCA to study macroeconomic fluctuations in emerging economies (Simonovska and Soderling (2008), Hevia (2009), Lama (2010)). Most applications used the same macroeconomic aggregates as CKM, i.e. they treated trade balance as part of the government spending wedge\(^4\). In our analysis, the behavior of trade balance is an important source of information about inter-temporal wedges.

Our paper is related to a number of studies that attempted to empirically evaluate competing models of business cycles in emerging economies. In two recent studies, Chang and Fernandez (2010) and Garcia-Cicco et al. (2010) presented small open economy models encompassing the two theories and used Bayesian methods to estimate relative importance of productivity and interest rate shocks. We view our work as complementary - we address similar question but with a different methodology.

We are not the first to add additional wedges to the original business cycle accounting of CKM.

---

\(^3\)Original sin refers to the situation when the country is not able to issue external debt in its own currency.

\(^4\)Hevia (2009) is one exception
Sustek (2010) adds an asset market and a monetary policy wedge to study the relationship between output fluctuations and inflation in the US economy. Lama (2010) and Hevia (2009) introduced a bond wedge to study the role of external shocks in emerging markets. Our paper is the first to introduce a wedge that is a manifestation of non-stationary shocks—a trend shock wedge.

2 Wedges

First, we clarify our interpretation of wedges. At the measurement level, a wedge is a gap between a frictionless model’s equilibrium conditions and their data counterparts. In a way, a wedge measures the extent to which a frictionless model fails along the dimension of interest. For example, consider the equilibrium condition from a frictionless model that equates wage with (i) marginal product of labor and with (ii) marginal rate of substitution between consumption and leisure:

\[- \frac{U_\ell(C_t, \ell_t)}{U_c(C_t, \ell_t)} = F_\ell(K_t, \ell_t).\] (2.1)

Assuming we have some estimates of capital stock in the data, we could plug the values for consumption, hours worked and capital stock directly from the data into the above equation. In general (2.1) will not hold—there will be a gap between the two sides of the equation: \(- \frac{U_\ell(C_{data}^t, \ell_{data}^t)}{U_c(C_{data}^t, \ell_{data}^t)} \neq F_\ell(K_{data}^t, \ell_{data}^t)\). A labor wedge, \((1 - \tau_{\ell,t})\), is a number such that:

\[- \frac{U_\ell(C_{data}^t, \ell_{data}^t)}{U_c(C_{data}^t, \ell_{data}^t)} = (1 - \tau_{\ell,t})F_\ell(K_{data}^t, \ell_{data}^t)\] (2.2)

We want to make three observations here. First, it is clear from (2.2) that the actual number for \(\tau_{\ell,t}\) we obtain from the data depends on the assumptions we make about functional forms of the utility function, and the production function: different models, matched with the same data, generate different wedges. In other words, a wedge is always in the context of a prototype model, and should only be interpreted as such.

This leads to our second observation. A wedge can also be interpreted as a gap between two models. For example, we can use a frictionless real business cycle model with quasi-linear preferences of Greenwood et al. (1988) to generate artificial macroeconomic data. If we then
look at this data using an RBC model with Cobb-Douglas preferences, we will need to introduce exogenous distortions to fully account for the series that have been generated earlier (even if they were generated from a frictionless, deterministic model). We will discuss this issue further in Section 5.3, where we describe the mapping between different models of emerging markets business cycles and our prototype economy with wedges.

Finally, we want to point out that a wedge can be treated as an empirical moment. A macroeconomic researcher often builds a model to understand certain regularities in the data or to analyze the effects of some policies. If the model admits a representative agent formulation, it will have certain implications for example for the behavior of the labor wedge in the neoclassical growth model. That wedge can be measured and used as a test of the model, in the same way as other empirical moments are used. Hence, wedge is a diagnostic tool.

3 Prototype Diagnostic Economy

We first describe the prototype economy with wedges. We use the adjective diagnostic to stress our interpretation of wedges as diagnostic tools. The prototype economy is a small open economy version of a standard real business cycle model of Kydland and Prescott (1982). Each period, the economy experiences an exogenous shock to the state of the world $s_t$. The history of the shocks up to and including time $t$ is $s^t := (s_0, s_1, ..., s_t)$. The unconditional probability of state $s^t$ is $\pi(s^t)$. There are 6 stochastic exogenous variables that are functions of the underlying state: (i) the efficiency wedge $A_t(s^t)$, (ii) the labor wedge $\tau_{\ell,t}(s^t)$, (iii) the investment wedge $\tau_{x,t}(s^t)$, (iv) the government spending wedge $g_t(s^t)$, (v) the trend-shock wedge $\gamma_t(s^t)$ and (vi) the debt price wedge $\tau_q,t(s^t)$.

In what follows, all allocations are expressed in per capita terms. A stand-in household maximizes expected utility over consumption and leisure

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(C_t(s^t), 1 - \ell_t(s^t))$$
subject to the budget constraint,

$$C_t(s^t) + [1 + \tau_{x,t}(s^t)]X_t(s^t) \leq [1 - \tau_{x,t}(s^t)]w_t(s^t)\ell_t(s^t) + r_t(s^t)K_t(s^{t-1}) + T_t(s^t) +$$

$$\frac{1}{R^*} \left[ 1 - \tau_{q,t}(s^t) \right] D_{t+1}(s^t) - D_t(s^{t-1}) - \frac{\psi}{2} \left( \frac{D_{t+1}(s^t)}{\Gamma_{t-1}(s^{t-1})d} - \bar{\gamma} \right)^2 \Gamma_{t-1}(s^{t-1}) \bar{d}$$

the law of motion for capital stock,

$$K_{t+1}(s^t) \leq (1 - \delta)K_t(s^{t-1}) + X_t(s^t) - \frac{\phi}{2} \left( \frac{K_{t+1}(s^t)}{K_t(s^{t-1})} - \bar{\gamma} \right)^2 K_t(s^{t-1}),$$

and the no-Ponzi condition:

$$\lim_{t \to \infty} D_{t+1} \Pi_{t=0}^T q_t \leq 0.$$

where $\bar{\gamma}$ is the growth rate of the economy along the balanced growth path. The last term on the RHS of the budget constraint (3.1) is the cost of debt holding and is introduced to ensure that the law of motion for debt in a linearized version of this economy is stationary\(^5\). In that term, $\bar{d}$ denotes a steady-state level of debt in a stationary version of the economy\(^6\). The remaining terms in the budget constraint are: consumption expenditure $C_t$; investment expenditure $X_t$; labor income $(1 - \tau_{\ell,t})w_t\ell_t$; capital income $r_tK_t$; lump-sum transfers from the government $T_t$; issuance of new debt $D_{t+1}$ at the price $\frac{1}{R^*}[1 - \tau_{q,t}(s^t)]$; and repayment of debt issued in previous period $D_t(s^{t-1})$. The term $[1 - \tau_{x,t}]$ is the investment wedge—any distortion that makes the relative price of investment be different from 1 (which is the price of consumption). The term $[1 - \tau_{\ell,t}]$ is the labor wedge—any distortion that makes marginal rate of substitution between consumption and leisure be different from marginal product of labor. The term $[1 - \tau_{q,t}]$ is the debt price wedge—any distortion that makes the price of debt be different from the inverse of the steady state world risk-free rate. Movements in the debt price wedge can represent movements in the world interest rate (e.g. in the return on a 3-month T-Bill in the United States). They can also represent movements in the country’s perceived probability of defaulting on the newly issued debt. Equation (3.3) rules out

---

\(^5\)See Schmitt-Grohe and Uribe (2003) for the discussion of different ways to avoid the unit root in the process for debt holdings.

\(^6\)In theory, the value of $\bar{d}$ is not determined. In our exercise, we will estimate it.
Ponzi schemes and equation (3.2) governs the law of motion of capital stock. We assume quadratic adjustment costs in capital accumulation.

