NAME: ____ **ALPHA**______________

INSTRUCTOR: _________________________________ **SECTION**___________

NO CALCULATORS ALLOWED ON THIS PART SHOW ALL WORK IN THIS EXAM PACKAGE

This exam consists of two parts. You must do all work on the exam package and turn in all parts.
Part I: multiple choice without calculator. (50%, 20 problems, 2.5% each)
Part II: long answer with calculator. (50% , 7 problems, approx. 7% each)

1. Write your name, alpha number, instructor, and section number on this test and your Scantron bubble sheet. Bubble in your alpha number and the **VERSION NUMBER** of Part I of your exam.

2. **Do Part I first. No calculators are allowed for Part I.** Show all work on your exam packet. You must turn in Part I of your exam, including the bubble sheet, before using a calculator. Part I and the bubble sheet are due **20 min prior to the end of the exam period.**

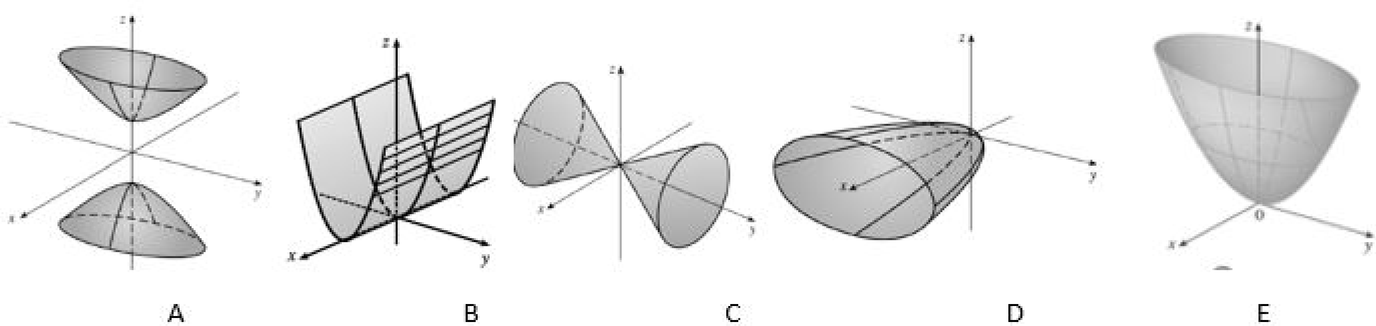
3. ALL communication devices (cell phones, smart watches, etc) are prohibited and must be put away during the exam. If you need to leave the classroom you must leave all said devices in the classroom.

4. TI-36X Pro calculators are allowed for Part II; they may not be shared. Show all work on your exam packet.

5. Conversion formulas (from spherical to rectangular coordinates) are provided on the last page of Part II, which you may use for any part of the exam. The rest of the page was intentionally left blank, in case you need extra paper to show your work.

PART I: MULTIPLE CHOICE (50%). NO CALCULATORS ALLOWED. Do all work on the exam packet. There is no extra penalty for wrong answers on the multiple choice part of the exam.

1. Match the equation $z = y^2$ with the appropriate graph below:



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2. The equation of a plane containing the point $(2, 1, 3)$ is:

a. $x^2 + y^2 + z^2 = 14$

b. $x = (2t+1)\vec{i} + (t+2)\vec{j} + (3t+1)\vec{k}$

c. $2x + y + 3z = 0$

d. $x + y + z = 6$

e. $2x^2 + y^2 + 3z^2 = 6.$

NAME: ______________________________

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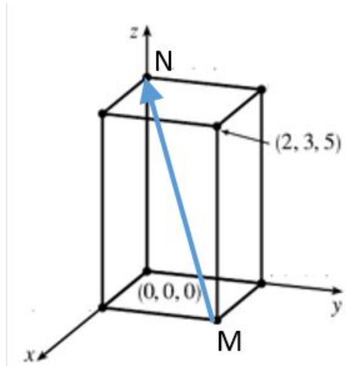
3. The rectangular box shown below has one vertex at the origin and the opposite vertex at $(2, 3, 5)$. The vector $\overrightarrow{MN}$ in the sketch is:



a) < 2, 3, 5>

b) < 2, 3, -5>

c) < -2, 3, -5>

d) <-2, -3, 5>

e) < 0, -3, 5>

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4. A curve C has the parametrization given by the vector $\vec{r}$ below.

