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Rise of the Machines Redux – Education, Technological Transition and Long-run Growth

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Abstract

We develop a growth model with over-lapping generations that endogenizes skill acquisition and two forms of technical change, one that automates existing production processes, and one that invents new production processes. The former kind of technological change obsoletes certain middle-range skills; the latter has the potential to increase such skills. This work suggests that 1) early industrialization generates greater automation; 2) employment polarization caused by automation also fosters education polarization, potentially affecting future growth; 3) the economy naturally transitions from automation to innovation; and 4) such a transition today will lessen wage inequality but may *not* bring back mid-skilled jobs as it had historically.

- *Keywords*: employment polarization; job polarization; directed technical change; human capital; unified growth theory; skill obsolescence; routinization; automation
- *JEL Codes*: J24, J31, O31, O33

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1 Introduction

This paper takes technological pessimists seriously. They have recently gained a large and ever-growing audience as new technologies replace skills thought previously to be immune to the forces of codification and mechanization, and as growth in wages and living standards continue to falter. These trends however are neither a-historical nor unprecedented — they have deep roots in the early industrialization of western Europe and North America.

Techno-pessimism these days come in one of three broad flavors. The first is the claim that most meaningful technologies have already been invented. According to this story, narrated compellingly by Tyler Cowen and Robert Gordon among many others, recent technologies are essentially derivative of earlier more substantive developments, contributing to aggregate productivity only marginally (Cowen 2011, Gordon 2015). The second suggests that technological developments are in fact meaningful, but that they come with the unpalatable bi-products of job and skill losses. Autor and Dorn (2013) develop the quintessential theory, while Frey and Osborne (2013) provide a much-discussed source of evidence, looking across over 700 contemporary industries and concluding that around half are in danger of this sort of job destruction. The final flavor is an oldie but goody — secular stagnation can befall the economy when technological growth is 'unbalanced,' as factors in technologically advancing sectors reallocate to technologically stagnant sectors (Baumol and Bowen 1966).

In a sense this paper carves out a techno-pessimist paradise, incorporating all three potentials in a growth model. Our primary goal is to understand how *education* decisions can co-evolve with such technological developments. To that end we develop a growth theory that carefully develops the co-evolution of technologies and education. Specifically, we develop a growth model with over-lapping generations that endogenizes skill acquisition and *two* forms of technical change, one that automates the production of existing goods, and one that increases the number of types of new goods. The former kind of technological change obsoletes certain middle-range skills but can raise the value of higher-range analytical skills. These technological changes in turn affect education decisions by households.

The framework allows us to consider the potential historical developments in technology and education over the last few centuries. It also provides the potential to unify our growth experiences in history with our current economic circumstances.[1]

Through solving and simulating the model, we produce a number of novel findings. First, by endogenizing the paths of technological progress, we suggest that early industrialization focused mostly on automation. This created employment and wage polarization as mid-level skills are obsoleted, leaving erstwhile routine workers little choice but to join manual occupations.

Second, employment polarization is consistent with *educational* polarization. Technological developments can raise the riskiness of mid-level education in terms of earnings, compelling some individuals either to get more education, or none at all. We both establish this empirically for the U.S. economy, and demonstrate this theoretically in the growth model. This can have implications for longer-term growth, as we demonstrate in latter sections.

Third, the early economy naturally transitions from an automative economy to an innovative one. This can reverse the polarizing trends created by automation. Here, unlike Autor and Dorn (2013), we show that technological progress need not *inevitably* lead to either employment or wage polarization. Our current era of automation however is fostering both, and is reminiscent of an earlier period of Western industrial history.

Finally, the transition to modernity can lessen inequality but can also create a de-skilling effect all around, due to the rising demand of jobs which require little to no education. This has implications for many factors not explicitly modeled here, such as lower effectiveness of democratic institutions (Glaeser et al. 2007). And since education can help maintain a vibrant middle class, de-skilling can stoke social strife. Routine jobs of the past, the source of such robust growth for twentieth-century middle-income households, may never again return.

Further, the suggested slowdown in accumulation of high-end skills also has the potential to slow down the overall economy (see for example Lucas 1988). We show however that given "reasonable" parameterization, we should not expect this. The model suggests that economic slowdown will arise mainly from unbalanced growth, not from loss of high-end skills.

[1]This work is however not intended to be a proper unified growth theory, as framed by Galor (2011). For example we abstract away from fertility here and therefore cannot comment on the nature or timing of the Demographic Transition.

Related Literature

This work fits in with a number of growth literatures. The first are growth models which generate technological transitions or cycles of economic activity. These include models focused on adoption and implementation of general purpose technologies such as Helpman and Trajtenberg (1998) and Howitt (1998). Other works generate cycles by differentiating between technological breakthroughs and improvements, such as Cheng and Dinopoulos (1992), or creating a delay in implementation of new technologies as in Felli and Ortalo-Magne (1997).

Another strain of literature aims to unify different phases of economic growth in one consistent model. These so-called "unified growth" theories endeavor to model Malthusian growth dynamics and also allow for a transition to modern economic growth (Galor and Weil 2000. See Galor 2005, 2011 for detailed summaries of this literature). This paper abstracts away from fertility, and so cannot comment on either Malthusian traps or the Demographic Transition. We do however attempt to understand long-run phases of technological progress and education in history. Specifically, we suggest that early industrialization was automative in nature, while this phase later gave way to more fundamental growth. This approach differs markedly from extant unified growth theories.

Our model also joins those that make a distinction between fundamental and secondary innovation. These include Young (1991,1993), Lucas (1993), Jovanovic and Nyarko (1996) and Redding (2002). In this last paper fundamental knowledge destroys a portion of secondary knowledge, where in our model secondary knowledge destroys certain skills.

Perhaps more closely related to this paper are two recent working papers. Acemoglu and Restrepo (2016) develop a theory where secondary innovation (which they call 'automation') can run ahead of more fundamental innovation, but this will tend to self-correct and reverse. In their model automation lowers labor costs, inducing newer labor-intensive technological developments. Here we suggest two differences that fundamentally change this outcome — endogenous skill acquisition, and the inclusion of a (relatively) unskilled service sector. In a world where mid-level education or training must be associated with a specific and established production process, automation will destroy not only jobs but *skills*, making investments in such middling skills less palatable. Further, new technologies eventually require new mid-level skills to operate them.

But if displaced mid-skilled workers end up in unskilled service occupations, education may not rise to meet this demand. Thus rather than self-correct, we demonstrate the potential for de-routinization to be self-reinforcing. Rather than fixate on balanced growth, we show that technological forces can be directed at one form of progress for a prolonged period of time.[2]

Hemous and Olsen (2014) also develop a growth model with two forms of technical progress. Here they have exogenous levels of high and low skilled labor, with automation replacing the low skilled. While they do include an extension with mid-level skills, the primary focus is on labor-replacing technologies, not skill-replacing ones. Their work also provides little insight into *historic* growth. The model suggests that fundamental innovation occurs first, giving way subsequently to automation. This perhaps fits some contemporaneous economies but does not conform with early industrialization, where few fundamental breakthroughs occurred, wage growth was limited or nonexistent, and processes were automated and mechanized causing mass displacement of many erstwhile valuable skills. In contrast our model endogenously allows for an initial takeoff in automation.

The rest of the paper proceeds as follows. Section 2 provides some background discussion and demonstrates some contemporary relationships between education, tasks and technology. Section 3 describes the model in steps, first describing production, then the two forms of technological changes, then the endogenous individual skill choice. Section 4 provides simulations of the model to help us understand both historic and contemporary growth patterns. To that end we show separate cases where the two forms of technologies grow either exogenously or fully endogenously. Section 5 provides some parting thoughts.

[2]Ray (2010) suggests that models focused on balanced growth paths are both unrealistic and not very helpful in understanding growth processes. And according to Temple (2003), "a balanced growth path is a special case which is likely to demand restrictive assumptions...it would be a mistake to assume that a good model of growth necessarily gives rise to a BGP." As it is, any model with a Baumol-like cost component will have an uninformative BGP of zero.

2 Background

2.1 Different Forms of Technological Change

This paper tackles some large and pressing questions. For example, why is there such pessimism over technological progress these days, even as we witness the rapid advancement of many new technologies? One strain of the argument that we are experiencing technological stasis (Cowen 2011 among others) hinges on the idea that the world has essentially run out of big ideas. Gordon (2000, 2015) suggests the big fundamental innovations had all been made by the late 19th and early 20th centuries.[3] Field (2011) is even more precise, declaring the 1930s as the most productive decade in terms of technological breakthroughs.

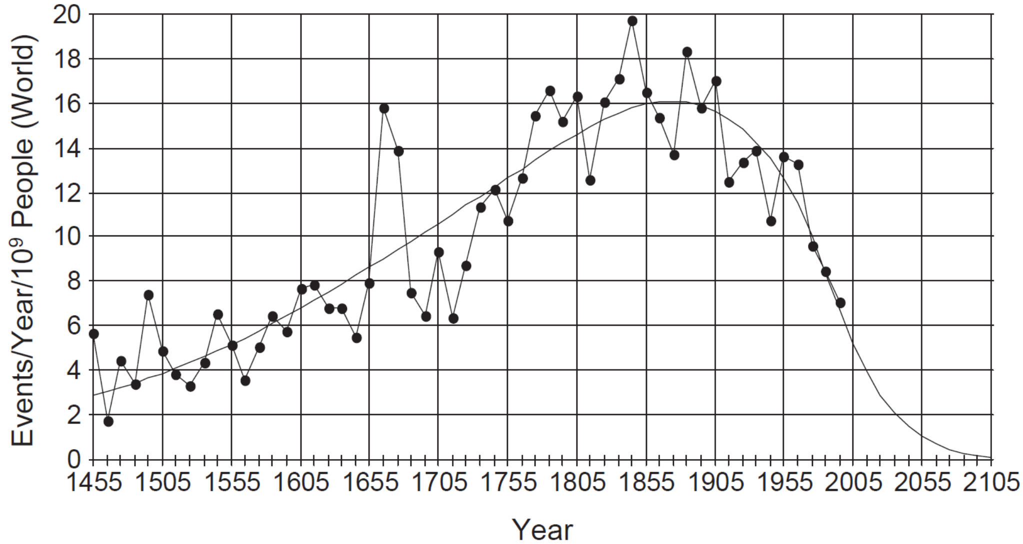
The techno-pessimistic sentiment is nicely captured by the top diagram in Figure 1 (replicated from Huebner 2005). It charts rates of innovation per capita for the world beginning in the fifteenth century and projected into the twenty-second. Here we see a steady rise in innovative activity, followed by a burst consistent with the Industrial Revolution. Innovations appear to peak in the late 19th century, after which point innovations start declining. At its current trend the world will fall below innovation rates achieved during the Middle Ages before mid-century!

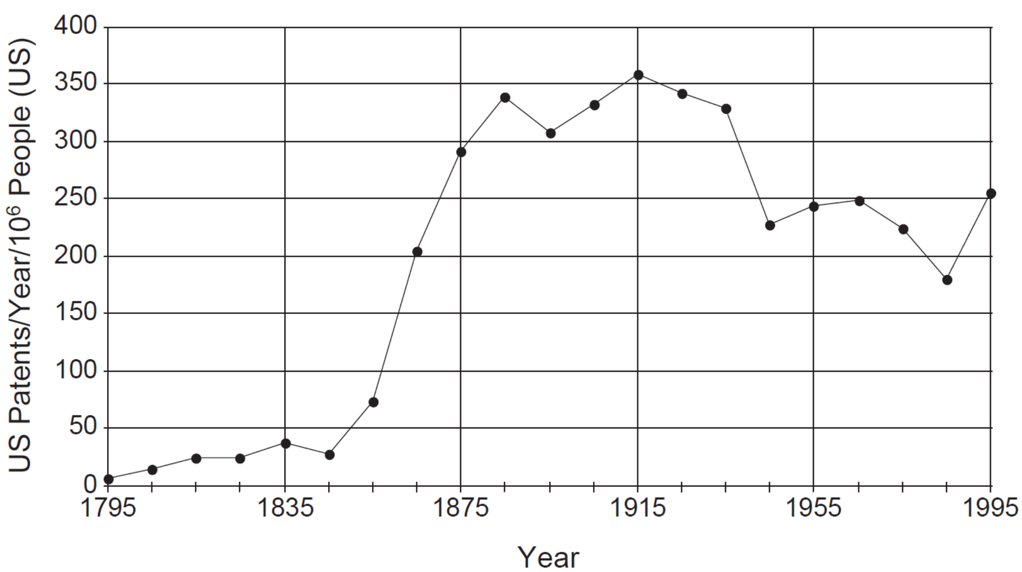
At the same time however, research activities appear to continue unabated. Taken from the same study, the bottom diagram in Figure 1 charts patents per capita for the United States over the past two centuries. While patent filings took a steep drop in the inter-war years, they have remained roughly consistent since the second World War. Jones (2002) in fact suggests patent filings rose during the second half of the twentieth century. Although there are more people in research oriented positions than ever before, each is far less innovative than their predecessors (Zoulay and Jones 2006; Bloom et al. 2016).