The total output in the economy is given by

\[ Y_t(s^t) = A_t(s^t) F(K_t(s^t - 1), \Gamma_t(s^t) \ell_t(s^t)) \]

where \( \log A_t(s^t) \) is the efficiency wedge and \( \Gamma_t(s^t) \) is the accumulated stock of productivity. The stock of productivity evolves according to

\[ \Gamma_t(s^t) = \gamma_t(s^t) \Gamma_{t-1}(s^{t-1}) \]

where \( \log \gamma_t(s^t) \) is the trend-shock wedge. A representative, competitive firm solves a static profit maximization problem:

\[ \max A_t(s^t) F(K_t(s^t - 1), \Gamma_t(s^t) \ell_t(s^t)) - r_t(s^t) K_t(s^t - 1) - w_t(s^t) \ell_t(s^t) \]

Similarly to CKM we assume the government balances its period-by-period budget constraint:

\[ TR_t(s^t) + G_t(s^t) = \tau_{x,t}(s^t) X_t(s^t) + \tau_{\ell,t}(s^t) w_t(s^t) \ell_t(s^t) + \frac{\tau_q}{R^*} D_{t+1}(s^t) \]

Resource constraint and external sector

The economy’s resource constraint is:

\[ C_t(s^t) + X_t(s^t) + G_t(s^t) + NX_t(s^t) = Y_t(s^t) \]

and \( \log G_t \) is the government spending wedge. Using the government and household’s budget constraint, we get that net exports are given by:

\[ NX_t(s^t) = D_t(s^{t-1}) - q_t(s^t) D_{t+1}(s^t) + \frac{\psi}{2} \left( \frac{D_{t+1}(s^t)}{\Gamma_{t-1}(s^{t-1}) d} - \tilde{\gamma} \right)^2 \Gamma_{t-1}(s^{t-1}) \tilde{d} \] (3.4)

The last term on the RHS of (3.4) follows from the portfolio adjustment costs. Introduction of these costs is important for two reasons. First, they are necessary for the debt’s law of motion to be stationary (Schmitt-Grohe and Uribe (2003)). Second, fluctuations of trade balance in a frictionless small open economy model are about 10 times greater than observed in the data. Without portfolio adjustment costs, matching the magnitude of the trade balance fluctuations would require huge movements in the debt price wedge. This would strip that wedge off any meaningful economic interpretation.

The current account in this economy is the reduction in its outstanding foreign debt:

\[ CA_t(s^t) = D_t(s^{t-1}) - D_{t+1}(s^t) \] (3.5)
**Functional forms**

We impose the following functional forms on utility and production functions. The utility function features constant relative risk aversion. We assume consumption and leisure enter the utility in a Cobb-Douglas aggregator. The production function is also assumed Cobb-Douglas.

\[
U(C, \ell) = \left[ C^{\eta}(1 - \ell)^{1-\eta} \right]^{1-\sigma} \\
F(K, \Gamma \ell) = K^\alpha (\Gamma \ell)^{1-\alpha}
\]

We do not consider alternative functional forms. The reasons behind this decision are discussed in Section 5.3.

**4 Accounting for Mexican Fluctuations: 1986-2009**

We now turn to the application of the method. We consider Mexican economy for two reasons. First, it is an emerging economy with relatively long series of consistent data available, going back to 1986. Second, it will allow us to compare our results with those in Garcia-Cicco et al. (2010) and Chang and Fernandez (2010).

**4.1 Data**

We use quarterly data, which gave us 90 observations that cover the period 1986:Q1 - 2009:Q2. The choice of the sample size was determined by the availability of quarterly series on hours worked. For Mexico, it goes back only to 1986. The national accounts data—GDP, investment and government spending—are in constant 2000 pesos, all deflated using GDP deflator. Trade balance over GDP is calculated as a ratio of nominal net exports to nominal GDP (in current pesos). All the above data is from OECD Main Economic Indicators (MEI) database. Current account data is from OECD Balance of Payments and is in current US dollars. Current account over GDP is calculated in nominal US dollars.
Employment data poses one important problem. Substantial fraction of Mexican labor force is employed in the informal sector and the official data pools the formal and the informal sector together. However, the GDP data uses is only for the formal sector. This is a potential problem, because labor tends to move between formal and informal sector, and we do not have a good measure of these movements. We will discuss this in more detail when we interpret our quantitative results in Section 4.5.2.

The data on interest rates is the most problematic. Interest rates on peso-denominated assets will be affected by the exchange rate risk. Interest rates on dollar-denominated assets will be stripped off this risk. However, given the national accounts data is in constant 2000 pesos we decided to consider the interest rate on the peso-denominated assets. This way, the interest rate and the quantities are in the same units, just like in the prototype model economy we consider. To construct the real interest rate, we divide the nominal gross interest rate by one plus the average inflation over the previous 4 quarters (implicitly assuming a particular form of adaptive expectations). We use the IFS data of the interest rates on Mexican 3-month Treasury Bills.

All the series (except for interest rates and inflation) have been seasonally adjusted.

4.2 Computation of wedges

Similarly to CKM, we assume that the mapping between the state of the economy and the vector of wedges is a bijection, i.e. \( s_t \equiv (\log A_t, \tau_{\ell,t}, \tau_{x,t}, \log g_t, \log \gamma_t, \tau_q) \). The business cycle accounting procedure consists of 3 steps.

In the first step we estimate the stochastic process governing the behavior of the vector of wedges \( \{s_t\} \).

In the second step, “knowing” the stochastic process for wedges, we identify the actual realizations of wedges (i.e. the values that the vector \( s_t \) has in each period \( t \)). We do it by solving for the values of the wedges, such that the model equilibrium conditions hold exactly in the data.

---

\(^7\)See Bosch and Maloney (2008) or Maloney (1999) for evidence on labor movements between formal and informal sector in Mexico.
In the last step we evaluate the contribution of different wedges to the fluctuations of different macroeconomic aggregates. We will now briefly summarize the first two steps, with particular focus on the identification of the trend-shock wedge \( \log \gamma_t \) and the debt price wedge \( 1 - \tau_{q_t} \).

**Step 1: Estimation of the stochastic process for wedges**

As in previous studies, stochastic process for wedges is assumed to be a 1-st order VAR:

\[
\begin{align*}
    s_t &= P_0 + P s_{t-1} + \epsilon_t, \\
    \epsilon_t &\sim N(0, \Sigma)
\end{align*}
\]

where \( s_t \) is a 6 \times 1 vector of wedges: \( s_t = [\log A_t, \tau_{\ell,t}, \tau_{x,t}, \log g_t, \log \gamma_t, \tau_{q,t}]' \). All the non-zero elements in \( P \) and in \( \Sigma \) as well as \( P_0 \) need to be estimated. The actual number of parameters to estimate will depend on the restrictions we impose on \( P \) and \( \Sigma \). Given our relatively short sample (25 years) and 6 wedges, we consider diagonal matrices \( P \) and \( \Sigma \) - each wedge is an AR(1) process\(^8\).

We make the economy stationary by de-trending (whenever necessary) by the aggregate stock of labor-augmenting productivity \( \Gamma_{t-1} \) (same way as in Aguiar and Gopinath (2007)). For each variable \( X \) that contains the trend, we define:

\[
\tilde{x}_t := \frac{X_t}{\Gamma_{t-1}}
\]

Equilibrium allocations are then characterized by the following conditions:

\(^8\)Considering fully unrestricted matrices \( P \) and \( \Sigma \) would imply estimation of 63 parameters using 90 observations.
\[ U_{c,t} = \hat{\beta}_t E_t \left\{ \frac{U_{c,t+1}}{} \right\} \]

\[ U_{c,t} = \hat{\beta}_t E_t \left\{ \frac{U_{c,t+1}}{} \right\} \]

\[ \bar{\epsilon}_t \frac{1 - \eta}{1 - \ell_t} \frac{1}{\eta} = (1 - \tau_{\ell,t})(1 - \alpha) \frac{\tilde{y}_t}{\ell_t} \]

\[ \tilde{y}_t = A_t \bar{k}_t^{\alpha} (\gamma_t \ell_t)^{1-\alpha} \]

\[ \bar{n}_t \bar{x}_t = \bar{d}_t - \frac{1 - \tau_q,t}{R^*} \gamma_t \bar{d}_t + \frac{\psi}{2} \left( \gamma_t \frac{\bar{d}_{t+1}}{d} - \bar{\gamma} \right) d \]

\[ \bar{y}_t = \bar{c}_t + \bar{v}_t + \bar{y}_t + \bar{n}_t \bar{x}_t \]

We solve our prototype economy by log-linearizing\(^9\) the equilibrium conditions around the steady state and then find the law of motion for endogenous variables via the method of undetermined coefficients. We use Uhlig’s Toolkit\(^10\) for this step. We then use the linearized laws of motion, and express the dynamic system governing the economy in the state-space form:

\[ X_{t+1} = AX_t + B \epsilon_{t+1} \]

\[ Y_t = CX_t + \eta_t \]

where

\[ X_t = \left[ \log \bar{k}_t, \log \bar{d}_t, \log \Gamma_t, \ell_t, 1 \right]^\prime \]

\[ Y_t = \left[ \log GDP_t, \log INV_t, \log \ell_t, \log GOV_t, \frac{NX_t}{GDP_t}, \log \frac{1}{R_t} \right]^\prime. \]

Matrices \( A \) and \( C \) depend on the parameters of our linearized laws of motion for endogenous and exogenous variables, which in turn depend on \( P_0 \) and \( P \). Matrix \( B \) depends on the variance

\(^9\)Except for the variables that can take on negative values. In that case we assume laws of motion are approximately linear in levels rather than in logs.