$$\vec{r}(t) = \ln t\,\vec{i} - \vec{j} + t^3\vec{k}$$

The equation of the line tangent to C at the point $(0, -1, 1)$ is

a. $x + 3z = 3$

b. $-(y + 1) + (z - 1) = 0.$

c. $x = \frac{1}{t}, \quad y = 0, \quad z = 3t^2 + 1$

d. $x = t, \quad y = -1, \quad z = 1 + 3t$

e. the tangent does not exist

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5. A particle starts from rest (zero velocity) and moves in the plane with acceleration $\vec{a}$ given at time t by

$$\overrightarrow{a}(t) =< 4\cos(\frac{1}{2}t),\ 3\sin t > .$$

The speed of the particle at $t = \pi$ equals:

a. $< 0, 3 >$

b. $< -2, 3 >$

c. $\sqrt{11}$

d. 8

e. 10

NAME: ____

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6. The level curves in the (x, y) plane for the function $f(x, y) = x^2 + y^2$ are:

a) cones b) paraboloids c) parabolas d) right triangles e) circles

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7. If

$$f(x, y, z) = \frac{x}{z}\sin(yz),$$

then $\frac{\partial^2 f}{\partial x \partial y}$ equals

a. 0

b. $\cos(yz)$

c. $\frac{1}{z}\cos(z)$

d. $\frac{1}{z}\sin(yz) + \frac{x}{z}\cos(yz) - \frac{x}{z}\cos(yz)$

e. $< \frac{1}{z}\sin(yz),\ \frac{x}{z}\cos(yz),\ -\frac{x}{z}\cos(yz) >$

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8. Let $f(x, y, z) = 2xe^z + 3y$. The maximum rate of change for f at $(3, 1, 0)$ is

a) $2e^z + 2xe^z + 3$

b) $< 2, 3, e^z >$

c) 7

d) 11

e) $\frac{72}{\sqrt{10}}$.

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9. Given the points $A(0, 2)$ and $B(1, 3)$ and a function f with gradient

$$\nabla f(x, y) =< x + 3y - 10, 3x - y >,$$

which of the following statements is true?

a) A and B are critical points for f; they are both local min.

b) Only A is a critical point for f. A is a saddle.

c) Only A is a critical point for f. A is a local max.

d) Only B is a critical point for f. B is a saddle.

e) Only B is a critical point for f. B is a local max.

NAME: ____

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10. A hill has the shape given by the equation

$$z = 100 - 0.05x^2 + 0.03y^2$$

where x, y and z are measured in meters, the positive x-axis points east, and the positive y-axis points north. If you are standing at the point at which $x = 1$ and $y = 1$ and you are walking east, are you ascending or descending and at what rate?

a) ascending at 0.05 m per horizontal m

b) ascending at 0.03 m per horizontal m

c) descending at 0.1 m per horizontal m

d) descending at 0.06 m per horizontal m

e) neither ascending nor descending

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11. Which integral corresponds to

$$\int_0^2 \int_{y^2}^{2y} f(x, y)\, dx\, dy$$

with reversed order of integration? Hint: sketch the regions of integration.

a) $\int_0^2 \int_{x^2}^{2x} f(x, y)\, dydx$

b) $\int_0^4 \int_{x/2}^{\sqrt{x}} f(x, y)\, dydx$

c) $\int_{y^2}^{2y} \int_0^2 f(x, y)\, dydx$

d) $\int_0^2 \int_{\sqrt{x}}^{x/2} f(x, y)\, dydx$

e) $\int_0^4 \int_{2x}^{x^2} f(x, y)\, dydx$

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12. Let D be the region *in the first quadrant*, between the circles $x^2 + y^2 = 1$, and $x^2 + y^2 = 9$. Sketch D. The integral of

$$f(x, y) = \frac{x}{x^2 + y^2}$$

over D is equal to

a) 2

b) $\pi \ln 3$

c) $\ln 2$

d) $\frac{1}{9}$

e) $\frac{8\pi}{9}$

NAME: ______________________________________

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13. Which of the following is a parametrization for a circle in the plane $x = 1$? All parameters are in $[0, 2\pi]$:

