NAME: ______________________________ ALPHA: _______________________

INSTRUCTOR: ________________________ SECTION: _____________________

CALCULUS II (SM122/122S) FINAL EXAMINATION

1330 – 1630, 13 December 2012 SHOW ALL WORK IN THIS EXAM PACKAGE

This exam is composed of three parts: Part I – long answer without calculator, Part II – multiple choice with calculator, and Part III – long answer with calculator. There is a table of series on the last page which you may use for any part of the exam. No calculators are allowed for Part I. You must turn in Part I of your exam before beginning Parts II and III. Calculators are allowed for Parts II and III. Calculators may not be shared. Part II, multiple choice, is 50% of the exam. Parts I and III are the other 50%. Write your name, alpha number, instructor, and section number on this test and your Scantron bubble sheet and bubble in your alpha number.

PART I: LONG ANSWER. NO CALCULATORS ALLOWED.
Show details of your work.

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21. (a) Evaluate

$$\int x\cos(x^2)\,dx.$$

(b) Evaluate

$$\int x\cos(2x)\;dx.$$

22. (a) Use a limit to determine whether the following integral converges or diverges. If it converges, find its value.

$$\int_1^\infty \frac{1}{x^2}\,dx$$

(b) Evaluate

$$\int \frac{12}{x^2+4x}\,dx.$$

23. Write ONE of the following two proofs.

Proof #1 Use the Mean Value Theorem and the Fundamental Theorem of Calculus to prove that if f is a function that is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\ dx.$$

OR

Proof #2 Let $\boldsymbol{a}$ be a nonzero vector and let $\boldsymbol{b}$ be another vector. Recall that the orthogonal projection of $\boldsymbol{b}$ with respect to $\boldsymbol{a}$ is defined to be

$$\mathrm{orth}_{\boldsymbol{a}}\boldsymbol{b} = \boldsymbol{b} - \mathrm{proj}_{\boldsymbol{a}}\boldsymbol{b}.$$

where $\mathrm{proj}_{\boldsymbol{a}}\boldsymbol{b}$ is the vector projection of $\boldsymbol{b}$ onto $\boldsymbol{a}$. Prove that $\mathrm{orth}_{\boldsymbol{a}}\boldsymbol{b}$ is orthogonal (perpendicular) to $\boldsymbol{a}$.

PART II: MULTIPLE CHOICE (50%). CALCULATORS ARE ALLOWED. CALCULATORS MAY NOT BE SHARED.

You must turn in Part I of the exam before beginning Part II. Be sure that you have put your name, alpha number, instructor, and section number on your bubble sheet and bubbled in your alpha number. These 20 problems are multiple choice. Fill in the best answer to each question on your Scantron bubble sheet. There is no extra penalty for wrong answers on the multiple choice part of the exam.

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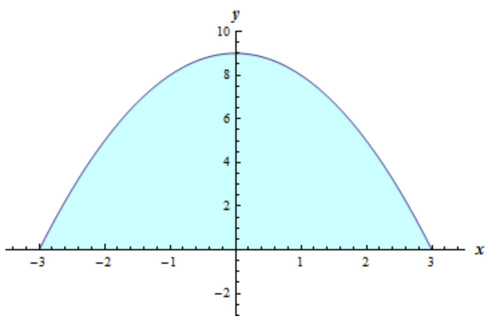
1. The area of the region below the parabola $y = 9 - x^2$ and above the x-axis is



a. 12 b. 24 c. 36 d. 48 e. 60

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2. The substitution $u = 1 + x^2$ transforms the integral

$$\int \frac{x}{1 + x^2}\, dx$$

into

a. $\frac{1}{2}\int \frac{1}{u}\, du$ b. $\int \frac{1}{u}\, du$ c. $2\int \frac{1}{u}\, du$ d. $\int \frac{\sqrt{u-1}}{u}\, du$ e. $\frac{1}{2}x^2 \tan^{-1}(x) + C$