\(^10\)Available at http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm
covariance matrix $\Sigma$. We then estimate parameters of $A$, $B$ and $C$ using maximum likelihood, where the likelihood function is calculated using the Kalman filter (see technical appendix in Chari et al. (2007) for details).

**Step 2: Identifying wedges**

Having estimated the process for wedges we need to measure the wedges in the data. We first compute model decision rules. The decision rules map the state of the economy to allocations. In each period $t$ the state of the economy is given by $(s_t, \bar{k}_t, \bar{d}_t, \Gamma_{t-1})$. Given the estimated process for $s_t$, we obtain model’s decision rules of the form: $Y(s_t, \bar{k}_t, \bar{d}_t, \Gamma_{t-1})$, etc. The wedges are chosen so that in each period $t$ we have $GDP(s_t, \bar{k}_t, \bar{d}_t, \Gamma_{t-1}) = GDP^\text{data}_t$, $\ell(s_t, \bar{k}_t, \bar{d}_t, \Gamma_{t-1}) = \ell^\text{data}_t$ and so on.

The observed values of labor wedge, debt price and government wedge are easy to obtain. Government wedge is taken directly from the data on government spending. Labor wedge can be obtained from (4.4). Debt price wedge is calculated as: $\log(1 - \tau_q) = \log R^* - \log R_t$, where $R_t$ is the real interest rate on Mexican 3-month T-bill (computed as described in Section 4.1 on page 8), and $R^*$ is the long-run average of the real gross return on the US 3-month T-bill. With such definition of $1 - \tau_q$, it is clear its movements do not need to result only from events specific to Mexican economy. They can also be affected by external factors, e.g. by the events in the United States.

Identification of the remaining three wedges, $\log A_t$, $\log \gamma_t$ and $1 + \tau_{x,t}$, is more complicated. First, we normalize the initial $\Gamma_0$ to be 1. The normalization will affect the estimate of the mean of $\log A$. Then, the three equations that jointly identify the three wedges are: (4.2), (4.3), (4.5). Equation (4.5) pins down $\log A_t + (1 - \alpha) \log \gamma_t$. Equation (4.3) helps separate the two, looking at the response of the trade balance. Finally, equation 4.2, together with production function (4.5), separate $\log \gamma_t$ from $\tau_{x,t}$ - both wedges are inter-temporal, in the sense they affect future return to capital (equation (4.2)), but $\log \gamma_t$ additionally affects the level of output (equation (4.5)).

What is the economics behind it? Trend shock raises permanent income, and so the response
of both investment and of consumption is stronger. This translates to a stronger response of the trade balance, which is why equation (4.3) is essential to distinguish between log $A_t$ and log $\gamma_t$. As in Aguiar and Gopinath (2007), this is simply an exploitation of the permanent income hypothesis. Since we have one more inter-temporal wedge—$\tau_{x,t}$—we also need to add one more inter-temporal equilibrium condition: the Euler equation for capital, (4.2).

4.3 Accounting

Our accounting procedure at the conceptual level is the same as in Chari et al. (2007). The output of Step 2 was the series of the estimated realizations of wedges—$\{\hat{s}_t\}_{t=1}^T$. Next, we evaluate the contribution of each wedge to the movements in macro aggregates. We do it by feeding in one wedge at a time, or in combinations. For example, in order to evaluate the contribution of the trend shock wedge alone to fluctuations during the whole sample, we set, for each $t$, log $A_t = \log \bar{A}$, $\tau_{\ell,t} = \bar{\tau}_\ell$, $\tau_{x,t} = \bar{\tau}_x$, log $\gamma_t = \log \hat{\gamma}_t$, i.e. the estimated realization of the trend shocks wedge. We then feed in such modified series of $s_t$ into the model, compute the allocations using model’s decisions rules and compare them with the data.

4.4 Estimation Results

Table 1 on page 14 presents the estimates of the model parameters. The first six rows are the parameters of the stochastic process for wedges. The last two rows are estimates of the two fixed parameters: steady state level of debt in the de-trended economy $\bar{d}$, and portfolio adjustment costs $\psi$.

4.4.1 Contribution of non-stationary shocks

Our results can be used to evaluate the contribution of non-stationary shocks to Mexican fluctuations, which was the central topic of the studies in Garcia-Cicco et al. (2010) and Chang and Fernandez (2010). Their models were very similar to ours: it was a small open economy which embedded (i) interest rate shocks with working capital constraint and (ii) trend shocks. In our
The estimates in Table 1 can be used to compute the trend component of the Solow residual. The variance of the trend component (relative to the variance of the Solow residual) can be calculated as follows (see Aguiar and Gopinath (2007)):

\[
\frac{\sigma^2_{\Delta t}}{\sigma^2_{\Delta sr}} = \frac{(1 - \alpha)^2 \sigma^2_{\log \gamma} / (1 - \rho_{\log \gamma})^2}{\frac{2}{1 + \rho_{\log A}} \sigma^2_{\log A} / (1 - \alpha)^2 \sigma^2_{\log \gamma} / (1 - \rho_{\log \gamma})^2}
\]

where \(\sigma^2_{\Delta t}\) is the variance of the trend growth, and \(\sigma^2_{\Delta sr}\) is the variance of the Solow residual growth.

Our estimates suggest the random walk component to be only 31.3% of the Solow residual. This is substantially smaller than 96% reported in Aguiar and Gopinath (2007). The intuition behind it is fairly clear. Trend shocks are the only shocks that could generate excess volatility of consumption.
and counter-cyclical trade balance in Aguiar and Gopinath (2007). They must be large, in order for the model to generate the right business cycle moments. We do a full accounting exercise and allow for other exogenous distortions to affect macroeconomic variables. Part of the trade balance movements result from movements in debt price wedge, and part of the consumption movements result from movements in the labor wedge (intuitively, the trend shock does not have to do all the job). The estimate is quite close to the one we can obtain from Garcia-Cicco et al. (2010). Using their estimates of the persistence and the variance of transitory TFP and trend shocks, the random walk component can be estimated to be 27% of the Solow residual. It is somewhat larger than in Chang and Fernandez (2010) who report the value of 20%.

4.5 Tequila Crisis

One of the most studied events in Mexico during the 1986-2009 period was the Tequila crisis in 1995. It featured large drop in real GDP, combined with a sudden stop of capital inflows. The Sudden Stop was manifested in the reversal of trade balance and the current account. It has become the major event studied by economists interested in the Tequila Crisis (see e.g. the analysis in Aguiar and Gopinath (2007), Kehoe and Ruhl (2009)). What is the quantitative contribution of different wedges to these outcomes?

We first look at the time series behavior of wedges during the crisis. Figure 1 on page 16 plots the estimated realizations of the 6 wedges. A few interesting observations can be made here. The first quarter of 1995 features a sharp drop in the debt price wedge, which is a manifestation of the well known fact that interest rates on Mexican bonds spiked at the onset of the Tequila Crisis. Interestingly, we also have a sharp decline in the labor wedge \( (1 - \tau_{\ell,t} \text{ drops}) \). This drop is necessary to simultaneously account for the excess response of consumption and for the right co-movement of output and hours worked. Finally, there is about a 5% point drop in trend-shock wedge in the first quarter of 1995.

The movements in the values of wedges are not enough to evaluate their importance as the drivers of fluctuations in macroeconomic variables. For that, we need to employ the third step of
Figure 1: Estimated realization of wedges during the Tequila Crisis; vertical line indicates the crisis date (1995:Q1).
the business cycle procedure. We simulate the model by (i) feeding in the estimated exogenous realizations of selected wedges (separately and in combinations) while (ii) keeping all other wedges at their steady state values. We now proceed to this step.