a) $\vec{r} =< u, 4\cos t, 4\sin t >$

b) $\vec{r} =< \cos t, \sin t, 4u >$

c) $\vec{r} =< 1, 4\cos t, 4\sin t >$

d) $\vec{r} =< 1, 1, 4 - t^2 >$

e) $\vec{r} =< u, v\cos t, v\sin t >$

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14. Let E be the solid cylinder of radius 2 and height 5 sitting on the $(x, y)-$ plane. Let $f(x, y, z) = \sqrt{x^2 + y^2}$. The integral of f over E is given by the integral

a. $\int_0^{2\pi} \int_0^2 \int_0^5 r^2 dzdrd\theta$

b. $\int_0^{2\pi} \int_0^r \int_0^5 rdzdrd\theta$

c. $\int_{-1}^1 \int_{-1}^1 \int_0^5 \sqrt{x^2 + y^2} dzdydx$

d. $\int_0^{2\pi} \int_0^\pi \int_0^5 \rho^3 \sin^2\phi \sin\theta d\rho d\phi d\theta$

e. $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_0^5 \sqrt{x^2 + y^2} dzdydx$

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15. Let E be the solid cylinder of radius 2 and height 5 sitting on the xy plane, and let S be its boundary with outward orientation. (Note that S consists of the circular side, the top and bottom disks). The flux of

$$\vec{F} =< -2xy, y^2, 4z + x^2 + y^2 >$$

across the surface S equals:

a) $< -2y, 2y, 4 >$

b) $< 2y, -2x, -2x >$

c) 0.

d) -25π

e) 80π.

NAME: ____

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16. Compute the line integral below for the curve C that is the part of the graph of the exponential curve $x = t,\ y = e^t$ starting from $(0, 1)$ and ending at $(1, e)$.

$$\int_C (ye^x)dx + e^x dy$$

a) $e^2 - 1$.

b) $e - 1$

c) $\frac{1}{2}e - 1$

d) $2e^2 - 2$

e) 0.

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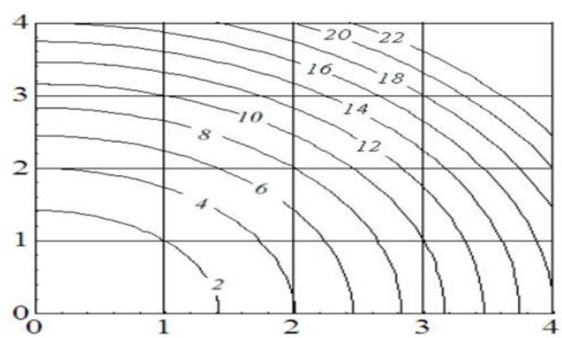
17. A conservative vector field $\vec{F}$ in the plane has potential f. Use the contour map for f provided below to find the work done by $\vec{F}$ in moving a particle along the curve $x = 4t - t^2,\ \ y = t$, with $1 \le t \le 4$.

a) -14
b) -6
c) 6
d) 14
e) 18



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18. Let S be the part of the surface $z = y^2 - x + 2$ that is above the unit square $0 \le x \le 1,\ 0 \le y \le 1$ in the $(x, y)-$ plane, with upward orientation. The flux of $\vec{F}$ across S for the vector field

$$\vec{F} = < z, 0, x - y^2 >$$

equals:

a) 2

b) $\frac{2}{3}$

c) $\frac{-8}{3}$

d) $< \frac{5}{6}, 0, \frac{1}{6} >$

e) $< \frac{2}{3}, \frac{-8}{3}, 2 >$

NAME: ____

NO CALCULATORS ALLOWED ON THIS PART SHOW ALL WORK IN THIS EXAM PACKAGE

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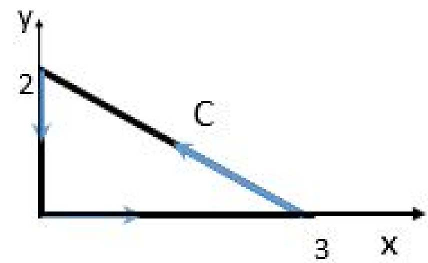
19. Evaluate the line integral below where C is the closed boundary curve of the triangle below, oriented counterclockwise.