To deepen the puzzle, lack of technological change would seem inconsistent with the recent notion of technological *unemployment* — the fear that machines and robots are starting to automate many of our jobs. Brynjolfsson and McAfee (2014) suggest machine intelligence has been on the rise for some time and will soon be everywhere, creating uncertainty over once stable

[3] These include electrification, internal-combustible engines, indoor plumbing, petro-chemicals, and the telephone.

Figure 1: Historical Rates of Innovation





Source: Huebner (2005). Points are an average over 10 years with the last point covering the period from 1990 to 1999.

middle-skill jobs.[4] By this line of reasoning, current technological change has been robust and transformative, even if a bi-product of this change has been unwelcome.

This work attempts to reconcile these different views on the past and future paths of technology by distinguishing between two types of technological change. In our model innovators can either "tinker" with existing forms of technologies, thus helping to automate existing production processes, or perform more basic research to develop altogether new processes. A key feature here is that successful tinkerers will improve old technologies but in the process obsolete the skills associated with the old technology. Basic researchers on the other hand create brand new industries, producing future opportunities for mid-skilled workers but leaving current groups of mid-skilled unharmed.[5] Here we echo the idea that society can potentially provide "too little" investment in basic research (Jones and Williams 2000), and provide a mechanism to help understand why.

2.2 The "Machine Question" in History

This approach has the benefit of connecting the past with the present. For example Khan (2015) suggests that early growth arose mainly from the efforts of tinkerers (resolving "perceived industrial problems"), but then this eventually gave way to the scientists, engineers and technicians performing more basic and breakthrough research. Characterizing this early period as the "tinkering economy" would be consistent with the discussions of the time, which focused far less on any "industrial revolution" than on the more alarming "machinery question."[6]

While the cry of mechanism sounds shrill from time to time (today being one such time), an

[4]An apt anecdote from their book is about the Dutch chess grandmaster Jan Hein Donner, who was asked how he would prepare for a chess match against a computer. Donner replied, much like a dour Luddite from centuries ago: "I would bring a hammer."

[5]This approach is similar in spirit to Aghion and Howitt (1994), which also looks at job destruction with technological growth. Their work looks however at unemployment, whereas we look at the effects to education. Young (1993) includes both invention and learning-by-doing in this model, without focusing on skill obsolescence.

[6]This was first posed by David Ricardo in 1821, concerning the 'influence of machinery on the interests of the different classes of society,' especially the laboring classes. "Thomas Carlyle (1839) railed against the 'demon of mechanism' whose disruptive power was guilty of 'oversetting whole multitudes of workmen.' A pamphlet written during this period reads: "Never until now did human invention devise such expedients for dispensing with the labor of the poor."

undercurrent of complaint has always existed since the dawn of industrialization.[7] For example, a New York Times headline in 1928 reads "March of the Machines Makes for Idle Hands." In the 1930s Keynes coined the term 'technological unemployment.' During the 1940s the New York Times periodically reported on the revival of these worries, labeled as the "old argument." In the early 1960s the U.S. Administration suggested a major challenge was the goal to 'maintain full employment at a time when automation....is replacing man.' Our approach demonstrates that while the potential for automation always exists, along with the concomitant voices of anxiety, so too does breakthrough knowledge to potentially countervail these effects.

The model here may shed some light on how these two apparent growth regimes are related. Further, the model suggests our current economy may resemble in some ways early industrialization and the age of tinkerers. Contrary to Brynjolfsson and McAfee (2014), who suggest that what we face in the 21st century is somehow new, we allege that the technological hurricane we currently see is one that we have weathered before.

This paper suggests there is value in decomposing technological progress into two distinctive forms. If we consider the second Industrial Revolution of the late 19th century, for example, we see that railroads replaced stagecoaches, steamships replaced sailboats, and mechanized cranes replaced rudimentary pulley systems. These technological advances rendered groups of stagecoach drivers, sailors, and pulley operatives obsolete. Yet at the same time technological changes involved newer production methods requiring machinists, engineers, repairmen, managers and financiers newly-trained in the new methods. Similarly, today we see digital technologies replace certain production processes, and thereby replace certain mid-skilled workers, while other more novel digital methods require new engineers and designers. There is indeed a rise of machines, but in some ways it is a familiar one.

2.3 Education and the Machine Question

Another large question somewhat neglected in the literature is the potential effects of technological change on education, both historically and recently. For example, it has been argued

[7] The following historical examples are taken from The Economist special report, "Artificial Intelligence," June 25, 2016.

that early industrialization could not have been consistent with larger demands for education because skill premia appear to be falling during this period (Clark 2004, Clark and Hamilton 2003). Our model demonstrates that early industrialization could have been both de-skilling (as routine labor is increasingly obsoleted by the efforts of tinkerers) and fostering growth in higher-end skills (as those on the higher end of the educational distribution shy away from risky mid-skills).

Yet at the same time, we will see that sustained innovation can be consistent with a slowdown in educational attainment overall. To the extent that highly educated individuals are necessary for sustained inventive activity, this may threaten a growth slowdown and a renewed emphasis on "tinkering."

Related to the effects of technologies on education are their potential effects on income inequality. Does their exist a growth-inequality tradeoff? Galor and Tsiddon (1997) find that polarization in the early stages of development may be necessary for a future growth takeoff. We similarly find wage polarization in the tinkering economy that reverses in the transition to more breakthrough growth, although the mechanism here is quite different.

We also acknowledge that there may be differences between skills and tasks (Acemoglu and Autor 2010). This distinction may be crucial when certain skills become obsolete. In these cases, one's acquired skill may not relate to one's current job, and this will naturally affect income inequality. Take for example a recent Deloitte study documenting a rise in "care-oriented" service jobs in the British economy.[8] Many of these workers studied to do something else in college, essentially becoming "displaced" into the service sector. A large portion of skills obtained in undergraduate programs are after all occupation specific, and students who major in technical fields appear to be *less* likely to work in a job related to their major than those who major in more general fields (Kinsler and Pavan 2015).

How is education affected by these two potentially distinct forms of technological changes for more contemporaneous economies? To provide a partial answer, and to help motivate the model, we take a brief empirical look, using U.S. industry-level data on occupations from some familiar

[8] "From brawn to brains: The impact of technology on jobs in the UK." The study documents, among changes in other professions, a 909 percent rise in nursing assistants and a 168 percent rise in care workers.

Table 1: Task Content and Job Characteristics

Dep. Variable - Task Content:	Analytical	Routine	Manual	Analytical	Routine	Manual
Required education	-0.24	0.87***	-0.11	-	-	-
	(0.16)	(0.17)	(0.10)			
Required education squared	0.03***	-0.04***	-0.0006	-	-	-
	(0.007)	(0.007)	(0.004)			
New job (Lin index)	-	-	-	5.04***	-1.30*	-0.90*
				(0.75)	(0.75)	(0.47)
Number of Observations	389	389	389	537	537	537
R-squared	0.44	0.07	0.10	0.08	0.01	0.01
F-stat	179.4	13.2	26.9	45.2	3.0	3.6

Robust standard errors in parentheses.
*** indicates significance at the .01 level. ** indicates significance at .05. * indicated significance at .1.
Dependent variable is the task content of each occupation based on Dictionary of Occupational Titles (Autor and Dorn 2013).
Required education measures the typical number of years of education needed for entry into each occupation (BLS).
New job measures the share of new job titles within each occupation (Lin 2011).

sources (see notes below tables 1 and 2).

Table 1 demonstrates some empirical relationships between task content and other occupational characteristics. In the first set of regressions we regress the task content of each occupation on the level of education typically required to enter into the occupation, as well as its squared value.[9] Estimated coefficients produced here naturally have no causal interpretations, only correlative ones. Still, an interesting pattern emerges — while education seems to have little relation with manual task jobs, there appear to be strong non-linearities for other tasks. Raising required education on average actually *lowers* the analytical skill content in jobs. At high levels of education, however, this reverses. An abundant education is necessary to join jobs with high amounts of analytical content. The opposite pattern emerges with routine content-oriented jobs — higher educated people on average join jobs with *more* routine-skill content, but the very highly educated will join jobs with less.

The latter set of regressions demonstrates the relationship between task content and *new* job content in occupations. We use Lin (2011)'s measure of the share of new job titles, meaning jobs

[9]The original information from BLS provides categories for degree requirements. Our measure is in years, ranging from "little to no formal education required" (we set this to 8 years) to Ph.D. required (we set this to 22 years). Alternative ranges produce similar results.

Table 2: Probability of Automation Across Jobs

Dep. Variable - Probability of Automation	(1)	(2)	(3)	(4)	(5)
Required education	-0.06***	0.07***	-	-	-
	(0.003)	(0.015)			
Required education squared	-	-0.005***	-	-	-
		(0.0006)			
Analytical task content	-	-	-0.09***	-	-0.087***
			(0.007)		(0.007)
Routine task content	-	-	0.02***	-	0.022***
			(0.007)		(0.007)
Manual task content	-	-	-0.02*	-	-0.023*
			(0.012)		(0.012)
New job (Lin index)	-	-	-	-1.03***	-0.36**
				(0.21)	(0.18)
Number of Observations	682	682	364	364	364
R-squared	0.32	0.37	0.37	0.06	0.37
F-stat	413.4	231.9	71.5	24.9	53.7

Robust standard errors in parentheses.
*** indicates significance at the .01 level. ** indicates significance at .05. * indicated significance at .1.
Dependent variable is the probability of automation (computerization) across occupations (Frey and Osborne 2013)
Required education measures the typical number of years of education needed for entry into each occupation (BLS).
Task content information for each job come from Autor and Dorn (2013).
New job measures the share of new job titles within each occupation (Lin 2011).

requiring new combinations of activities or techniques that have emerged in the labor market, within each occupation. The share of new job titles can proxy for those sectors newly created through technological change (Acemoglu and Restrepo 2015). Here we find that technologically advanced occupations are those with more analytical content and less routine and manual content.

Table 2 demonstrates the relationships between the probability of automation (using Frey and Osborne (2013)'s measure of the likelihood of computerization across occupations) and other job characteristics. We see in the first specification, unsurprisingly, that in general joining jobs requiring more education lowers the probability of job automation. When we include the squared education term, we are startled to find the reverse — rising educational requirements are associated with *greater* likelihood of skill obsolescence! Only with much higher educational levels does the likelihood of automation decline.

This finding is echoed when observing other factors. We see that automation is less likely for positions with more analytical content *and* more manual content, and for positions with newer job titles. Automation is more likely for positions with more routine content.

There are a number of new ideas that emerge from these empirics. One is that building from a low skill level to a moderate skill level raises the riskiness of job loss. Only by acquiring a considerable amount of education does one become more insured against such loss. Another finding is that getting high levels of education gives individuals access to the jobs newly invented — these tend to be positions that are neither routinized nor automated.

While nothing causal can be claimed here, it does appear that we are living in an age of job, wage, and *educational* polarization. Can we expect this to be the new normal, and can history guide us? Such questions motivate us to develop a model with different forms of technological developments, and different *levels* of education (rather than a binary or continuous measure).

3 The Model

We begin with a utility and production structure with unbalanced technological change reminiscent of Baumol (1967), with extensions by Autor et al. (2003), Weiss (2008), and Autor and Dorn (2013). The economy consists of two sectors which produce goods or services. These products are imperfectly substitutable in utility. For any period t (suppressing this subscript for now), the planner's problem is to maximize an aggregate consumption bundle given by

$$C = \left(\rho C_s^{\frac{\sigma-1}{\sigma}} + (1-\rho) C_g^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where C_s are services, C_g are manufactured products, and ρ is the relative weight placed on services in utility. $\sigma \leq 1$ determines the elasticity of substitution between goods and services. We will assume throughout that $\sigma > 0$, so that goods and services grossly complement each other in utility (a standard assumption in the literature).