4.5.1 Sudden Stop and Excess Volatility of Consumption

In our analysis of the Tequila Crisis we focus mainly on the trade balance reversal. Figure 2 plots three series for trade balance. The blue line plots the original data. The green line plots the series from the model where fluctuations are driven by only two wedges: efficiency and trend shock. We denote that series with AG since this corresponds exactly to the model of Aguiar and Gopinath (2007) (with Cobb-Douglas preferences). The red line plots the series from the model where fluctuations are driven by only three wedges: efficiency, labor and debt price. We denote that series NP as it most closely corresponds to the independent country risk model of Neumeyer and Perri (2005). The first two columns of Table 2 show the decomposition of the Sudden Stop event into all six wedges. We present the decomposition of two outcomes: (i) the trade balance reversal and (ii) the excess drop in consumption. We focus on those two outcomes because they are the two distinct features of emerging markets business cycles emphasized in the literature.

Trend-shock wedges alone account for about 22% of the Sudden Stop. Combination of the labor and the debt price wedge accounts for 65% of the Sudden Stop. The three wedges combined account for 87% of the Sudden Stop. While trend-shock wedge alone captures “only” 22% of trade balance reversal, it has a substantial contribution to the excess response of consumption. Trend-shock wedge alone accounts for 43% of consumption drop. The contribution of labor and debt price wedges is even greater. The movements in the two wedges account for 86% of consumption drop. The movement in all three wedges (absent other shocks) would have generated a 29% larger fall in consumption than observed.

The combination of efficiency, labor, debt price and trend shock wedges account for almost 87% of the Sudden Stop. The exact NP model with GHH preferences maps to our prototype economy with a combination of all but government spending wedge (see the discussion in Section 5.3 on page 27).
entirety of the trade balance reversal. These four wedges correspond to the combination of the two theories of emerging markets business cycles. Our interpretation is the two models should not be viewed as competing, but rather as complementary explanations. Aguiar and Gopinath (2007) explanation relies on movements in permanent income. The explanation in Neumeyer and Perri (2005) and Uribe and Yue (2006) relies on the movements in relative price of today’s and future’s output (the interest rate). While our results indicate the quantitative importance of the theory relying on the movements in the interest rate might be larger, the movements in permanent income still appear to be an important driver of the events in Mexico during the Tequila Crisis (about 20-25% of the Sudden Stop can be accounted for by the shock to the non-stationary component of TFP, alone).

Finally, notice the contribution of the six wedges to the movements in the trade balance, and in consumption during the Tequila Crisis, is somewhat different to their contribution during the
Table 2: Decomposition of movements in trade balance, and consumption

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>∆NX</td>
</tr>
<tr>
<td>Efficiency</td>
<td>7%</td>
</tr>
<tr>
<td>Labor</td>
<td>18%</td>
</tr>
<tr>
<td>Investment</td>
<td>-5%</td>
</tr>
<tr>
<td>Government</td>
<td>12%</td>
</tr>
<tr>
<td>Trend</td>
<td>22%</td>
</tr>
<tr>
<td>Debt price</td>
<td>47%</td>
</tr>
</tbody>
</table>

whole period we analyze. Columns 3 and 4 of Table 2 shows the variance decomposition of the HP-filtered consumption and the HP-filtered ratio of trade balance to GDP.

The first striking result is the very large contribution of government spending wedge to fluctuations in consumption. We believe the reason behind that is a steady decline in the share of government expenditure in Mexican GDP during the time period covered, indicating a fairly large substitutability between private and government consumption expenditures. The second big difference is that the major driver of fluctuations in trade balance are shocks to permanent income, rather than the shocks to interest rates. The common feature of the two parts of Table 2 is a fairly large role of the labor wedge. It appeared as the most important driver of consumption movements during the Tequila Crisis, and it is the second most important wedge driving the fluctuations during the whole period we analyze. It also remains an important driver of movements in the ratio of trade balance to GDP.
4.5.2 Labor wedge

There is a few important things to discuss in relation to the behavior of the labor wedge during the Tequila Crisis. As we can see from Figure 1, the labor wedge had a sharp drop in the first quarter of 1995. That drop can be better understood if we write down the FOC for consumption-leisure trade-off:

\[
\frac{1 - \eta}{\eta} \frac{C_t}{1 - \ell_t} = (1 - \tau_{\ell,t})(1 - \alpha) \frac{Y_t}{\ell_t}
\]

or, equivalently:

\[
(1 - \tau_{\ell,t}) = \frac{1 - \eta}{(1 - \alpha) \eta} \cdot \frac{C_t}{Y_t} \cdot \frac{\ell_t}{1 - \ell_t}.
\]

Since consumption dropped more than GDP, we have a fall in \(\frac{C_t}{Y_t}\). At the same time, hours worked fell, so \(\frac{\ell_t}{1 - \ell_t}\) also falls. That means, labor wedge must have fallen as well. In general, in a model with Cobb-Douglas preferences and Cobb-Douglas production function, pro-cyclical labor wedge is necessary to simultaneously account for (i) excess volatility of consumption and (ii) pro-cyclical hours worked. In our estimates, the correlation of the labor wedges with output is 0.14 (Lama (2010) estimates that correlation to be 0.4).

What theories of business cycles in emerging markets imply time-moving labor wedge? A working-capital constraint models of Neumeyer and Perri (2005) and Uribe and Yue (2006) is one such class. There is also a growing number of studies that focus on the role of informal sector, in particular on the movements of labor in and out of informal sector. For example, Restrepo-Echavarra (2011) considers movements of labor in- and out of informal sector to account for the excess movements in consumption. Such movements would be reflected as movements in the labor wedge in our prototype economy. Finally, Li et al. (2010), develop a search-model of labor market frictions that exploits the fact that labor market income is more volatile than GDP (which then translates to excess volatility of consumption).

We want to point out that our discussion does not indicate the trend-shock wedge is not important. The labor wedge is the single most important distortion that accounts for 58% of consumption drop in the first quarter of 1995. However, the trend shock wedge is a close second, as it accounts
for 43% of that same consumption drop.

5 Equivalence results

Since we added two new wedges to the CKM original method, we provide theoretical results mapping them into detailed economies.

5.1 From R&D subsidies to non-stationary efficiency wedge

We start by suggesting one possible interpretation of the non-stationary efficiency wedge. We will show that NIPA-measured allocations in a Schumpeterian model, with time-varying subsidies to R&D expenditures, are equivalent to NIPA-measured allocations in the detailed economy with appropriately defined capital income and non-stationary efficiency wedges.

5.1.1 A detailed economy with time-varying R&D subsidies

Consider a simplified version of the economy in Aghion and Howitt (1992). There is a constant measure 1 of sectors \(i \in [0,1]\). Final good is produced by a large number of identical competitive firms. The output of a representative firm is:

\[
Y_t = \int_0^1 A_t(i)x_t(i)^\alpha \ell^{1-\alpha} di
\]

where \(x(i)\) is the input of the capital good produced in sector \(i\); \(A(i)\) measures how much that capital good contributes to the production of the final good; and \(\ell\) is labor supplied inelastically by a stand-in household. The profit maximization of a representative producer of the final good is:

\[
\max_{\ell, [x_t(i)]_{i \in [0,1]}} \int_0^1 A_t(i)x_t(i)^\alpha \ell^{1-\alpha} di - w_t \ell - \int_0^1 p_t(i)x_t(i)di \tag{5.1}
\]

Production of capital goods Each capital good \(x(i)\) is produced by a monopolist. The monopolist \(i\) faces a downward sloping demand curve for its production given by 

\[
p(i) = \alpha A(i) \cdot (\ell/x(i))^{1-\alpha}.
\]

The demand curve is derived from the profit maximization problem (5.1) of final good producers.
A monopolist $i$ rents capital from a representative household to produce capital good $x(i)$. It solves the following maximization problem:

$$\max \alpha x(i)^{\alpha} A(i) \ell^{1-\alpha} - rK(i)$$

subject to $x(i) \leq \frac{K(i)}{A(i)}$. The latter constraint specifies the technology to produce capital good $x$. Solving this problem yields:

$$x_t(i) = \ell \cdot \left( \frac{\alpha^2}{r_t} \right)^{1/(1-\alpha)}, \quad \forall i$$