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3. Which of the following is a Cartesian equation for the curve given in polar coordinates by $r = 3$?

a. $x = 3$
b. $y = 3$
c. $x + y = 3$
d. $x^2 + y^2 = 3$
e. $x^2 + y^2 = 9$

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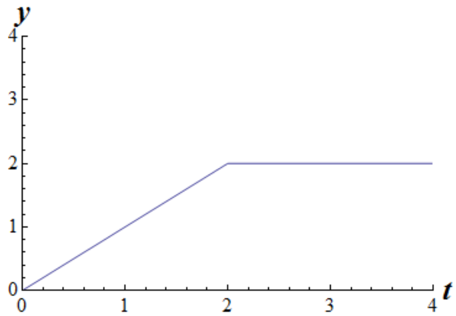
4. The graph of $y = f(t)$ is shown below. Suppose that

$$g(x) = \int_0^x f(t)dt.$$

Then the value of $g(4)$ is



a. 0 b. 2 c. 4 d. 6 e. 8

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5. A force of 30 N is required to hold a spring stretched 0.1 meters beyond its natural length. How much work is done in stretching it from its natural length to 0.2 meters beyond its natural length?

a. 1.5 J b. 3 J c. 6 J d. 60 J e. 300 J

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6. A table of values of a function f is given. Use the Trapezoidal Rule to estimate $\int_0^2 f(x)dx$.

x	0	0.5	1.0	1.5	2.0
$f(x)$	0	2	8	9	12

a. 9.5 b. 10 c. 12 d. 12.5 e. 15.5

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7. For which value of r is $y = e^{rt}$ a solution of the differential equation

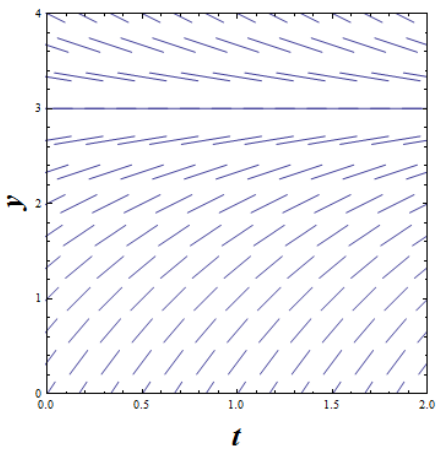
$$\frac{dy}{dt} - 3y = 0?$$

a. -3 b. 0 c. $\frac{1}{3}$ d. 3 e. for no value of r

8. The direction field of the differential equation

$$\frac{dy}{dt} = 3 - y$$

is shown below. If $y(0) = 0$, then $y(1)$ is closest to



a. 0 b. 1 c. 2 d. 3 e. 4

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9. The solution of the initial value problem $\frac{dy}{dt} = 3 - y$ and $y(0) = 0$ is

a. $3y - \frac{1}{2}y^2$ b. $3 - 3e^{-t}$ c. $3 - 3e^{t}$ d. $3 - e^{-t}$ e. $1 - e^{t}$

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10. Which of the following is ***not*** an improper integral?

a. $\int_{-1}^{1} \frac{1}{x}\, dx$

b. $\int_{0}^{1} \frac{1}{2-x}\, dx$

c. $\int_{0}^{1} \frac{1}{1-x}\, dx$

d. $\int_{0}^{1} \frac{1}{\sqrt{1-x}}\, dx$

e. $\int_{-\infty}^{0} \frac{1}{1+x^2}\, dx$

11. Suppose that

$$a_n = \frac{2n+1}{5n+3}.$$

Then

a. the sequence $\{a_n\}$ converges to $\frac{1}{3}$ and the series $\sum_{n=1}^{\infty} a_n$ converges.

b. the sequence $\{a_n\}$ converges to $\frac{2}{5}$ and the series $\sum_{n=1}^{\infty} a_n$ converges.

c. the sequence $\{a_n\}$ diverges and the series $\sum_{n=1}^{\infty} a_n$ converges.

d. the sequence $\{a_n\}$ converges to $\frac{1}{3}$ and the series $\sum_{n=1}^{\infty} a_n$ diverges.

e. the sequence $\{a_n\}$ converges to $\frac{2}{5}$ and the series $\sum_{n=1}^{\infty} a_n$ diverges.