There are three basic forms of labor. Purely unskilled workers (L_u) use no human capital and can only work in the service industry (as manual laborers). Those who invest in human capital can invest either in mid-level routine skills, or high-level analytical skills. A key feature in this model is that mid-level routine skills are specific to a particular manufacturing sector, and face potential obsolescence. High-level analytical skills on the other hand are not tied to any specific task and face no potential for obsolescence.

There are two potential types of technological changes — those that improve upon the production of existing goods, and those that produce altogether new goods. There are thus two (potential) types of capital goods. K^* is capital used in the production of machines which in turn produce types of goods that already exist. K^{new} on the other hand is capital devoted to constructing machines from brand new blueprints, which in turn produce types of goods which have been newly invented.

Production of service and manufacturing goods involve the following respective production functions:

$$C_s = Y_s = L_s, \tag{2}$$

$$C_g = Y_g - pK - p^* K^*, \tag{3}$$

$$Y_g = \int_0^N y(i)^\gamma di + \int_0^{N^*} y(j)^\gamma dj + \int_0^{N^{new}} y(m)^\gamma dm, \tag{4}$$

where Y_s is total production of services, Y_g is total production of manufactured goods, and $[p^*, p]$ is a vector of capital prices. N is the number of old routine sectors, N^* is the number of automated sectors, and N^{new} is the number of newly-invented sectors. These sectors are described more fully in the next section. Finally, to ensure there are determinate levels of capital and labor employment in each sector, we maintain $0 < \gamma < 1$.

3.1 Production

Factors are paid their marginal products. Unskilled service workers earn w_u, mid-skilled workers earn w_r times the mid-level human capital they accumulate, and analytical workers earn w_a times the high-level human capital they accumulate. We will see in section 3.3 that human capital amounts will differ across individuals.

Capital is used only in the manufacturing sector. A portion of the final consumption good must be allocated to the production of capital for the building of automating machines and/or new machines. Capital fully depreciates after each period. Sector-specific production in old (routinized) sectors, automated sectors, and newly-invented sectors, are respectively given by

$$y(i) = A_r l_r(i), \tag{5}$$

$$y(j) = A^* l_a^*(j)^\beta k^*(j)^{1-\beta}, \tag{6}$$

$$y(m) = A^{new} l_a(m)^\alpha k(m)^{1-\alpha}. \tag{7}$$

where factors are now written in lowercase to indicate sector-specific levels of employment.

Note that capital is only utilized in automated or new sectors. We will assume throughout that $\alpha > \beta$ — that is, the importance of analytical labor relative to capital is larger for new production than for automated production. This captures the notion that new processes are more complicated, requiring a set of skills flexible enough to address complex tasks. Automated processes on the other hand are more mechanized, requiring skilled workers to manage mechanized production but not requiring much in the way of deeper problem-solving skills.

We will also assume that $A^{new} < A_r < A^*$. This captures the intuitive idea that production processes improve over time. As a sector transitions from new to routinized to automative, it climbs up the quality ladder.

Note also that mid-level routine skills can only be employed in old manufacturing sectors, and that these skills are assigned to a specific sector. Each mid-level skill can only be used in its assigned sector. This assumption reflects the idea that mid-level skills are typically linked to a specific industry or production process. Historical examples include the major areas of growth during the Industrial Revolution such as textile production and steam engineering — education for workers in these industries typically took the form of training in industry-specific tasks.[10] Of course there are many examples of industry-specific skills used today in production

[10] Economic historians have suggested that more general skills like basic literacy were not particularly important compared with task-specific skills during this period (Mitch 1982).

and craft occupations, operative and assembler occupations, and transportation, construction, mechanical, mining, and farm occupations, many which are currently threatened by technological obsolescence through automation (Autor and Dorn 2013).

On the other hand, we will assume that analytical skills are not tied to any specific industry. Those who have acquired analytical skills can work in any extant sector across $(0, N^*]$ *or* $(0, N^{new}]$. This captures the idea that these skills are flexible enough to work with any type of capital, even those whose blueprints have been newly invented.

Given the fact that analytical labor can work in any automated or new sector, and capital designed for either automation or new production can be employed across all sectors within each specialization, we will have equal amounts of labor and capital across sectors in each production area. This means

$$\int_0^N y(i)^\gamma di = \int_0^N \left(A_r l_r(i)\right)^\gamma di = N \left(A_r l_r\right)^\gamma , \tag{8}$$

$$\int_0^{N^*} y(j)^\gamma dj = N^* \left(A^* l_a^{*\beta} k^{*1-\beta}\right)^\gamma , \tag{9}$$

$$\int_0^{N^{new}} y(m)^\gamma dm = N^{new} \left(A^{new} l_a^\alpha k^{1-\alpha}\right)^\gamma , \tag{10}$$

where now l_r, l_a^*, l_a, k_a^* and k_a denote the labor and capital amounts hired in each sector.

Given this, perfectly competitive manufacturers demand different types of capital and labor, given technological levels, machine prices and wages. We thus have six first order conditions:

$$p^* = \frac{\partial Y_g}{\partial k^*(j)} = \frac{\partial Y_g}{\partial y(j)} \frac{\partial y(j)}{\partial k^*(j)}, \tag{11}$$

$$p = \frac{\partial Y_g}{\partial k(m)} = \frac{\partial Y_g}{\partial y(m)} \frac{\partial y(m)}{\partial k(m)}, \tag{12}$$

$$w_r = \frac{\partial Y_g}{\partial l_r(i)} = \frac{\partial Y_g}{\partial y(i)} \frac{\partial y(i)}{\partial l_r(i)} = \gamma y(i)^{\gamma-1} A_r, \tag{13}$$

$$w_a = \frac{\partial Y_g}{\partial l_a^*(j)} = \frac{\partial Y_g}{\partial y(j)} \frac{\partial y(j)}{\partial l_a^*(j)} = \gamma y(j)^{\gamma-1} \beta A^* l_a^*(j)^{\beta-1} k^*(j)^{1-\beta}, \tag{14}$$

$$w_a = \frac{\partial Y_g}{\partial l_a(m)} = \frac{\partial Y_g}{\partial y(m)} \frac{\partial y(m)}{\partial l_a(m)} = \gamma y(m)^{\gamma-1} \alpha A^{new} l_a(m)^{\alpha-1} k(m)^{1-\alpha}, \tag{15}$$

$$w_u = \left(\frac{\rho}{1-\rho}\right) \left(\frac{C_s}{C_g}\right)^{\frac{-1}{\sigma}}. \tag{16}$$

Equations (11) – (12) are machine prices, which will be charged by machine producers. Equations (13) – (16) are *ex ante* spot wages for raw labor or human capital. Aggregate levels of analytical and routine labor are given by

$$L_a = N^{new} l_a + N^* l_a^*, \tag{17}$$

$$L_{r,ex-post} = N l_r. \tag{18}$$

3.2 Technological Change

We will assume an over-lapping generations framework where individuals live for two time periods. On the technology side, we assume that "young" individuals can decide to invest resources to invent new-type machines (which would produce brand new goods), or to invest resources 'tinkering' with existing production processes and invent machines to automate production. We further assume that they can enjoy the fruits of their labors when they are "old" — that is, for one time period only.

A tinkerer who automates an existing production process has monopoly rights to the machine-type used in that production for one period. This production was part of N in $t-1$ (and linked with specific mid-skilled workers) and becomes part of N^* during t. Given iso-elastic demand, this machine producer charges a constant mark-up over marginal cost of machine production, $p^* = (1/\beta)$.[11]

[11] We assume that the monopoly price is charged in perpetuity. This produces the mildly awkward case of rents being earned by erstwhile tinkerers long since dead. This motivates us to suggest possible redistribution schemes in a later section.

An innovator who invents a *new* type of machine also has monopoly rights to it for one period. These machines become part of N^{new} during this one period t. In the period $t+1$ however, the machine becomes obsolete, and the production of that product automatically becomes routinized, raising N. This captures the idea that it takes a certain amount of time for new production processes to become streamlined enough for mid-skilled workers to operate them. At time t young individuals who decide to acquire mid-level skills can observe the new technologies and adapt their skills to use them at $t+1$ (so long of course as that process had not become automated in the meantime). At time t however new machine producers have monopoly rights over their machines and charge a markup of $p = (1/\alpha)$.

Note that given these capital prices, we can rewrite our expressions of k and k^* as

$$k = (1-\alpha)^{\frac{2}{1+\alpha\gamma-\gamma}} \gamma^{\frac{1}{1+\alpha\gamma-\gamma}} A^{new\,\frac{\gamma}{1+\alpha\gamma-\gamma}} l_a^{\frac{\alpha\gamma}{1+\alpha\gamma-\gamma}}, \tag{19}$$

$$k^* = (1-\beta)^{\frac{2}{1+\beta\gamma-\gamma}} \gamma^{\frac{1}{1+\beta\gamma-\gamma}} A^{*\,\frac{\gamma}{1+\beta\gamma-\gamma}} l_a^{*\,\frac{\beta\gamma}{1+\beta\gamma-\gamma}}. \tag{20}$$

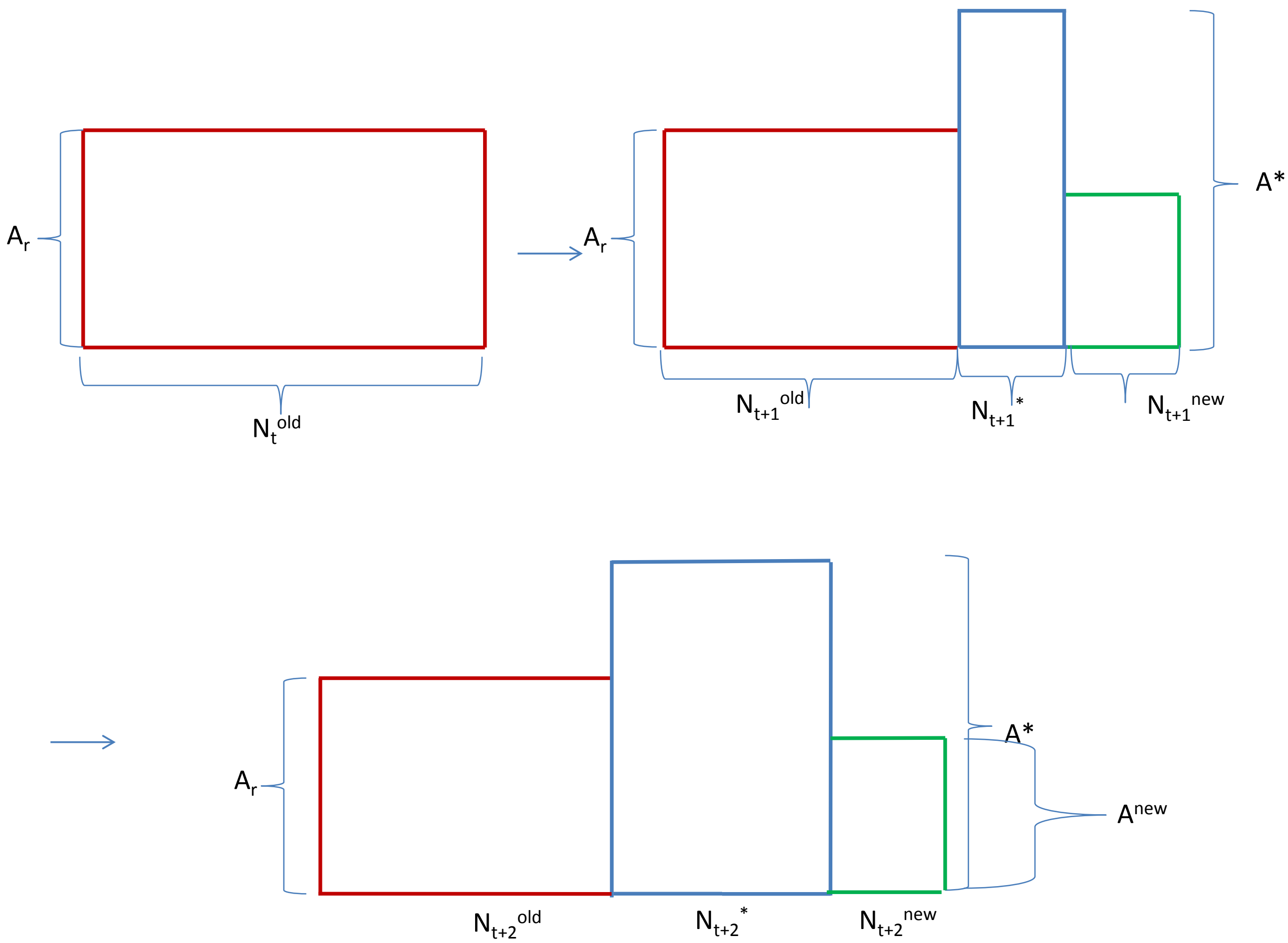
At any given period there may be both new automation that takes away from routine production, and also new machine blueprints that will eventually lead to more routine production. The law of motion for the total amount of routinized production can be thus written as

$$N_t = N_{t-1} - \left(N_t^* - N_{t-1}^*\right) + N_{t-1}^{new}. \tag{21}$$

Note that if automative machines for a sector are successfully developed by tinkerers, that sector can no longer employ routine labor, the idea being that that particular mid-level skill is no longer applicable since now capital and analytical labor are newly combined to produce the same product. Thus a fraction of erstwhile routine laborers will be unable to use their human capital (more on this in next section). Let us define ϕ_t as the fraction of sectors that have become automated in period t across all routine sectors. This is given by

$$\phi_t \equiv \frac{N_t^* - N_{t-1}^*}{N_{t-1} + N_{t-1}^{new}}. \tag{22}$$

Figure 2: Illustration of Growth in Both Quality of Machines and New Machines



Vertical growth (from A_r to A^*) implies automation, which comes from a portion of existing routine production. Horizontal growth (increase in N^{new}) implies new innovation.