$$\int_C <\ 3y,\ 4x + y^2\ > \cdot d\vec{r}$$

a) 0
b) 1
c) 2
d) 3
e) 6



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20. Consider the surface S of equation $2x + y + z = x^2 + y^2$ and the point $P(1, 3, 5)$ on that surface. The equation of the plane tangent to S at the point P is:

a) $-5(y - 3) + z - 5 = 0$

b) $2(x - 1) + (y - 3) + (z - 5) = 10$

c) $2x(x - 1) + 2y(y - 3) + 5 = z$

d) $x + 3y + 5z = 35$

e) $4 - 2x - 2y = 0$

NAME: ____

YOU SHOULD TURN IN PART I and the SCANTRON FORM PRIOR TO USING THE CALCULATOR ON THIS PART Show details of your work and box your answers.

PART II: LONG ANSWER. TI-36X Pro calculators are allowed. You must turn in Part I and the bubble sheet before using your calculator. Part I and the bubble sheet are due **20 min prior to the end of the exam period.** Show details of your work and box your answers.

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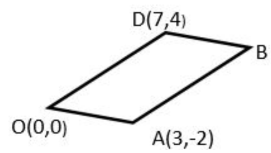
21. The parallelogram $OABD$ below has three of its vertices located at $O(0,0)$, $A(3,-2)$, and $D(7,4)$.



a) Name two pairs of equal vectors.

b) A particle starts from the origin O and moves on a metal plate toward the point D. The temperature T of the plate at location (x, y) is given by

$$T(x,y) = 8x - 30y + \frac{1}{49}x^2y^3.$$

At what rate is the temperature of the plate increasing or decreasing as the particle starts moving?

c) Use operations with vectors to find $\overrightarrow{OB}$.

d) Find the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

NAME: ______________________________________

YOU SHOULD TURN IN PART I and the SCANTRON FORM PRIOR TO USING THE CALCULATOR ON THIS PART Show details of your work and box your answers.

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22. The table below gives the speed of sound C (in meters per second) traveling through sea water of salinity S (in grams per liter, gr/l), at the depth D below the ocean surface (in hundred meters, hm)

S \ D	0	0.5	1	1.5
25	1482	1482.8	1483.6	1484.2
30	1488.2	1489	1489.8	1490.6
35	1494.4	1495.2	1496	1496.8

a) Approximate $\frac{\partial C}{\partial S}$ (give units) at $(S, D) = (35, 1)$.

b) Explain the physical meaning of $\frac{\partial C}{\partial S}$.

c) Approximate $\frac{\partial C}{\partial D}$ (give units) at $(S, D) = (35, 1)$.

d) A diver is moving underwater; at some time (t in hours) the salinity of the water is $S = 35$ grams per liter and decreasing at a rate of 0.05 grams per liter per hour, and the depth of the diver is $D = 1$ hundred meters and increasing at a rate of 0.2 hundred meters per hour.

Find the rate of change with respect to time of the speed of sound experienced by the diver.

NAME: ____

YOU SHOULD TURN IN PART I and the SCANTRON FORM PRIOR TO USING THE CALCULATOR ON THIS PART Show details of your work and box your answers.

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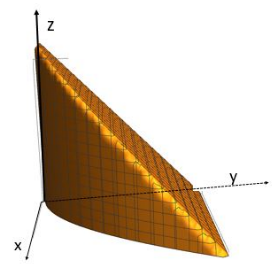
23. Let E be the solid bounded by the planes $z = 0, x = 0$, and $y + z = 4$ and the cylinder $y = x^2$, sketched below. The density of the solid E is approximately equal to $\rho(x, y, z) = \frac{1}{4-y}$.



a) Set up iterated integrals equal to the mass of the solid, in TWO of the orders listed below:

$$dz\ dx\ dy \qquad dz\ dy\ dx \qquad dx\ dz\ dy \qquad dz\ dr\ d\theta$$

b) Find the mass.

NAME: ____

YOU SHOULD TURN IN PART I and the SCANTRON FORM PRIOR TO USING THE CALCULATOR ON THIS PART Show details of your work and box your answers.