Notice that $x_t$ is independent of $i$, i.e. each sector has the same value of $x_t(i) = \frac{K_t(i)}{A_t(i)} = x_t(j)$, all $i \neq j$. The rental rate on capital is then $r_t = \alpha^2 x_t(i)^{\alpha-1} \ell^{1-\alpha} = \alpha^2 x_t^{\alpha-1} \ell^{1-\alpha}$. Plugging this into the objective function of the monopolist, and using the fact that $x(i) = \frac{K(i)}{A(i)}$ we get that the profit of the $i^{th}$ monopolist is $\Pi(i) = \alpha(1 - \alpha) A(i) x_t \ell^{1-\alpha}$. The sum of all the profits of all capital goods producers is $\Pi_t = \int \alpha(1 - \alpha) A_t(i) x_t^\alpha \ell^{1-\alpha} di = (1 - \alpha) \cdot \alpha Y_t$. Income of capital rented to the $i^{th}$ firm is $r_t K_t(i) = r_t \frac{A_t(i)}{x_t(i)} = \alpha^2 x_t(i)^{\alpha-1} A_t(i)$. Total capital income in the economy becomes: $r_t K_t = \int r_t K_t(i) di = \alpha \cdot \alpha Y_t$. To summarize, we get:

Profits: \hspace{1cm} \Pi_t = (1 - \alpha) \cdot \alpha Y_t \hspace{1cm} (5.2)

Capital income: \hspace{1cm} r_t K_t = \alpha \cdot \alpha Y_t \hspace{1cm} (5.3)

**R&D** In each period $t$ a new entrepreneur can develop a new generation of capital good $x(i)$ whose contribution to the final output would be $A_{t+1}(i) = A_t(i) \cdot (1 + \gamma)$ where $\gamma > 0$ is a constant.

A new entrepreneur $i$ decides how many units of a final good to invest in the R&D activity in sector $i$. If he invests $N_t(i)$ units, then with probability $\lambda \cdot \frac{N_t(i)}{A_t(i)}$ he will have access to technology $A_{t+1}(i) = A_t(i) \cdot (1 + \gamma)$, starting in period $t+1$. From then on, he has exclusive right to produce capital good $x(i)$. The cost of investing $N_t(i)$ units into R&D is $(1 - \psi_t) N_t(i)$, where $\psi_t$ is a measure of government policy that reduces the costs R&D investments. It can be a tax cut for firms investing in new technologies, but it can also be a reduction in barriers to entry, if new entrants bring in better technologies. We assume the process for $\psi_t$ is i.i.d.
In period $t$, the entrepreneur $i$ solves the following problem:

$$\max PV_t(N_t(i)) - (1 - \psi_t)N_t(i)$$

where $(1 - \psi_t)N_t(i)$ is the cost of investing $N_t(i)$ units while $PV_t$ is the expected payoff from that investment which is given by:

$$PV_t = \lambda \left( \frac{N_t(i)}{A_t(i)} \right) \cdot A_t(i)(1 + \gamma) \sum_{\tau > t} \frac{\prod_{j=\tau}^{\tau-1} \{ \int_{\Psi} (1 - \lambda n_j(\psi_j)) d\mu(\psi_j) \}}{R^{*\tau-t}} \Pi_{\tau}(i).$$

The expected payoff is a present discounted value of the stream of monopolist profits from producing and selling the capital good $x(i)$ in subsequent periods. The flow of profit in period $\tau > t$ is discounted by $(\frac{1}{R^*})^{\tau-t}$ multiplied by the probability that no other innovation in sector $i$ will be made before period $\tau$.

First order condition for that problem yields:

$$\lambda (1 + \gamma) \sum_{\tau > t} \frac{\prod_{j=\tau}^{\tau-1} \{ \int_{\Psi} [1 - \lambda n_j(\psi_j)] d\mu(\psi_j) \}}{R^{*\tau-t}} \Pi_{\tau} = (1 - \psi_t)$$

The above equation implicitly defines

$$\frac{N_t(i)}{A_t(i)} := n(\psi_t)$$

Notice that the ratio $\frac{N(i)}{A(i)}$ is the same for all sectors. Resources spent on R&D in sector $i$ are $N_t(i) = A_t(i)n(\psi_t)$. Total resources used for R&D are:

$$N_t = A_t \cdot n(\psi_t)$$

**Productivity growth** Productivity in sector $i$ in period $t + 1$ is:

$$A_{t+1}(i) = \begin{cases} A_t(i)(1 + \gamma), & \text{w.p. } \lambda n_t(\psi_t); \\ A_t(i), & \text{w.p. } (1 - \lambda n_t(\psi_t)). \end{cases}$$

while the aggregate $A_{t+1}$ is:

$$A_{t+1} = \int A_{t+1}(i)di = \lambda n_t(\psi_t)(1 + \bar{\gamma}) \int A_t(i)di + (1 - \lambda n_t(\psi_t)) \int A_t(i)di$$
which yields $A_{t+1} = ((1 + \gamma n_t(\psi_t)) A_t$. Hence, productivity growth in period $t$ is:

$$\gamma_t = \gamma n_t(\psi_t)$$

From the above equation we see that variation in government policy (which affects innovations in the economy) will have an effect on the growth rate of total factor productivity.

**Household** A stand-in household has preferences over consumption. It rents capital $K$ to monopolists that produce capital goods $x$. It has $\ell$ units of labor available that it supplies inelastically to final good producers\textsuperscript{12}. The utility maximization problem of the household is the following:

$$\max \sum \beta^t U(C_t)$$

s.t.

$$C_t + X_t \leq w_t \ell + r_t K_t + R^* \cdot B_{t-1} - B_t + T_t + \tilde{\Pi}_t$$

$$K_t \leq (1 - \delta) \cdot K_t + X_t$$

$$B_t \geq -\bar{B}$$

where $\tilde{\Pi}_t$ is the flow of net profits from the intermediate good producers which is equal to the gross profits of operating producers of capital goods minus the expenses on R&D from new entrants: $\tilde{\Pi}_t = \int \tilde{\Pi}_t(i) di = \int [\Pi_t(i) - (1 - \psi_t)N_t(i)]di$. Besides the flow of net profits, household has 4 more sources of income: wages ($w_t \ell$), capital income ($r_t K_t$), return on bonds ($R^* B_{t-1}$) and transfers from the government ($T_t$).

**National Accounts** GDP in this economy is the total output of the final good: $GDP_t = \int A_t(i) x_t(i)^{\alpha} \ell di$ and it equals the expenditures on consumption $C$, investment $I$ and net exports $NX$:

$$C_t + I_t + NX_t = GDP_t$$

\textsuperscript{12}Inelastic supply of labor is assumed for ease of exposition and does not alter the results derived in this section.
where aggregate investment $I$ consists of expenditures on $R$&$D$ and of investment in the stock physical capital: $I_t = X_t + N_t$.

5.1.2 A prototype economy with trend shock and capital income wedges

Now we will show that, given the allocations in the detailed economy, we can construct a series of trend shock and capital income wedges in the prototype economy, so that the allocations in the two economies are identical. The way we will proceed is the following. We start with equilibrium allocations in the detailed economy with time-varying $(\lambda_t, \psi_t)$. We will then construct trend shock and capital income wedges such that those allocations that are measured in NIPA are also equilibrium allocation in the prototype economy with wedges.

What do we mean by allocations that are measured in NIPA? For example, both investment and capital stock series are equilibrium allocations in the detailed economy. However, only investment is measured in NIPA. Capital stock is computed using the series for investment and given a particular model (e.g. a neoclassical growth model). In the detailed economy with Schumpeterian growth there are two types of investment expenditures. The first is $N$—expenditure on $R$&$D$ which does not add to the stock of physical capital, but which increases productivity. The second is $X$—investment which adds to the stock of physical capital. In the data, investment expenditure is $I_{data} = X + N$. If we look at the data through the lenses of a neoclassical model, then $X + N$ is what is added to capital stock. If we look at the data through the lenses of the Schumpeterian model described in the previous section, then only $X$ adds to physical capital stock. More precisely, we will prove the following Proposition:

**Proposition 5.1.** Let $(GDP_t^*, C_t^*, X_t^*, N_t^*, B_t^*, NX_t^*, \ell_t^*)_{t=1}^\infty$ be a sequence of equilibrium allocations in the economy described in Section 5.1.1, for a given sequence of $R$&$D$ subsidies $\psi_t\}_{t=1}^\infty$. There exists a sequence of capital income and efficiency trend wedges $(\tau_{k,t}, \Gamma_t)_{t=1}^\infty$ such that the sequence $(GDP_t^*, C_t^*, I_t^*, B_t^*, NX_t^*, \ell_t^*)_{t=1}^\infty$ is also a sequence of equilibrium allocations in the prototype economy with the two wedges, where $I_t^* = X_t^* + N_t^*$. 