Figure 2 gives a diagrammatical representation of technological growth when *both* automation and new machine inventions occur. Automation improvements raise a certain fraction of machines vertically *à la* Aghion and Howitt (1994) — this vertical rise shows up within a sub-group of old routine production processes the following period. Newly invented machines on the other hand push technology out horizontally *à la* Romer (1990).

It is important to note that these forms of technological changes can be both skill-replacing and skill-enhancing. Skills are obsoleted through automation, but new-updated skills are employed when new technologies become seasoned. These technological developments will alter individual incentives to invest in human capital. Often ignored in the literature of directed technical change, we next turn to that problem.

3.3 Endogenous Skill Choice

As individuals live for two time periods, we suggest that they maximize an expected utility function given by

$$EU\left(c_{t-1}, c_t\right) = \left(c_{t-1}^{\omega} + E\left(c_t\right)^{\omega}\right)^{1/\omega}, \tag{23}$$

where c_{t-1} is per person consumption when the person is young, c_t is per person consumption when the person is old, and $\omega < 1$.[12] Individuals are indexed over a number line of constant size $\overline{L}$. They are born with a pair of potential endowments that rise linearly with this index.[13] That is, individual $l \in [0, \overline{L}]$ is born with a pair of potential endowments $[a_l^r, a_l^a]$ — when young they can choose to invest in one type of skill, and receive the endowment in the next time period. At time $t-1$ the individual chooses what kind of worker they would like to be at time t. If the individual chooses not to get any education, she will work as an unskilled laborer for both time periods and earn $1 + w_u$ each period. If the individual chooses to become a mid-skilled worker she will invest her time at $t-1$ getting an education, thus earning no wages (she instead receives a normalized value of 1). Her education is devoted to a specific sector i — since all routine

[12]Note that while individuals seek to maximize the dynamic problem given by (23), the social planner's problem is to solve the static problem given by (1) each period. In essence skilled workers are paid by competitive firms in manufactured product, then spend a portion of this product hiring unskilled service workers via (16).

[13]Other functional forms would work just as well, but may not be as analytically tractible.

sectors are symmetrical, we will treat this as randomly chosen. At time t she acquires a_l^r units of mid-skilled human capital and earns $w^r a_l^r$ *provided* that i remains a routine sector at time t. If production from sector i has been tinkered with and automated, l's routine skills become obsolete and she becomes an unskilled worker, earning $1 + w_u$ for only one period. Finally, if the individual chooses to become a high-skilled worker she will invest her time at $t - 1$ getting a high-level education, also earning no wages. At time t she acquires a_l^a units of analytical human capital and earns $w_t^a a_l^r$ with certainty.

Given these payoffs, the individual who chooses to be unskilled receives a utility of

$$U_u = \left((1 + w_u)^\omega + (1 + w_u)^\omega\right)^{1/\omega}, \tag{24}$$

The individual who chooses to be semi-skilled worker faces an *ex ante* utility of

$$U_r = \left(1 + (1 + E_l)^\omega\right)^{1/\omega}, \tag{25}$$

where E_l is the *expected* wage that individual l can earn when investing in mid-skills. Finally, the individual who chooses to be a high-skilled analytical worker receives a utility of

$$U_a = \left(1 + (1 + a_l^a w_a)^\omega\right)^{1/\omega}. \tag{26}$$

Given these potential utilities, agent l decides whether to work two periods as an unskilled worker, to invest their time at $t - 1$ to receive a_l^r units of mid-skill human capital to potentially use in period t, or to invest their time at $t - 1$ to receive a_l^a units of high-skill human capital to use with certainty in period t.

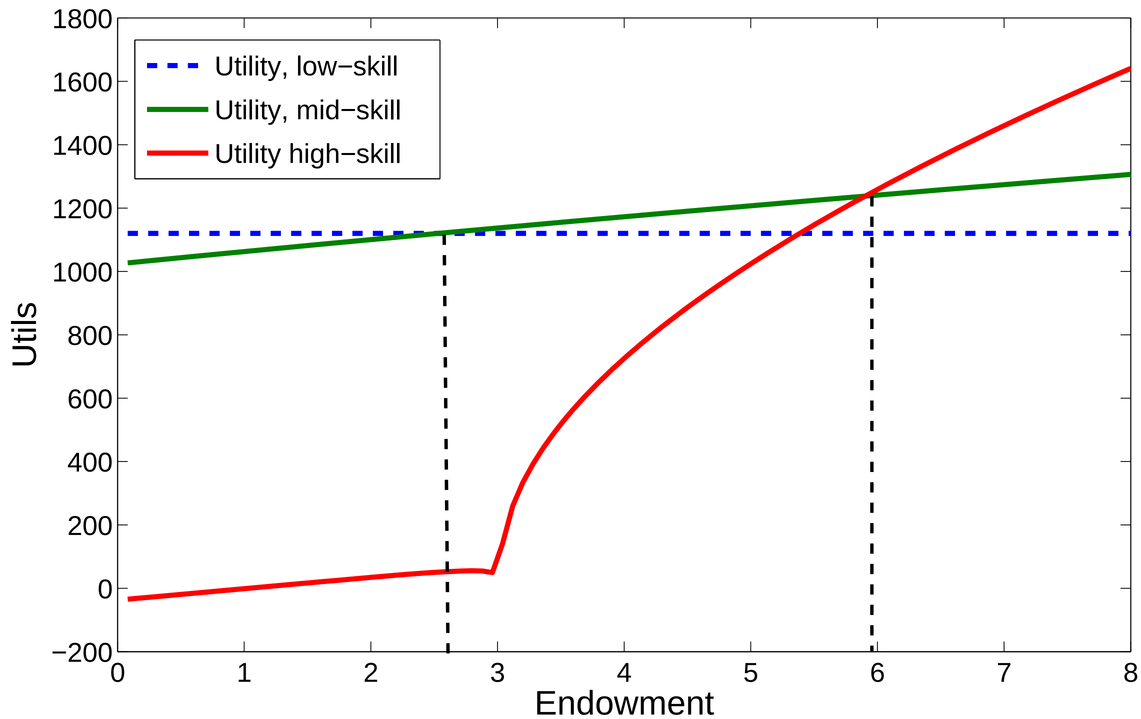
Note that there is uncertainty over whether one's investment in mid-skilled human capital will actually pay off at t, since there is a possibility that those skills will become obsolete with automation. Here we treat one's expected wage for routine labor linearly as

$$E_{l,t-1} = (1 - \phi_t) a_{l,t}^r w_{r,t} + \phi w_{u,t}, \tag{27}$$

where ϕ_t is given by (22). That is, with probably ϕ individual l finds that his mid-level human capital is obsolete, and he instead works as an unskilled worker for one period.

Finally, assume that potential skill endowments rise at linear rate γ_1 along the labor index for mid-level routine skills, and at linear rate $\gamma_2 > \gamma_1$ along the index for high-level analytical skills. Further assume that for a range of $[0, \hat{L}]$, a_l^a is zero. That is, those under $\hat{L}$ can never become analytical workers, as they are not endowed with any potential analytical skills. This suggests that individual l is endowed with $\gamma_1 l$ units of potential mid-level human capital and $\gamma_2(l - \hat{L})$ units of potential high-level human capital.

Figure 3: Utility from Different Endowments



Here $\overline{L} = 8$, wages are constant, and $w_u < w_r < w_a$. In this illustrative example, those below $L_1 = 2.6$ will be unskilled, while those above $L_2 = 5.95$ will be highly skilled. Those in the middle will opt to be routine-skilled. Note that wage changes would shift these utility curves, changing threshold levels L_1 and L_2.

With this set-up, we can solve for equilibrium levels of each type of worker by calculating *threshold points* where individuals would be indifferent between two outcomes. We illustrate these two points in Figure 3. First, define L_1 as the worker who is indifferent between being an unskilled worker and investing in routine skills. This individual's expected return to routine skill investment is $E_{L_1} = (1 - \phi)\gamma_1 L_1 w_r + \phi w_u$, and her utilities are such that $U_{u,L_1} = U_{r,L_1}$. Solving for L_1 we get

$$L_1 = \frac{\left(\left(1+w_{u,t-1}\right)^{\omega}+\left(1+w_{u,t}\right)^{\omega}-1\right)^{1/\omega}-1-\phi_t w_{u,t}}{\gamma_1\left(1-\phi_t\right)w_{r,t}}. \tag{28}$$

Next we define L_2 as the worker who is indifferent between being a routine worker and being an analytical worker. This individual's expected return to routine skill investment is $E_{L_2} = (1-\phi)\gamma_1 L_2 w_r + \phi w_u$, and her utilities are such that $U_{r,L_2} = U_{a,L_2}$. Solving for L_2 we get

$$L_2 = \frac{\gamma_2 \hat{L} w_{a,t} + \phi_t w_{u,t}}{\gamma_2 w_{a,t} - \left(1-\phi_t\right)\gamma_1 w_{r,t}}. \tag{29}$$

Given wages, we can use these thresholds to solve for equilibrium aggregate levels of each type of human capital. Figure 4 shows how this looks. The area under each linear line between the thresholds indicate the total mass of each type of human capital, at least *ex ante*. *Ex post* of course there will be less L_r, as fraction ϕ will be reallocated to unskilled jobs — this will be uniformly random across all endowment levels. *Ex post* labor amounts are thus given by

$$L_u = L_{1,t-1} + L_{1,t} + \phi\left(L_2 - L_1\right), \tag{30}$$

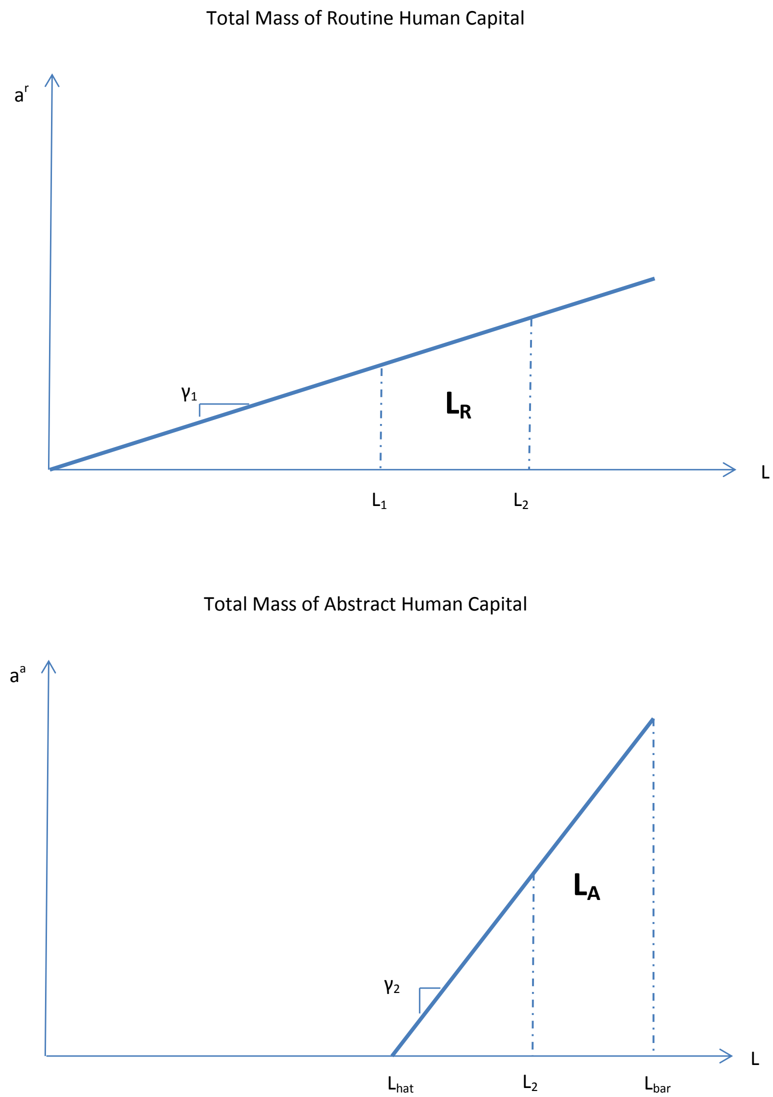
$$L_r = (1-\phi)\left(\left(L_2-L_1\right)\gamma_1 L_1 + \frac{1}{2}\left(L_2-L_1\right)\gamma_1\left(L_2-L_1\right)\right), \tag{31}$$

$$L_a = \left(\bar{L}-L_2\right)\gamma_2\left(L_2-\hat{L}\right) + \frac{1}{2}\left(\bar{L}-L_2\right)\gamma_2\left(\bar{L}-L_2\right). \tag{32}$$