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12. The interval and radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$$

are

a. interval $(-\infty, \infty)$, radius ∞

b. interval $(-\infty, \infty)$, radius 0

c. interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$, radius $\frac{1}{2}$

d. interval $(-2,2)$, radius 2

e. interval $(-2,2)$, radius $\frac{1}{2}$

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13. The sum of the geometric series $1 + x^2 + x^4 + x^6 + \cdots$, for $|x| < 1$, is

a. $\frac{1}{1+x^2}$ b. $\frac{1}{1-x^2}$ c. $\cos(x^2)$ d. e^{x^2} e. ∞ (series diverges)

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14. Which of the following gives the first few terms of the Maclaurin series for $\cos(3x)$?

a. $1 - \frac{3^2x^2}{2!} + \frac{3^4x^4}{4!} - \cdots$

b. $1 - \frac{3x^2}{2!} + \frac{3x^4}{4!} - \cdots$

c. $3 - \frac{3x^2}{2!} + \frac{3x^4}{4!} - \cdots$

d. $\cos(3x) - 3\sin(3x)\,x - 9\cos(3x)\frac{x^2}{2!} + \cdots$

e. $3x - \frac{3x^3}{2!} + \frac{3x^5}{4!} + \cdots$

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15. Suppose that f is a function such that $f(2) = 5$, $f'(2) = 3$, and $f''(2) = -2$. Which of the following is the 2nd degree Taylor polynomial of f centered at $x = 2$?

a. $5 + 3x - x^2$

b. $5 + 3x - 2x^2$

c. $5 + 3(x-2) - (x-2)^2$

d. $5 + 3(x-2) - 2(x-2)^2$

e. $2 + 2(x-3) + 2(x+2)^2$

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16. A point is given in polar coordinates by $r = -2, \theta = \frac{\pi}{3}$. The Cartesian (rectangular) coordinates (x, y) for the point are

a. $(-2,0)$ b. $\left(\frac{1}{4}, \frac{\sqrt{3}}{4}\right)$ c. $\left(-\frac{1}{4}, -\frac{\sqrt{3}}{4}\right)$ d. $(1, \sqrt{3})$ e. $(-1, -\sqrt{3})$

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17. A sled moves along a horizontal path. The sled is pulled by a force of 40 pounds acting at an angle of 20° above the horizontal. The work done by the force in pulling the sled 100 feet is approximately

a. 0 ft-lbs b. 1370 ft-lbs c. 1630 ft-lbs d. 3760 ft-lbs e. 4000 ft-lbs

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18. The dot product of the vectors $\boldsymbol{a} = 3\boldsymbol{i} + 0\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{b} = 2\boldsymbol{i} + 2\boldsymbol{j} - 1\boldsymbol{k}$ is

a. $6\boldsymbol{i} + 0\boldsymbol{j} + 4\boldsymbol{k}$ b. $8\boldsymbol{i} - 5\boldsymbol{j} + 6\boldsymbol{k}$ c. 9 d. 10 e. 15

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19. Which of the following vectors is perpendicular to both $\boldsymbol{v} = \langle 2, -1, 3 \rangle$ and $\boldsymbol{w} = \langle 1, -1, 2 \rangle$?

a. $\langle 0,3,1 \rangle$ b. $\langle 1,-1,-1 \rangle$ c. $\langle 2,0,-1 \rangle$ d. $\langle 2,1,6 \rangle$ e. $\langle 3,-2,5 \rangle$

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20. Which of the following describes the line through the point $(2,4,2)$ which is parallel to the vector $\langle 5,-3,4 \rangle$?