25
Proof. See Appendix for the proof.

5.2 From terms of trade shocks to debt price wedge

5.2.1 A detailed economy with terms of trade shocks

Consider a version of an economy with domestic and foreign tradable goods a’la Backus et al. (1994). Consumption and investment are composites of domestically produced tradable good \( a \) and an imported tradable good \( b \):

\[
c_t + x_t = G(a_t, b_t)
\]

where \( c \) is aggregate consumption, \( x \) is aggregate investment and the function \( G \) is assumed to be homogenous of degree one. Production function of good \( a \) is Cobb-Douglas:

\[
y_t^a = A_t k_t^{\alpha_t} \ell_t^{1-\alpha_t}
\]

The price of the domestically produced good is normalized to 1. The price of the foreign tradable good is \( q_t^b \) and the price of the aggregate consumption/investment good, \( p_t \), is the value of the following minimization problem:

\[
p_t \equiv \min_{a_t, b_t} a_t + q_t^b \cdot b_t, \quad \text{subject to } G(a_t, b_t) = 1.
\]

The economy has access to an international risk-free bond, denominated in units of the foreign good\(^{13}\). A stand-in household maximizes expectation of the flow of utility from consumption and leisure:

\[
E_0 \sum_t \beta_t [c_t^\eta (1 - \ell_t)^{1-\eta}]^{1-\sigma} \frac{1}{1 - \sigma}
\]

subject to (i) the budget constraint

\[
p_t c_t + p_t x_t + q_t^b d_t \leq w_t \ell_t + r_t k_t + \frac{1}{R^*} \cdot q_t^b d_{t+1},
\]

\(^{13}\)Many emerging economies suffer from “original sin”—large fraction of their debt is denominated in foreign currency (Eichengreen et al. (2003)).
(ii) law of motion for capital stock: \( k_{t+1} \leq (1 - \delta)k_t + x_t \); and (iii) a no-Ponzi condition. For simplicity we are considering the case of the economy with no adjustment costs on both capital and debt accumulation.

### 5.2.2 A prototype economy with wedges

We will now show the equilibrium allocations in the economy described in Section 5.2.1 are identical to allocations in a prototype economy with labor, capital income and debt price wedges. The intratemporal consumption-leisure trade-off in the detailed economy is \( -U_{\ell,t}/U_{c,t} = \frac{1}{p_t} \cdot F_{\ell,t} \). The equivalence requires labor wedge to be:

\[
1 - \tau_{\ell,t} = \frac{1}{p_t}
\]

Next, the Euler equation for capital in the detailed economy is \( U_{c,t} = \beta E_t U_{c,t+1} [1 - \delta + \frac{1}{p_{t+1}} F_{k,t+1}] \).

We will then define capital income wedge to be:

\[
1 - \tau_{k,t} = \frac{1}{p_t}
\]

Finally, the Euler equation for debt in the detailed economy is \( U_{c,t} = R^* E_t U_{c,t+1} \frac{1}{\hat{q}_{t+1}} \frac{p_t}{q_t} \). In the prototype economy (without adjustments costs on debt accumulation) that equation is \( (1 - \tau_q) U_{c,t} = R^* E_t U_{c,t+1} \). Simple algebra shows the two conditions are equivalent if the debt price wedge, \( \tau_q \) is:

\[
\tau_q = R^* E_t \frac{U_{c,t+1}}{U_{c,t}} \left[ \frac{q_{t+1}}{q_t} \frac{p_t}{p_{t+1}} - 1 \right]
\]

where the RHS is evaluated at the equilibrium allocations and prices of the detailed economy.

### 5.3 Emerging Markets through the Lenses of a Diagnostic Economy

Finally, we would like to formally lay out the relationship between our prototype economy with wedges and the literature on emerging markets business cycle. We will consider two theories. First is the model with trend shocks of Aguiar and Gopinath (2007). The second is the model with interest
rate shocks and financial frictions of Neumeyer and Perri (2005) and Uribe and Yue (2006). We chose these two theories because they have attracted much attention in the literature. Also, recent studies that confronted models of emerging markets business cycles with the data (Garcia-Cicco et al. (2010), Chang and Fernandez (2010)), focused their attention on these two models.

We have to stress, however, that our exercise did not formally test the two models against each other. Nor did, in any way, assess which theory was “better”, or closer to the data. The purpose of this section is to ease the interpretation of our quantitative results from Section 4 in the context of the particular strand of literature.

5.3.1 Non-stationary Productivity Shocks

Aguiar and Gopinath (2007) (AG henceforth) exploit permanent income hypothesis to account for excess volatility of consumption and counter-cyclical trade balance in emerging economies. In addition to traditional shocks to the level of TFP, they introduce shocks to the growth rate of TFP. The growth shocks make a big difference, because their effects are permanent. A model with large relative variance of the growth shocks accounts successfully for the behavior of consumption and trade balance in emerging economies.

The benchmark specification of the AG economy with Cobb-Douglas preferences is a special case of our prototype economy with only two wedges: the efficiency wedge and the trend-shock wedge. However, AG also consider a specification with quasi-linear preferences represented by the GHH utility function (Greenwood et al. (1988)): \( U(C, \ell) = \frac{[C_l - \psi(\ell)]^{1-\sigma}}{1-\sigma} \). Theoretically, there are two equivalent ways to incorporate this latter specification in our exercise. We can consider the prototype economy from Section 3 but with GHH utility function. In that case, the AG economy will again be a special case of our prototype economy. Following this strategy will require re-estimation of the process for wedges. Alternatively, we can derive a mapping between the AG economy with GHH preferences and the prototype economy with Cobb-Douglas preferences. We decided to follow the latter strategy. The reason follows from our discussion of wedges in Section 2: a wedge is always within a context of a particular prototype economy. Considering alternative
prototype economy would, in our view, make the analysis more opaque.

**Mapping: from AG economy with GHH preferences to wedges**  In the AG economy with GHH preferences, the allocations solve the following social planner problem:

$$\max E_0 \sum \beta^t [C_t - \psi \Gamma_{t-1} \ell_t^{\nu}]^{1-\sigma}/(1-\sigma)$$

subject to:

$$C_t + X_t + D_t = A_t K_t^\alpha (\Gamma_t \ell_t)^{1-\alpha} + q_t D_{t+1}$$

$$K_{t+1} = (1 - \delta) K_t + X_t$$

where the laws of motion for exogenous shocks are:

$$\log A_t = \rho \log A_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2)$$

$$\log \Gamma_t = \log \Gamma_{t-1} + g_t$$

$$g_t = (1 - \rho_g) \mu + \rho_g \log g_{t-1} + \epsilon_{g,t}, \quad \epsilon_{g,t} \sim N(0, \sigma_{g}^2)$$

The proposition we will prove is the following.

**Proposition 5.2.** Consider the sequence of equilibrium allocations in the AG economy with GHH preferences. There exists a sequence of wedges \( \{ \log A_t, \tau_{\ell,t}, \tau_x,t, g_t, \log \gamma_t, \tau_{q,t} \} \) such that the allocations from the AG economy are also the equilibrium allocations in the prototype economy with wedges.