3.4 General Equilibrium

A static general equilibrium follows from the above discussion. Given existing technological levels, equilibrium is given by solving (13), (14), (15), (16), (19), (20), (28), (29), (30), (31), and (32) for equilibrium values of wages, capital, threshold education levels, and labor amounts. Note that in the purely static case ϕ is zero, as there are no sectors being newly automated and so no uncertainty over whether or not certain mid-skilled workers will find their human capital obsoleted.

Figure 4: Equilibrium Amounts of Human Capital



The total amount of routine human capital, L_r, is given by the trapezoid in the top diagram. The total amount of analytical human capital, L_a, is given by the trapezoid in the bottom diagram.

With technological changes will come changes to marginal products of factors and changes to the factors themselves through changes in education choice. We can anticipate some of the changes by observing our equilibrium conditions. First, observe the equilibrium threshold person indifferent to investing in mid-skill human capital or remaining an unskilled worker (equation 28). If w_u falls or w_r rises, it is clear that L_1 will unambiguously fall. That is, the threshold person falls down the spectrum of workers, fewer people choose to remain an unskilled worker, and more people choose to invest in mid-level skills.

Other relationships may appear more ambiguous. To help clarify we provide the following lemmas to help us understand the growth path of the economy (see appendix A for all proofs):

Lemma 1 *For a given ω and ϕ, there is a threshold unskilled wage $\tilde{w_u}$ such that $\frac{\partial L_1}{\partial \phi} > 0$ for all $w_u > \tilde{w_u}$. Further, $\frac{\partial^2 L_1}{\partial \phi \partial w_u} > 0$ for all $w_u > 0$ and $\phi > 0$.*

This suggests that for all reasonable values of unskilled wages, automation by itself will raise the unskilled labor-routine labor threshold and thus lower investments to routine education. Furthermore, the higher is the unskilled wage, the greater is the labor response to any automation. Thus as unskilled wages rise (as we will observe in the simulations), automation becomes increasingly more de-skilling.

Lemma 2 $\frac{\partial L_2}{\partial w_a} < 0$, $\frac{\partial L_2}{\partial w_r} > 0$ *for all* $\phi < 1$, *and* $\frac{\partial L_2}{\partial w_u} > 0$ *for all* $\phi > 0$.

The second lemma suggests that *ceteris paribus* increases in analytical wages always increases investments in analytical education, while *ceteris paribus* increases in routine wages will always decrease investments in analytical education, and *ceteris paribus* increases in manual wages will likewise decrease investments in analytical education when there is some automation occurring (since automation makes mid-skills risky, a higher unskilled wage makes the downside risk less painful). As we will observe in the next section, *relative* wage changes will then be a major factor in determining net changes in education accumulation.

Lemma 3 *A sufficient condition for $\frac{\partial L_2}{\partial \phi} < 0$ is $w_u < \gamma_1 \hat{L} w_r$.*

The final lemma simply states that so long as manual wages are low enough, increases in automation should be analytical-skill enhancing. However, as we will see in the simulations, as manufacturing productivity continues to improve, these unskilled wages can rise enough so that this relationship reverses.

Of course we ultimately wish to know what happens to levels of education with different types of technological changes. For this we turn to simulation.

4 Simulating Technological Change[14]

Here we simulate the economy described in the prior section to demonstrate how different forms of technological change influence our variables of interest. For the first few cases we will *exogenously* grow both N^* and N^{new} for a case with low initial technological levels (labeling this the 'historical growth' case), and for a case with higher initial technological levels (labeling this the 'contemporary growth' case). By exogenously changing technologies we are able to isolate the effects of each type of change.

We then endogenize growth in N^* and N^{new} by introducing innovators and tinkerers to the economy. This allows us to highlight the inevitable transition from the tinkering economy to the innovating economy in historic growth, as well as observe the overall growth implications from such transitions.

Note that we are more focused on the qualitative direction of variable changes than on quantitative magnitudes. So we do not perform a careful parameter calibration exercise, which would produce little new insights. Qualitative results described below appear to hold for all positive parameter values where $\sigma > 0$, $\gamma_1 < \gamma_2$, $\alpha > \beta$, and $\hat{L} < \bar{L}$ (a fuller set of simulation results for a range of parameter values are available upon request). For all diagrams we normalize initial values to one; in this way we can compare changes in magnitudes between different cases.

[14]For all simulations, parameter values are set as follows. $\rho = 0.5$, $\gamma = 0.5$, $\sigma = 0.5$, $\alpha = 0.5$, $\beta = 0.33$, $\omega = 0.1$, $\gamma_1 = 0.5$, $\gamma_2 = 0.75$, $\hat{L} = 3$, $\bar{L} = 10$. Qualitative directions of variable changes appear insensitive to specific parameterizations, provided $\sigma > 0$ (that is, provided goods and services are grossly substitutable). We set technological levels $A^{new} = 1$, $A_r = 1.5$, and $A^* = 2$.

4.1 Historical Growth — Rise of the Machines in History

In the first case we set initial levels of N and N^* low, simulating a pre-industrial economy. We then exogenously grow N^* relatively fast and N^{new} relatively slowly, suggesting that early industrialization was characterized more by automation than by true breakthrough innovation. Over time we allow N^{new} to grow faster and faster, in the end outpacing automation, mimicking the transition from early growth to the more robust growth of the second Industrial Revolution.[15] In section 4.3 we endogenize this transition, suggesting it is natural to think of an economy evolving from a tinkering to an innovative economy. To pinpoint certain features we simply impose this transition here.

For exogenous growth in the economy, we can propose the following:

Proposition 1 *For starting values where N is sufficiently higher than N^*, exogenous growth in N^* will raise L_a, lower L_r, and raise L_u.*

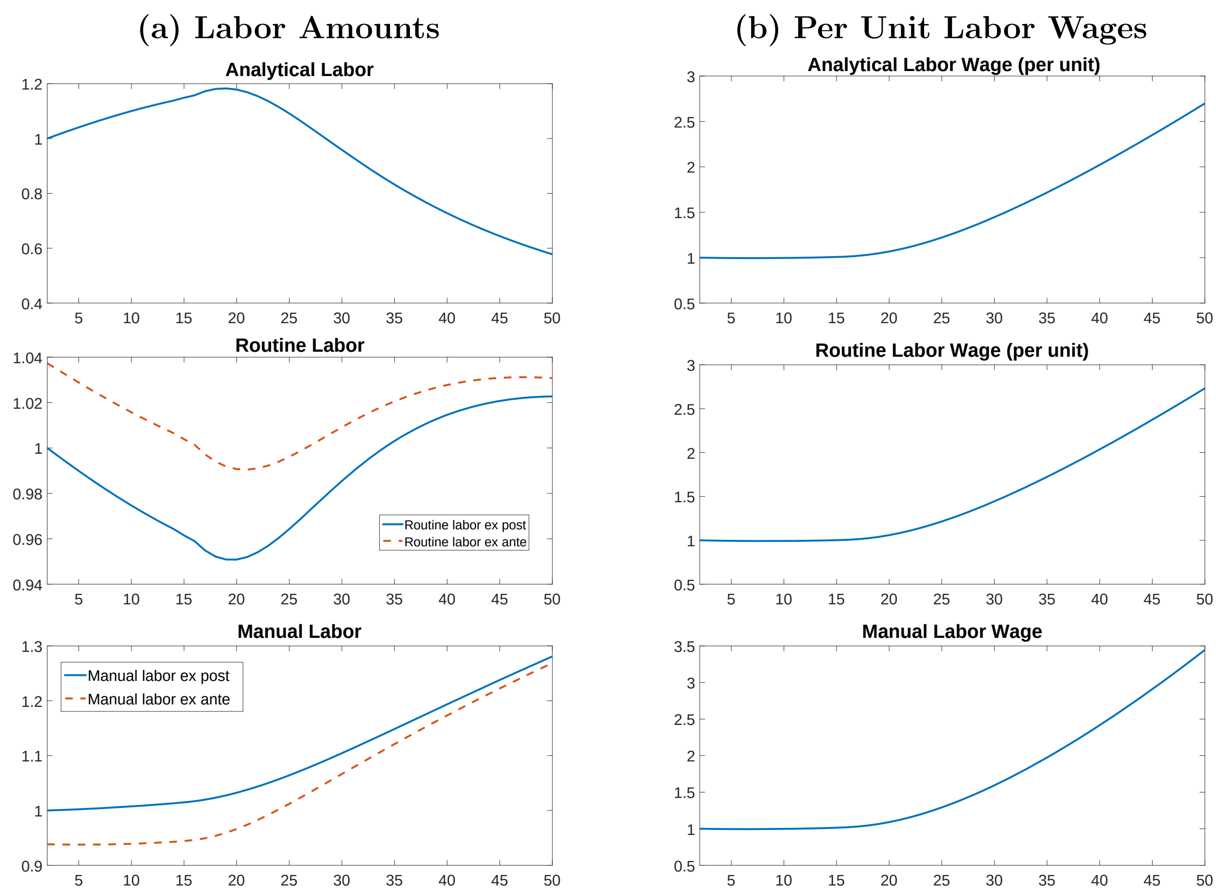
Proposition 2 *For values where N and N^* are sufficiently low, exogenous growth in N will lower L_a, raise L_r, and raise L_u.*

In short, Proposition 1 suggests that automation produces educational polarization, while Proposition 2 suggests that new technological growth reverses this polarization at the high educational end, but continues it at the low end.

Figure 5 demonstrate the results from this simulation. We see that in the early part of the simulation, when the economy is dominated by automation overall, wage growth is very low. While automation raises the qualities of production and so increases productivity, the loss of erstwhile productive human capital lowers it. As in Autor and Dorn (2013), this type of technological progress is consistent with employment and wage polarization (although the latter effect can not be seen in the figure since wages are very flat — this will be more discernible in section 4.3). This also creates educational polarization, as more and more individuals decide to become either totally unskilled workers or highly skilled analytical workers.

[15] Specifically, $N_1 = 1$ and $N_1^* = 1$. N^* grows 1 percent throughout all time periods. Increases in N^{new} exogenously rises linearly and incrementally, from 0.002 at the beginning of the simulation to 0.5 at the end. Growth is thus dominated early on by automation, and later on by growth in new technologies.

Figure 5: Rise of the Machines in History — from automation to new technologies



Here we simulate the case where automation dominates early on, while new innovation exogenously gathers steam and dominates later on. In this case we start with virtually stagnant wages and declining mid-skills, followed by more roubst wage growth and rising mid-skills.

This is broadly consistent with early growth in Europe during the latter 18th and early 19th centuries, which was characterized by disruptive technological changes, sluggish wage growth and ambiguous effects on overall education. It reinforces the pessimistic view of early industrialization widely held among economic historians, since working hours rose (Voth 1998) even as heights declined (Cinnirella 2008) and overall wages stagnated (Clark 2007). These factors have led to debates among economic historians over whether or not there was an Industrial Revolution at all (Landes 1999). Eric Hobsbawm (1962) argues for example that economies generally became worse until the 1840s, and only then improved.