24. Consider the solid E described in spherical coordinates as:

$$0 \leq \rho \leq 4, \ \ 0 \leq \phi \leq \pi/4, \ \ 0 \leq \theta \leq 2\pi.$$

a) Name the surfaces that enclose the solid. For one of them, give its equation in rectangular coordinates (simplify your answer).

b) Sketch the solid E.

c) Is the point with rectangular coordinates $(2, 2, 2\sqrt{2})$ on any of the surfaces you identified in a)?

d) Set up a triple integral in spherical coordinates equal to the volume of E. Do NOT evaluate it.

e) Set up a triple integral in cylindrical coordinates equal to the volume of E. Do NOT evaluate it.

NAME: ____

YOU SHOULD TURN IN PART I and the SCANTRON FORM PRIOR TO USING THE CALCULATOR ON THIS PART Show details of your work and box your answers.

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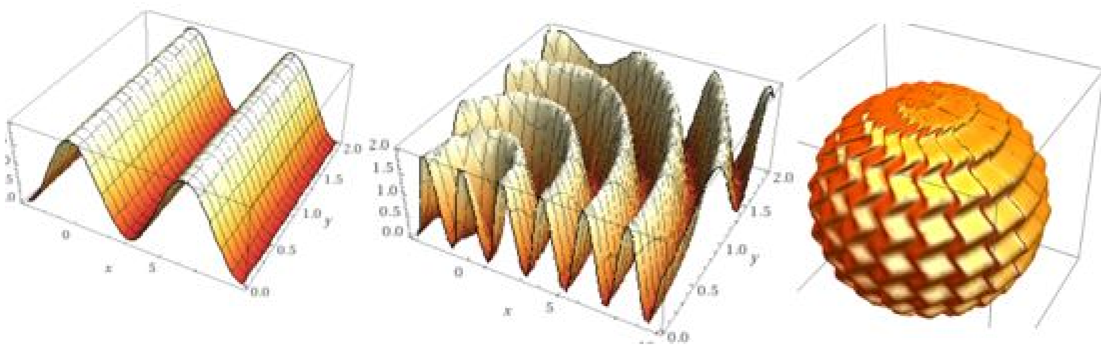
25. Consider the vector field $\vec{F} =< -y, z - y, 2021^{2022z} >$. Let the surface S be the part of $z = 1 + \cos x$ corresponding to $-\pi \le x \le 3\pi,\ \ 0 \le y \le 2$, oriented to point up, and let C be the boundary curve of S, with positive orientation.

a) Find curl $\vec{F}$, also denoted $\nabla \times \vec{F}$.

b) Which of the following sketches matches S? Circle the right one.



c) Use a named theorem to identify an integral below that equals $\int_C \vec{F} \cdot d\vec{r}$. Circle the answer.

$$\int_C \nabla \times \vec{F} \cdot d\vec{r} \qquad \int\int_S \vec{F} \cdot d\vec{S} \qquad \int\int_S \nabla \times \vec{F} \cdot d\vec{S} \qquad \int\int\int_E \nabla \times \vec{F}\, dV.$$

d) Find $\int_C \vec{F} \cdot d\vec{r}$.

NAME: ____

YOU SHOULD TURN IN PART I and the SCANTRON FORM PRIOR TO USING THE CALCULATOR ON THIS PART Show details of your work and box your answers.

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26. Consider the conservative vector field

$$\overrightarrow{F}(x,y,z) = < 1+y^2,\ 2xy+e^{3z},\ 3ye^{3z}\ > .$$

Find a potential for $\vec{F}$.

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27. Finish the statement of the Fundamental Theorem for Line Integrals included below, and provide a proof. Use proper notations and careful English for the proof.

Let C be a smooth curve from A to B, given by the vector function $\vec{r}(t),\ a \leq t \leq b$. Let $\vec{F}$ be a conservative vector field, continuous on a domain containing C, with potential f. Then

$$\int_C \vec{F} \cdot d\vec{r}\ =$$

NAME: ____

YOU SHOULD TURN IN PART I and the SCANTRON FORM PRIOR TO USING THE CALCULATOR ON THIS PART Show details of your work and box your answers.

Conversion formulas (from spherical to rectangular coordinates):

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho \cos\phi.$$