*Proof. See Appendix.*

### 5.3.2 The Role of Interest Rate Shocks and Financial Frictions

Neumeyer and Perri (2005) (NP henceforth) and Uribe and Yue (2006) develop small open economy models where shocks to interest rates coupled with financial frictions account for output, consumption and trade balance movements in emerging economies. In this section we will derive the realizations of wedges in the prototype economy necessary to account for the allocations arising in the NP economy.
Detailed economy with Interest Rate Shocks and Working Capital Constraints

In the NP model a stand-in household has quasi-linear preferences represented by the GHH utility function: \( U(C, \ell) = \frac{(C - \psi \ell)^{1-\sigma}}{1-\sigma} \). The budget constraint for the household and the law of motion for capital are standard:

\[
C(s^t) + I(s^t) + R(s^{t-1})D(s^{t-1}) \leq w(s^t)\ell(s^t) + r(s^t)K(s^{t-1}) + D(s^t)
\]

\[
K(s^t) = (1 - \delta)K(s^{t-1}) + I(s^t) - \left(\frac{K(s^t)}{K(s^{t-1})} - \bar{\gamma}\right)^2 K(s^{t-1})
\]

The departure from the neo-classical growth model comes on the firms’s side. Firms are “financially” constrained in the following sense. A fraction \( \theta \) of the wage bill must be paid before the production takes place. Because of that firms must borrow the total amount of \( \theta w_t \ell_t \) at the beginning of period \( t \) and repay it at the end. The firms borrow at the gross interest rate of \( R(s^t) \), which is determined at the beginning of period \( t \), after the realization of all shocks. With this assumption about the timing, the profit maximization problem of a typical firm is static:

\[
\max \tilde{A}_t F(K_t, \Gamma_t \ell_t) - w_t \ell_t - r_t K_t - (R_t(s^t) - 1) \theta w_t \ell_t
\]

where the last term comes from interest the firms must pay on the loans taken to finance the wage bill. The interest rate \( R \) consists of two components—the world interest rate \( R^* \) and a country spread \( CS: R(s^t) = R^*(s^t) \cdot CS(s^t) \). The economy grows at a constant rate \( \bar{\gamma} \), so that \( \Gamma_t = \bar{\gamma} \cdot \Gamma_{t-1} \).

There are stationary shocks to the level of total factor productivity (TFP): \( \log \tilde{A}_t = \rho \log \tilde{A}_{t-1} + \epsilon_t \).

**Proposition 5.3.** Consider the sequence of equilibrium allocations in the NP economy described in Section 5.3.2. There exists a sequence of wedges \( \{\log A_t, \tau_{t, \ell}, \tau_{x,t}, g_t, \log \gamma_t, \tau_{q,t}\} \) such that the allocations from the NP economy are also the equilibrium allocations in the prototype economy with wedges, with the exception of net external household debt being: \( D_t+1(s^t) = R(s^t)\bar{D}(s^t) \), where the LHS corresponds to the prototype economy and the RHS corresponds to the NP economy.

**Proof.** See Appendix.

\( \square \)
6 Summary and Conclusions

We extended the method developed in Chari et al. (2007) (Business Cycle Accounting) to study fluctuations in a small open economy. In particular, we used the behavior of trade balance as a source of information about the possible origins of fluctuations. In order to do so we added two more wedges - a debt price wedge and a trend shock wedge. We provided equivalence results. First, we showed the NIPA-measured allocations in a detailed economy with time varying policies which foster or inhibit innovation are identical to NIPA-measured allocations in the prototype economy with movements in trend shock and capital income wedge. Second, we showed the allocations in a detailed economy with terms of trade shocks are identical to allocations in the prototype economy with movements in debt price wedge and in capital income wedge.

We applied the method to understand sources of fluctuations in emerging economies, with a particular focus on two theories: (i) trend shocks of Aguiar and Gopinath (2007) and (ii) interest rate shocks of Neumeyer and Perri (2005) (NP) and Uribe and Yue (2006). We showed analytically, the NP framework with GHH preferences, interest rate shocks and working capital constraint, is equivalent to a prototype model with Cobb-Douglas preferences and 3 wedges: investment, trend shock, labor and debt price. Thus, the model in Aguiar and Gopinath (2007) with its restriction to only two sources of fluctuations, captures part of the frictions emphasized in Neumeyer and Perri (2005) and Uribe and Yue (2006).

Empirically, we studied fluctuations of the Mexican economy during the period 1987-2010. In particular, we performed the accounting exercise on the Sudden Stop that occurred during the Tequila Crisis. Our results indicate that trend shocks alone account for about 22% of the Sudden Stop, while movements in the labor wedge combined with movements in the debt price account for 65% of the Sudden Stop. Together, trend shock wedge, labor wedge and debt price wedge account for 94% of the Sudden Stop. We interpret it as an indication of complementarity between two major theories of Emerging Markets Business Cycles. Each theory captures important mechanisms that contributed to the Sudden Stop during the Tequila Crisis.
Our results indicate the major role played by the labor wedge as a driver of the sudden stop and of the excess response of consumption, *in our diagnostic economy*. We think that future studies of emerging markets business cycles should emphasize their implications for the behavior of the labor wedge (this conclusion refers more generally to studies that focus on the excess volatility of consumption).

The BCA has attracted much attention (both positive (Inaba and Nutahara (2012)) and critical (Christiano and Davis (2006))). We believe BCA is a very useful and informative tool. However, it would be helpful if future studies (both empirical and theoretical) restricted their analysis to the same prototype economy. This will facilitate the comparison across papers and make the literature more transparent. While a-priori there is no reason to choose one functional form over any other, we chose to work with Cobb-Douglas utility and production function which are probably the most common benchmark specifications and are analytically very tractable.

Naturally, the CKM method can be further extended, in order to exploit the information contained in other important macroeconomic variables. From the perspective of international macroeconomics, a useful extension to consider would be one that exploits the information contained in movements of international prices. It may be helpful as a guidance for theories that would help us understand the behavior of perhaps the most puzzling variables: real exchange rates and terms of trade.

References


A Proofs of Equivalence Results

A.1 Proof of Proposition 5.1

Euler Equation We start with inter-temporal Euler equations. The Euler equations in the two economies are:

\[
U_{c,t}^d = \beta E_t U_{c,t+1}^d \left(1 - \delta + r_{t+1}^d\right)
\]
\[
U_{c,t}^P = \beta E_t U_{c,t+1}^P \left[1 - \delta + (1 - \tau_{k,t+1}) r_{t+1}^P\right]
\]

The two conditions are equivalent if:

\[
\begin{align*}
U_{c,t}^d &= U_{c,t}^P \quad (A.1) \\
U_{c,t+1}^d &= U_{c,t+1}^P \quad (A.2) \\
r_{t+1}^d &= (1 - \tau_{k,t+1}) r_{t+1}^P \quad (A.3)
\end{align*}
\]

The first two conditions hold by construction. The third condition yields the following relationship between the capital income wedge, stock of productivity, capital stock in the prototype economy and amount of capital good produced in the detailed economy:

\[
(1 - \tau_{k,t}) K_t^{\alpha - 1} \Gamma_t^{1-\alpha} = \alpha x_t^{\alpha - 1},
\]

which is equivalent to:

\[
\Gamma_t^{1-\alpha} = \alpha x_t^{\alpha - 1} K_t^{1-\alpha} \frac{1}{1 - \tau_{k,t}} \quad (A.4)
\]

GDP and capital stocks Next we look at the definition of the Gross Domestic Product (GDP). In the detailed economy it is \(Y_t^d = \int A_t(i) x_t^{\alpha} \ell^{1-\alpha} di\), while in the prototype economy it is \(Y_t^P = K_t^{\alpha} (\Gamma_t \ell)^{1-\alpha}\). Equivalence requires that \(A_t x_t^{\alpha} \ell^{1-\alpha} = K_t^{\alpha} (\Gamma_t \ell)^{1-\alpha}\) where \(A_t = \int A(i) di\). The last condition, combined with (A.4) gives that \(A_t x_t = \alpha K_t^P \frac{1}{1 - \tau_{k,t}}\). Since \(A_t x_t = K_t^d\), we get the following relationship between stocks of physical capital in the two economies:

\[
K_t^P = K_t^d \frac{1 - \tau_{k,t}}{\alpha} \quad (A.5)
\]
Investment  Investment expenditure in the prototype economy is $I_t^P = X_t^P = K_{t+1}^P - (1 - \delta)K_t^P$, while investment expenditure in the detailed economy is $I_t^d = X_t^d + N_t$, where $I$ is investment expenditure (net of taxes) while $X$ is the investment in physical capital. Notice that we explicitly take into account the fact that R&D expenditures are classified as investment expenditures. They do not add to the stock of physical capital (machines) but rather to the know-how and development of new generation of capital goods. Equivalence of the NIPA-measured allocations requires $I_t^d = I_t^P$ which means $X_t^P = X_t^d + N_t$.