The model echoes these historic sentiments — automation is disruptive, destroying an erstwhile valuable amount of mid-level skills even as more productive methods are introduced. The rise in N^* represents for example the "wave of gadgets" that erupt over Britain after 1760 (Ashton 1848). But, as Jones (1988) and others have argued, there was very little aggregate growth generated from these gadgets.

And while automation drives those at the higher ends of the endowment spectrum to choose greater education (by Lemma 3 — part of the value of getting a high-level education is the insurance it provides against obsolescence), it drives those at the lower end of the spectrum to choose less (by Lemma 1). This polarization also appears well documented historically — while wealthy families provided higher levels of education to children, formal education or apprenticeship programs for the masses appeared to fall or remain stagnant from 1780 — 1830 (Mitch 1982).

However, as breakthrough technological growth picks up steam, educational trends reverse, routine-skill loss lessens, and wage growth becomes far more robust. Growth in wages is most pronounced for mid-level skills. Newly invented technologies become newly routinized at a faster rate than routinized processes become automated — this drives up the overall demand for routine workers. Further, this drives down incentives to invest in analytical skills, since routine wage growth outpaces the probability of obsolescence. The consequence is skill *compression*, a reversal of what had transpired during early industrialization.

Thus we suggest that the second Industrial Revolution involved the rejuvenation of mid-level skills. The rise in education was here focused on specific tasks for jobs. But by 1850 for the first time engineers focused in many areas on "research and development," as opposed to being engaged in mere tinkering (Fox and Guagnini 1999). This case highlights the important insight that a transition from automation to new innovation can "save" the economy from skill and wage polarization. This however may not happen in our current economic environment, as we demonstrate below.

4.2 Contemporary Growth — Rise of the Machines Redux

For whatever reason (which we admit we do not attempt to explain here) we find ourselves once again in a time of automation and polarization. To simulate a more modern case, we redo the above exercise, where just like before automation dominates at first but then gives way to faster and faster breakthrough technological growth. The only difference now is that we begin with higher technological levels N and N^*. This is thus a more advanced economy, one that begins with much higher levels of GDP and wages.[16]

The results of this exercise are demonstrated in Figure 6. The start of the economy resembles our earlier case suggested by Proposition 1, but here is intended to suggest a tableau of *contemporaneous* economic malaise. Despite a great deal of technological change based on automation, we again observe virtually stagnant wages. We further observe employment and wage polarization, along with educational polarization. Thus we suggest one possible explanation for a widely-discussed contemporary puzzle — we can witness transformative technological changes that destroy skills and jobs, yet simultaneously produces anemic wage growth.

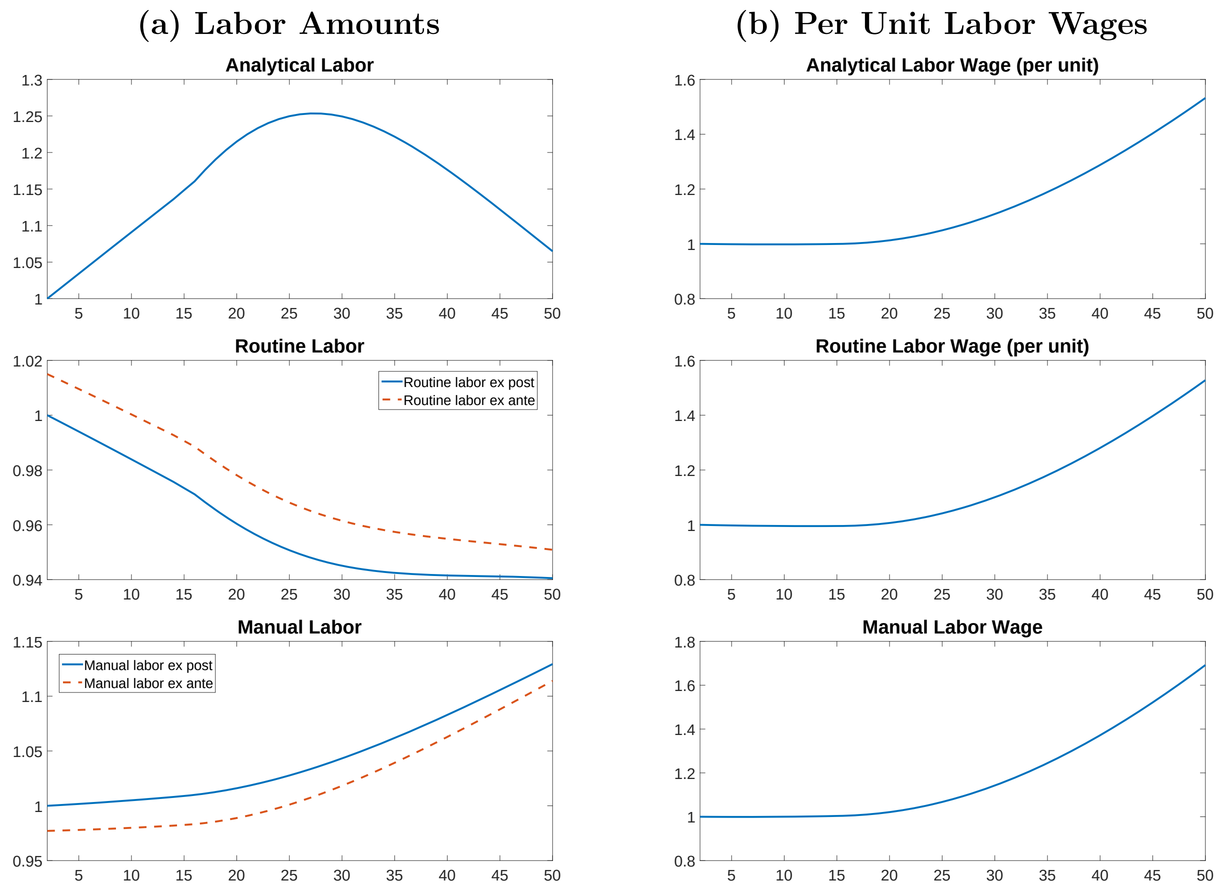
As the economy transitions to a more technologically advancing one, we do *not* observe the kind of mid-skill rebound that we had before — the routine-analytical skill threshold rises only modestly, and aggregate routine labor continues to fall. Why? Here we propose the following:

Proposition 3 *For values where N and N^* are sufficiently high, exogenous growth in N will lower L_a, lower L_r, and raise L_u.*

In a more modern society, manufacturing is already a fairly large and productive part of the economy. Further technological progress then compels labor to join the stagnant service sector.

[16] Specifically, $N_1 = 5$ and $N_1^* = 2$, translating roughly into a doubling of manufactured output compared with the prior case. Growth in N^* and N^{new} are the same as in the previous simualation.

Figure 6: Rise of the Machines Redux — from today's automation to tomorrow's new technologies



Again, we have automation dominate early on, giving way eventually to new innovation. While wage growth is similarly flat early on and more robust later on, mid-level routine skills continue to fall throughout.

That is, Lemma 3 no longer holds, and the large unskilled wage creates an overall de-skilling effect on the economy. Consequently overall growth in the economy is lower in this case compared to the prior case.

In this case the "tinkering" economy remains, and so polarization continues. New breakthrough machines can produce much more output than automative machines, but in this case there is not as much demand for output (demand rather is channeled more towards services). And less innovation also means less routinization, limiting opportunities for mid-skilled workers. This can help explain why robust automation can be accompanied by less than robust overall growth.

4.3 Endogenous Growth

The next sections show that our discussion above is robust to more endogenous treatments of technological changes. Specifically, we highlight two new impediments to long-run growth as an economy transitions from a tinkering economy to an innovative one. One relates to the incentives to innovate — if the implementation of new technologies increasingly requires highly-skilled workers, innovation itself becomes less profitable. The other relates to the drop in high-level education as an economy transitions (Proposition 3). Endogenous growth theory suggests that this can slow down growth overall. We discuss each of these in subsequent sections.

If technological changes arise from the micro-inventive activities of researchers and tinkerers, education and employment adjustments can in turn affect both the direction and extent of future technological developments. Past research has demonstrated how factors of production and technologies "directed" at different factors can interact in economically important ways (Acemoglu 1998, O'Rourke 2013, Rahman 2013). Here we show a simple endogenous treatment of technological changes to observe interactions in this framework.

To accomplish this we now develop rudimentary technology sectors. Assume free entry into either research or tinkering. Individuals can expend resource c to deterministically invent the blueprint to a new machine, or expend resource c^* to deterministically invent the blueprint to a machine that automates an existing routine process. After machines are invented, owners expend unitary marginal costs to build these machines and charge the monopoly price to competitive producers.

Similar to Rahman (2013), we assume costs to research and tinkering change in technological factors. Specifically, we assume that research costs rise in the relative number of existing blueprints N, but fall in some measure of "basic knowledge" for research B. Tinkering costs on the other hand rise in the relative number of automated production processes, and also fall in basic knowledge.[17]

The value of a new innovation or a new automated process, V and V^* respectively, is the ability to charge a mark-up to manufacturers for their machines for one time period. Specifically, the value and cost of innovation are given by

$$V = \left(\frac{1}{1-\alpha} - 1 \right) k, \tag{33}$$

$$c = \left(\frac{N/\left(N+N^*\right)}{B} \right)^{\nu}. \tag{34}$$

Using similar logic the value and cost of automation are given by

$$V^* = \left(\frac{1}{1-\beta} - 1 \right) k^*, \tag{35}$$

$$c^* = \left(\frac{N^*/\left(N+N^*\right)}{B} \right)^{\nu}. \tag{36}$$

Notice that basic research costs rise in the number of old routinized sectors in the economy relative to *all* old sectors $(N + N^*)$. Thus when N is low (relative to N^*) the numerator in the cost function is close to zero, and innovation costs are low. But as N grows relative to N^*, the numerator approaches one and costs grow. This captures the idea that it becomes harder to successfully innovate as the pool of potential new ideas grows shallower (the so-called "fishing-out" effect). A parallel cost framework exists for tinkerers — having lots of automated sectors relative to routinized sectors raises the cost of successful tinkering.

[17] These assumptions evoke Mokyr (2002)'s notion of "propositional" knowledge fueling growth in "prescriptive" knowledge. The idea of "useable" knowledge affecting the cost of innovation have been employed in growth theories such as Barro and Sala-i-Martin (2003), O'Rourke et al. (2013) and Rahman (2013).

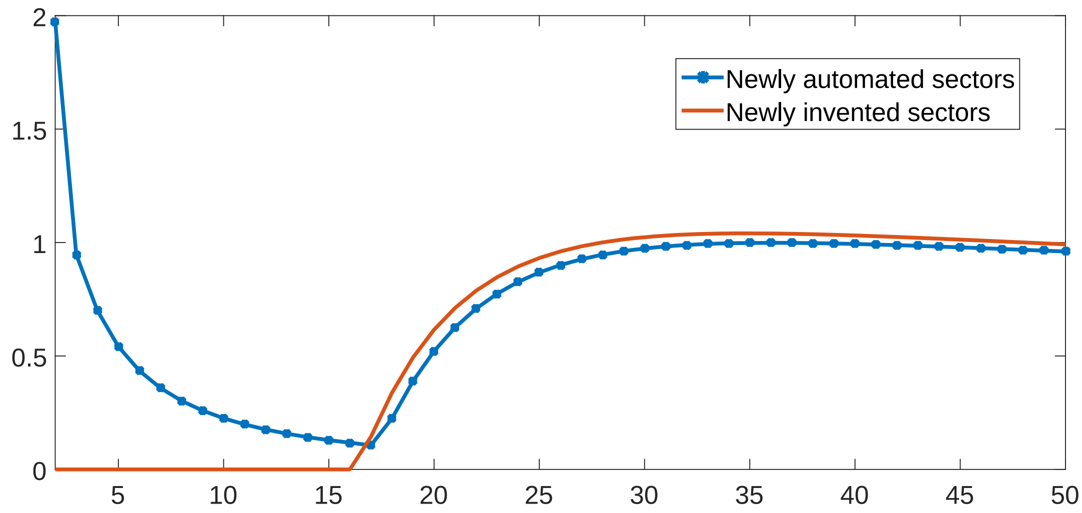
Finally, we allow basic levels of knowledge B to itself grow endogenously. Innovative or tinkering activities continue so long as the value of the activity exceeds the costs. Thus, so long as basic knowledge is large enough, we have $V = c$ and $V^* = c^*$, in which case both innovative and tinkering activities occur.

To motivate growth in B we use an "R&D" equation used in Jones (1995) and many other works:

$$\Delta B = \delta L_a^{\lambda} B^{-\phi} \tag{37}$$

where $0 < \lambda < 1$ and $\phi > 0$. Positive changes in knowledge depend both on the number of analytical workers and on the stock of existing workers. Here we do not consider L_a as explicit researchers and scientists per say, but rather as a group of highly-skilled that contribute to the stock of knowledge either explicitly or by happenstance. We discuss the parameterization of this growth equation later on.

Figure 7: Direction of Endogenous Technological Change



The vertical axis measures ΔN and ΔN^*. When we endogenize the scope and direction of technological progress, automation occurs first, followed by innovation.

4.3.1 Baseline Simulation

For the case of endogenous growth we provide two final propositions:

Proposition 4 *For initial values where $N > N^*$, early endogenous growth will involve growth in N^*, while $\frac{\Delta N}{N} = 0$.*

Proposition 5 *For all t greater than some t^*, $\frac{\Delta N}{N} > 0$.*

Let us step through the historical import of these propositions. In the pre-Industrial world, basic scientific knowledge is low such that $V < c$ and $V^* < c^*$, so sustained inventive activity is infeasible (Mokyr 2002, O'Rourke et al. 2013). Most manufacturing in the economy is routine in nature (N is large relative to N^*, and most skills are of the semi-skilled sector-specific variety. The semi-skilled here are weavers, craftsmen, carpenters, blacksmiths, scribes and so on. Education here comes mainly in the form of narrowly-defined, task-specific apprenticeships. Meanwhile, because L_a starts out quite low, B grows glacially.

What is the Industrial Revolution in this world? Since our pre-industrial world is one where there are many more routinized sectors N than capital-intensive automative sectors N^*, $c > c^*$. So, when science finally catches up to become industrially useful, it first becomes useful in automation. This is borne out by history (Mokyr 2002 describes it as "industrial enlightenment"). This is the manifestation of Proposition 4. During this period capital grows, but overall growth remains anemic.

Our suggestion that early industrialization will inherently be an automative one hinges on our understanding of pre-industrial societies. Such economies typically have many routine sectors employing semi-skilled workers (trained in crafts or guilds on a set of narrow tasks). They also tend to have small levels of employed capital or highly-skilled workers. In our framework this suggests a structure where technological levels are such that compel tinkerers to begin industrial growth as automative growth.

From a more historical perspective, it is important to understand the reasons why industrialization occurred in certain western European regions but not in other areas. While do not explicitly simulate this, the model leads us to an important insight — high wage costs for semi-skilled workers *relative* to skilled workers should promote automation. This is a slight amendment to Robert Allen's high-wage interpretation for why England was the first to modernize (Allen 2009). We suggest that *relative* wages may matter more, since automation still requires the hiring of workers, albeit with different skill sets. It will be the relatively low costs of high-skilled workers that will spur tinkerers to greater efforts.

Proposition 5 suggests that eventually, the economy will transition to modernity. This happens when $V = c$. At this point new (breakthrough) innovations occur, which raise both N and N^* (since new sectors eventually become routinized sectors).

Figures 7 and 8 demonstrate the simulation evolution of this economy.[18] Figure 7 shows the evolution of our two forms of technological change. When V^* ends up equaling c^* once B reaches a critical mass (this happens at $t = 2$), the economy automates, but no new inventions are made. Automation slows down as the number of sectors in which to automate shrink. At time $t^* = 16$, B finally reaches the point where $V = c$, and innovation can happen as well. Eventually growth in both stabilize.

How this evolution in technologies affect labor and wages are demonstrated in Figure 8. The overall patterns echo those demonstrated for the exogenous case shown in Figure 6. As before, the early tinkering economy is one of wage and employment polarization. There is much mid-skilled displacement early on, which gets smaller and smaller. Then, when the transition to breakthrough innovation occurs, wages rise more robustly. But routine labor displacement again picks up pace and mid-level skills continue to shrink. This is suggested by Lemma 1 — the higher is the unskilled wage (and for more advanced economies it will be rather high), the greater will de-routinization be with greater automation. Because greater innovation allows for greater automation, mid skills continue to fall.

Note also that higher-level analytical skills reverse its prior rise. With endogenous growth this should slow down th economy, since analytical workers contribute to growth in basic knowledge B. But given our parameterization, this doesn't much matter for growth. We can see this as the growth of wages do not change even as the growth in L_a slows down.

Are we justified in this conclusion, as it is sensitive to our parameter choices? The important

[18]Parameter values are the same as those used in sections 4.1 and 4.2. As for new parameters, $\delta = 20$, $\lambda = 0.1$, and $\phi = 0.5$. Value for δ is simply to scale the terms appropriately, and values for λ and ϕ are motivated from Jones (1998). We discuss these values more below.

parameter is λ from (37), since it dictates how much the scale of high-skilled workers influences growth in basic knowledge. Here Jones (1998) provides us some guidance. If we take logs of both sides of (37) and differentiate with respect to time, we can solve for the growth rate of B as

$$\frac{\Delta B}{B} = \frac{\lambda}{\phi}\left(\frac{\Delta L_a}{L_a}\right). \tag{38}$$

Here we see that if the growth of L_a falls, the growth of basic knowledge falls. However, empirical evidence suggests λ/ϕ is quite low. We set λ/ϕ to 0.2 (Jones 1998). Although on one hand this is dispiriting, since researchers and high-skilled individuals in general do not seem to help much with growth overall. On the other hand, we might feel more sanguine about the growth prospects of a transition to innovation — any drop in the growth of highly skilled workers should do little to slow down growth in wages.

4.3.2 Potential for Slowdown — The Role of Analytical Skills in New Production

As mentioned earlier, the endogenous model of tinkering and innovation demonstrated here allows us consider two new areas of potential slowdown. The first of these relate to the role analytical labor plays in implementing new technologies. From (19) we can see that the demand for capital is a positive function of the amount of analytical labor available to implement the new technology. And from (10) we see that the larger is α, the more important is analytical labor in the production of new goods. For our simulations, we suggest that it makes sense for $\alpha > \beta$, since implementation of brand new processes should be more human capital-intensive than implementation of automative processes.

But, the larger is analytical labor's role in implementing new technologies (that is, larger α), the worse growth prospects become. This may be unclear from looking at equation (33). On the one hand a larger importance for l_a in new goods production necessarily lowers the importance for new capital goods, lowering the incentive to invent them. On the other hand such an increase also raises the mark-up the inventor can charge for newly invented machines. This has potential relevance for us today, as there have been recent suggestions that such markups have been rising over the last few decades (De Loecker and Eeckout 2017).

But, as one can demonstrate from simulations, a higher α generally slows down the innovation economy (not demonstrated here). And for very high levels of α, growth collapses entirely:

Lemma 4 *As α approaches one, V approaches zero.*

See appendix for proof.

4.3.3 Potential for Slowdown — Redistribution Schemes

One big takeaway from this work is that routine-skill destruction can continue even as the economy transitions to fundamental breakthrough technologies, which means skill losses will continue. Can policy do anything about it, and should it?

We end our discussion with one possible approach to dealing with consistent skill-loss through technological obsolescence. The key is to acknowledge that there is value in a piece of capital (k^*) that replaces a specific skill (l_r). The value is accrued as perpetual monopoly rents. In the model rents are perpetually generated, while people are finitely lived. This provides an opportunity — value from automating machinery can be used to compensate those who lose from *future* automation. This would be one step towards something like a universal basic income, where income is paid to everyone regardless of economic circumstance (Munger 2012), or where profits from automative machinery are taxed and redistributed (Guerreiro et al. 2017). In reality of course patent protections are finite, creating opportunities for other enterprises. The point however is that improvements in production can be captured (perhaps with clever policies) and redistributed. Such schemes are increasingly being considered in the face of mass automation and skill-loss.

A benefit from building the general equilibrium model here is it can help us understand what the long-run impacts on employment and wages might be from such a transfer scheme. Let us consider the total lost routine wages for those who have been displaced from automation at time t as $loss_t$, and consider the earnings accrued from all automative machinery in the economy at this time as π_t.

$$loss_t = \phi_t \left[\left(L_{2,t-1} - L_{1,t-1} \right) / 2 \right] w_{r,t}, \tag{39}$$

$$\pi_t = \left(\frac{\beta}{1-\beta}\right) k_t^* \left(N_{t-1}^* - N_1^*\right). \tag{40}$$

where $(N_{t-1}^* - N_1^*)$ are all extant sectors that use automative machinery. Figures 9 and 10 demonstrate this final case. We can see that redistribution exacerbates de-skilling at the higher end of the skill spectrum. But, two things. First, as already mentioned, λ is likely to be very low, so any growth decline in analytical skill would have minimal impact on growth overall. Second, there actually is no long-run growth decline in analytical labor from the redistribution, only a shorter-term downward shift in analytical labor. In this scenario then the growth impacts from such a scheme would be minimal. This redistribution does however generate a negative pecuniary externality for routine laborers who do not get displaced. Because now mid-level skills have been rendered "safe," more choose to be routine workers, depressing w_r.

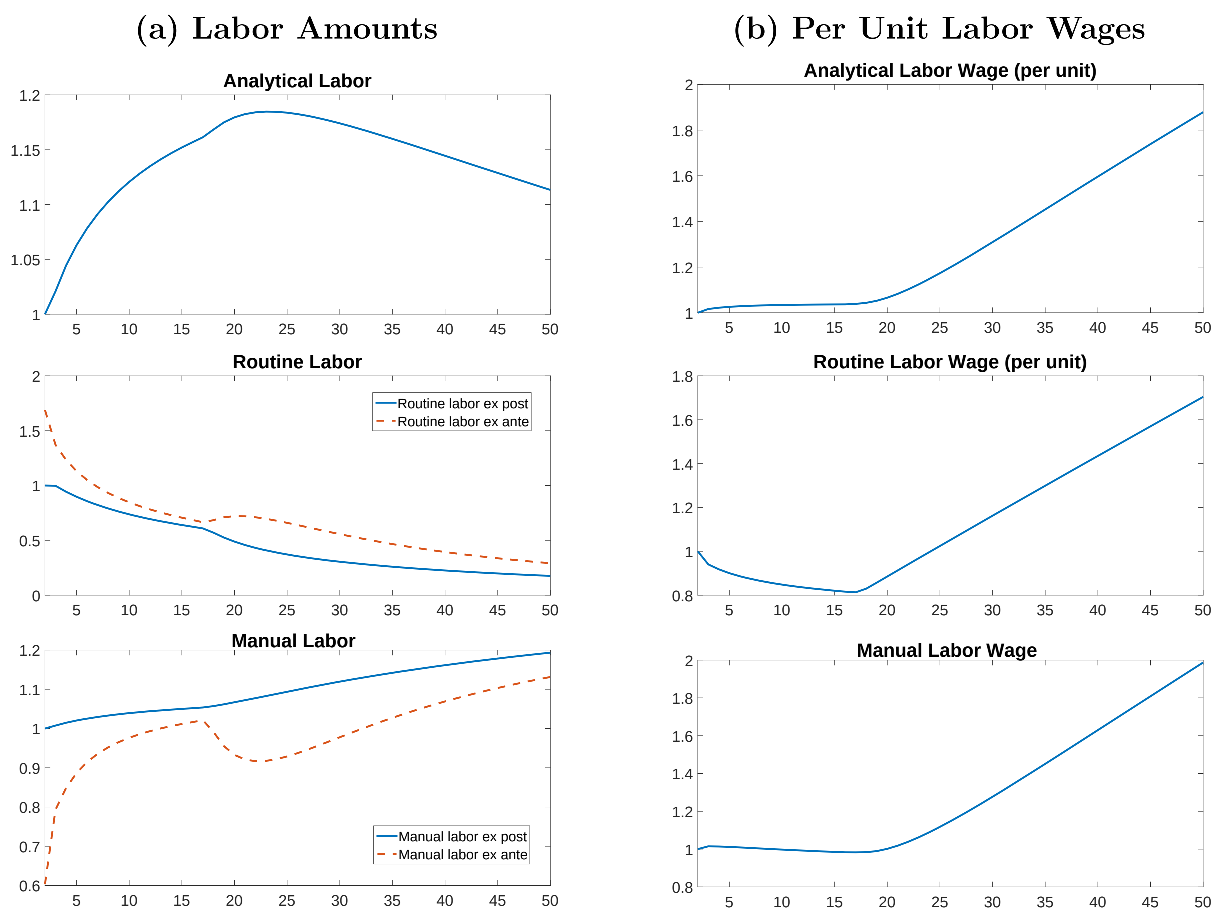
5 Conclusion

This paper explores the interactions between different forms of technological changes and different types of skilled labor. We derive a number of new insights in order to better understand the evolution of technologies and human capital over the last few centuries.