Recovering wedges  The law of motion for capital stock in the prototype economy is:

$$K_{t+1}^P = (1 - \delta)K_t^P + X_t^d + N_t$$

Using (A.5) we can write:

$$\frac{1 - \tau_{k,t+1}}{\alpha} K_{t+1}^d = (1 - \delta) \frac{1 - \tau_{k,t}}{\alpha} K_t^d + X_t^d + N_t$$

which gives an expression for $\tau_{k,t+1}$ as a function of $\tau_{k,t}$ and allocations in the detailed economy:

$$\tau_{k,t+1} = f\left(\tau_{k,t}, K_t^d, X_t^d, N_t\right)$$

(A.7)

Given the sequence of $\{\tau_{k,t}\}$ we obtain the sequence of $\{\Gamma_t\}$ using (A.5) and (A.4):

$$\Gamma_t = \left(\frac{\alpha}{1 - \tau_{k,t}}\right) \frac{\tau_{k,t}}{\alpha} K_t^d$$

(A.8)

Verifying budget constraint  By construction, NIPA-measured allocations are the same in the two economies. We then used the equilibrium Euler equations (together with definition of GDP and
law of motion for capital stock), to construct wedges such that the allocations satisfy the equilibrium
Euler equations in the prototype economy. What remains to be shown is that, given the sequence
of wedges computed using (A.7) and (A.8), the allocation satisfies the budget constraint in the
prototype economy. The budget constraints in the two economies are:

\[
C^*_t + X^*_t = w_t\ell + r^d_t K^d_t + R^* B^*_{t-1} - B^*_t + T^d_t + \Pi_t - (1 - \psi_t) N^*_t^d \quad \text{(detailed)}
\]
\[
C^*_t + I^P_t = w_t\ell + (1 - \tau_{k,t}) r^P_t K^P_t + R^* B^*_{t-1} - B^*_t + T^P_t \quad \text{(prototype)}
\]

Subtracting the two equalities from each other we get the following condition must hold:

\[
-N^*_t = r^d_t K^d_t - (1 - \tau_{k,t}) r^P_t K^P_t + T^d_t + \Pi_t - (1 - \psi_t) N^*_t - T^P_t
\]

Since in the prototype economy we have \( T^P_t = \tau_{k,t} r^P_t K^P_t \) that condition becomes: \(-\psi N^*_t = r^d_t K^d_t - r^P_t K^P_t + T^d_t + \Pi_t \). Using (5.2) and (5.3) and the fact that by construction \( Y^d_t = Y^P_t \) we get that \( \Pi_t + r^d_t K^d_t = r^P_t K^P_t \). Hence, budget constraint is satisfied, because we know that in the
detailed economy we have: \( T^d_t = -\psi_t N^*_t \), since R&D subsidies (taxes) are financed (spent on)
lump-sum taxes (transfers).

### A.2 Proof of Equivalence Result: the AG economy

We will now construct the sequences of wedges that ensure the equilibrium allocations in the two
economies are the same. We start with the labor wedge, which is obtained from the equilibrium
conditions equating \( MRS_{c,\ell} \) with \( MPL \) in the two economies:

\[
\psi \nu \ell_t(s^{t})^{\nu-1} = (1 - \alpha) \frac{Y_t(s^t)}{\ell(s^t)}, \quad \text{(AG with GHH preferences)}
\]
\[
\frac{C_t(s^t)}{1 - \ell_t(s^t) - \eta} = (1 - \tau_{\ell,t}(s^t))(1 - \alpha) \frac{Y_t(s^t)}{\ell(s^t)}, \quad \text{(prototype)} \quad (A.9)
\]

Simple algebra shows the allocations from the AG economy with GHH preferences satisfy (A.9)
when the labor wedge is:

\[
1 - \tau_{\ell,t}(s^t) = \frac{C^*(s^t)}{1 \ell^*(s^t)} \frac{1 - \eta}{\eta} \frac{1}{\nu \psi \ell^*(s^t)^{\nu-1}}
\]
Next, consider the inter-temporal capital Euler equation in the two economies:

\[ e^{\eta t} U_{c,t} = \beta E_t U_{c,t+1} \left[ 1 - \delta + \alpha Y_{t+1}/K_{t+1} \right], \quad (\text{AG with GHH preferences}) \]

\[ \gamma_t^{1-\eta(1-\sigma)}(1 + \tau_{x,t}) U_{c,t}^P = \beta E_t U_{c,t+1}^P \left[ (1 - \delta)(1 + \tau_{x,t+1}) + \alpha Y_{t+1}/K_{t+1} \right], \quad (\text{prototype}) \quad (A.10) \]

For the AG allocations to satisfy (A.10) it is sufficient the following conditions hold across all states:

\[ e^{\eta t} U_{c,t} = \gamma_t^{1-\eta(1-\sigma)}(1 + \tau_{x,t}) U_{c,t}^P, \quad t = 1, 2, 3, ... \quad (A.11) \]

\[ U_{c,t} [1 - \delta + \alpha Y_t/K_t] = U_{c,t}^P \left[ (1 - \delta)(1 + \tau_{x,t}) + \alpha Y_t/K_t \right], \quad t = 2, 3, 4, ... \quad (A.12) \]

The second equation defines \( \tau_{x,t} \) for every \( t = 2, 3, 4, ... \). Then, the trend efficiency wedge for \( t = 2, 3, 4, ... \) in the prototype economy can be computed using (A.11). Notice, that in order to calculate the implied sequence for the trend efficiency wedge \( \{\gamma_t\} \) we would have to know the realizations of the original trend shocks \( \{g_t\} \) from the AG economy.

### A.3 Proof of the Equivalence Result: the NP economy

We construct the sequences of wedges that ensure the equilibrium allocations in the two economies are the same. We start with the labor wedge, which is obtained from the equilibrium conditions equating \( MRS_{c,\ell} \) with \( MPL \) in the two economies:

\[ \psi \nu \ell_t(s^t)^{\nu-1} = \frac{1}{1 + \theta(R(s^t) - 1)} (1 - \alpha) \frac{Y_t(s^t)}{\ell(s^t)}, \quad (\text{NP}) \]

\[ C_t^*(s^t) \frac{1 - \eta}{1 - \ell_t(s^t)} = (1 - \tau_{x,t}(s^t))(1 - \alpha) \frac{Y_t(s^t)}{\ell(s^t)}, \quad (\text{prototype}) \quad (A.13) \]

Simple algebra shows the allocations from the NP economy satisfy (A.13) when we labor wedge is:

\[ 1 - \tau_{x,t}(s^t) = \frac{C^*(s^t) \frac{1 - \eta}{1 - \ell^*(s^t)}}{\eta \left[ 1 + \theta(R(s^t) - 1) \right]} \]

Next, consider the inter-temporal capital Euler equation in the two economies:

\[ e^{\eta t} U_{c,t} = \beta E_t U_{c,t+1} \left[ 1 - \delta + \alpha Y_{t+1}/K_{t+1} \right], \quad (\text{NP}) \]

\[ \gamma_t^{1-\eta(1-\sigma)}(1 + \tau_{x,t}) U_{c,t}^P = \beta E_t U_{c,t+1}^P \left[ (1 - \delta)(1 + \tau_{x,t+1}) + \alpha Y_{t+1}/K_{t+1} \right], \quad (\text{prototype}) \quad (A.14) \]
For the NP allocations to satisfy (A.14) it is sufficient the following conditions hold across all states:

\[ \bar{\gamma}^\sigma U_{c,t} = \gamma_t^{1-\eta(1-\sigma)} (1 + \tau_{x,t}) U_{c,t}^P, \quad t = 1, 2, 3, ... \] (A.15)

\[ U_{c,t} [1 - \delta + \alpha Y_t/K_t] = U_{c,t}^P [(1 - \delta)(1 + \tau_{x,t}) + \alpha Y_t/K_t], \quad t = 2, 3, 4, ... \] (A.16)

Then, for every \( t = 2, 3, 4, ... \) we use the latter condition to define \( 1 + \tau_{x,t} \) to be:

\[ 1 + \tau_{x,t} = \frac{U_{c,t}}{U_{c,t}^P} \left[ 1 + \alpha \frac{Y_t/K_t}{1 - \delta} \right] - \alpha \frac{Y_t/K_t}{1 - \delta} \]

and we use (A.15) to define \( \gamma_t \) for \( t = 2, 3, 4, ... \):

\[ \gamma_t = \left[ \frac{1}{1 + \tau_{x,t}} \frac{\bar{\gamma}^\sigma U_{c,t}}{U_{c,t}^P} \right]^{\frac{1}{1-\eta(1-\sigma)}} \]

Finally, the debt price wedge is given by:

\[ 1 - \tau_{q,t}(s^t) = \frac{1}{Q \cdot R(s^t)} \]