Taking the pessimists seriously, we see they may have a point. Several even. We suggest that the return to breakthrough technological growth of the kind rhapsodized and longed for by Tyler Cowen and Robert Gordon will likely not regenerate mid-level jobs the way they did historically. Growth slows down not due to a slowdown of high-level human capital accumulation, but rather due to the mid-skilled migrating to unproductive service jobs. In an extreme version of this bleak tableau, we all end up uneducated, working in stagnant occupations, even as the potential for innovation expands to unprecedented heights.

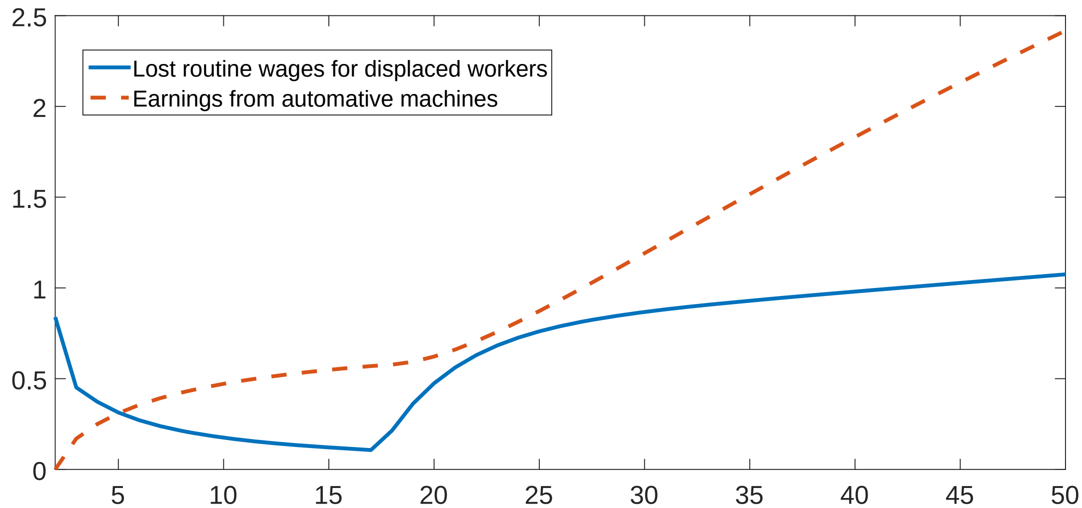
Given the technological transitions suggested by the model, one might wonder about the possible phase of economic growth we currently find ourselves. In truth we see aspects of both types. On the one hand, techno-pessimistic grumblings about low productivity growth and job insecurity suggests the presence of a "tinkering" economy. Yet we also see evidence of skill deterioration and burgeoning reliance on service production, which in our model is consistent

Figure 8: Endogenous Scope and Direction of Technological Change



With endogenous automation and innovation, we observe similar patterns to the exogenous case.

Figure 9: Redistributing Earnings to Displaced Workers



After a certain point, the earnings from automation can always compensate those displaced by new automation. This produces an opportunity for compensation schemes.

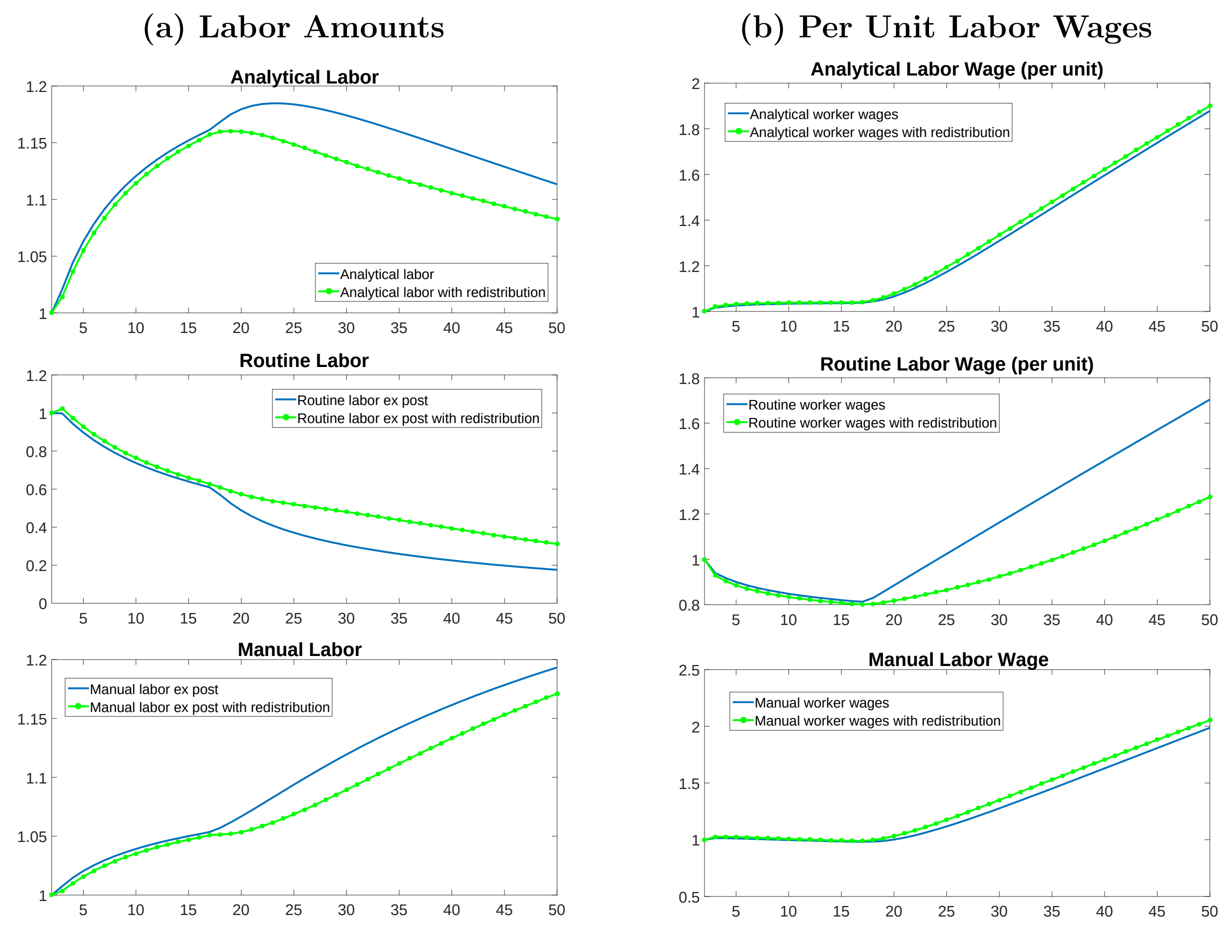
with more robust technological activity.

While what we present here is a closed macro model designed to represent the overall economy, countries might be better characterized as a series of economies spatially integrated (Autor and Dorn 2013). Such an extension may help us further explore which regions are better characterized as tinkering economies (marked by burgeoning informal sectors and job displacement) and which as fundamentally innovating ones (with robust *overall* wage growth).

Another aspect is to look at the economic evolutions of different economies to see if there is any evidence of such technological cycles over the long run. Evidence of technological slowdowns and re-emphasis on tinkering may motivate educational policies designed to help spur virtuous feedback between high-level human capital and innovative breakthroughs.

There are also potential lessons for educational reform. Joel Mokyr suggests that since 1945 education has encouraged greater specialization, where students learn more and more about less and less (in this case education is like clay: "shape it, then bake it, and that's the way it stays."). As production processes become obsolete however, it becomes more important to learn to *relearn* (education here would be like putty, which can be reshaped). A key educational challenge then

Figure 10: Endogenous Scope and Direction of Technological Change with Redistribution to Displaced Workers



Redistribution lowers both analytical and manual labor. This however does not really affect growth. Why?

is in figuring out how to inject analytical skills to the whole workforce, so that we might avoid having clay-baked human capital shattered by the future forces of automation.

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APPENDIX A — Proofs

Proof of Lemma 1

To show that the no skill/routine skill threshold (L_1) rises with rises in the probability of automation (ϕ), we use the quotient rule:

$\frac{\partial L_1}{\partial \phi} = [-\gamma_1(1-\phi)w_r\phi + X\gamma_1 w_r] \,/\, (\gamma_1(1-\phi)w_r)^2$

where $X \equiv ((1+w_{u,t-1})^{\omega} + (1+w_{u,t})^{\omega} - 1)^{1/\omega} - 1 - \phi w_{u,t}$. The above expression is positive when $X > \phi(1-\phi)$. As $\frac{\partial X}{\partial w_u} > 0$ for all $w_u > 0$ and $\phi \leq 1$, we can see that 1) there exists some threshold level of w_u such that $\frac{\partial L_1}{\partial \phi} > 0$ for all w_u above that threshold, and 2) that $\frac{\partial^2 L_1}{\partial \phi \partial w_u} > 0$.

Proof of Lemma 2

To show that the routine skill/analytical skill threshold falls with rises in analytical wages, we use the quotient rule:

$\frac{\partial L_2}{\partial w_a} = \left[(\gamma_2 w_a - (1-\phi)\gamma_1 w_r)\,\gamma_2\hat{L} - \left(\gamma_2\hat{L}w_a + \phi w_u\right)\gamma_2\right] / \,(\gamma_2 w_a - (1-\phi)\gamma_1 w_r)^2$

The denominator must be positive by construction. The numerator can be simplified to:

$-(1-\phi)\gamma_1\gamma_2\hat{L}w_r - \gamma_2\hat{L}w_a$

which is always negative for all positive parameter values. Thus $\frac{\partial L_2}{\partial w_a} < 0$.

The relationships $\frac{\partial L_2}{\partial w_r} > 0$ and $\frac{\partial L_2}{\partial w_u} > 0$ are manifest by the expression for L_2.

Proof of Lemma 3

To show a case where the routine skill/analytical skill threshold falls with increases in automation, we again use the quotient rule:

$\frac{\partial L_2}{\partial \phi} = \left[(\gamma_2 w_a - (1-\phi)\gamma_1 w_r)\,w_u - \left(\gamma_2\hat{L}w_a + \phi w_u\right)\gamma_1 w_r\right] / \,(\gamma_2 w_a - (1-\phi)\gamma_1 w_r)^2$

Again we focus just on the numerator, which can be simplified to

$\gamma_2 w_a w_u - \gamma_1 w_r w_u - \gamma_1\gamma 2\hat{L}w_a w_r$

This expression will clearly be negative whenever the third term exceeds the first term. This suggests

If $w_u < \gamma_1\hat{L}w_r, \;\; \frac{\partial L_2}{\partial \phi} < 0$

Proof of Lemma 4

The value of inventing a new machine blueprint can written by combining (19) and (33):

$V = \left(\frac{\alpha}{1-\alpha}\right)(1-\alpha)^{\frac{2}{1+\alpha\gamma-\gamma}}\,\gamma^{\frac{1}{1+\alpha\gamma-\gamma}}\,A^{new\,\frac{\gamma}{1+\alpha\gamma-\gamma}}\,l_a^{\frac{\alpha\gamma}{1+\alpha\gamma-\gamma}}$

We can take the limit of this expression as α approaches one and focus on the coefficient term, as the other terms converge to finite values. To the limit of the coefficient, we apply L'Hopital's Rule:

$\lim_{\alpha\to 1}\left[(1-\alpha)^{\frac{2}{1-\gamma-\alpha\gamma}}\right]/\,(1-\alpha) = \lim_{\alpha\to 1} f(\alpha)/g(\alpha) = 0/0$

$\lim_{\alpha\to 1} f'(\alpha)/g'(\alpha) = (2/1-\gamma+\alpha\gamma)\,(1-\alpha) + \left[(1-\alpha)^{\frac{2}{1-\gamma+\alpha\gamma}}\,ln(1-\alpha)\gamma/\,(1-\gamma+\alpha\gamma)^2\right] = 0$

Thus the value of new innovation approaches zero as α approaches one.