a. $5(x-2) - 3(y-4) + 4(z-2) = 0$

b. $x = 2 + 5t, y = 4 - 3t, z = 2 + 4t$

c. $2(x-5) + 4(y+3) + 2(z-4) = 0$

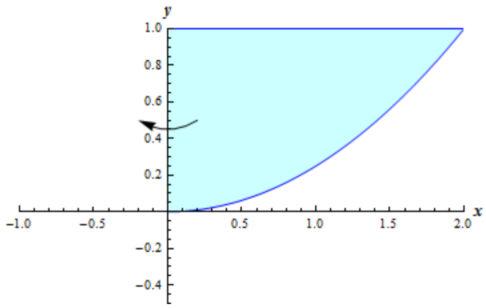
d. $x = 5 + 2t, y = -3 + 4t, z = 4 + 2t$

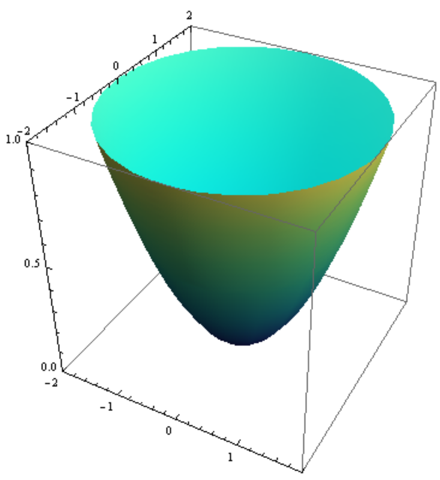
e. $3\boldsymbol{i} - 7\boldsymbol{j} + 2\boldsymbol{k}$

PART III: LONG ANSWER. CALCULATORS ARE ALLOWED. CALCULATORS MAY NOT BE SHARED.

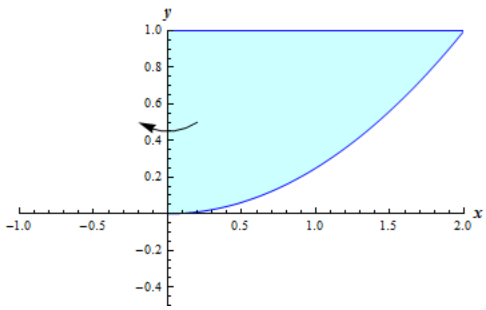
You must have turned in Part I of your exam before beginning this part. Show details of your work and box your answers.

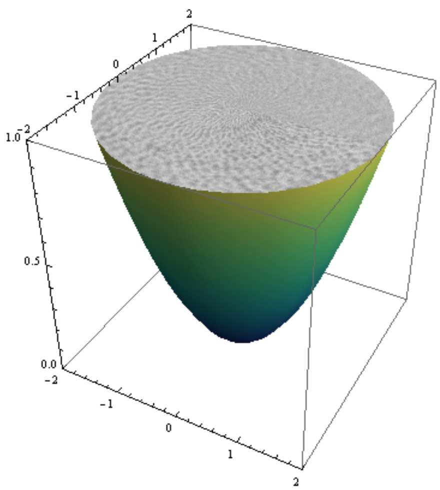
24. Let R be the region in the first quadrant bounded by the graph of $x = 2\sqrt{y}$, the y-axis, and the line $y = 1$. Find the volume of the solid obtained when the region R is revolved about the y-axis.





25. A tank full of water has the shape of the solid of revolution of the previous problem, i.e., the tank has the shape generated when the graph of the equation $x = 2\sqrt{y}$, for $0 \leq y \leq 1$, is revolved about the y-axis. Assume that distances are measured in feet, and recall that water weighs 62.5 pounds per cubic foot. Find the work required to pump all of the water out of the top of the tank.





26. (a) Given the vectors $\boldsymbol{v} = \langle 1, -2, 3 \rangle$ and $\boldsymbol{w} = \langle 4, 0, 2 \rangle$, evaluate the cross product $\boldsymbol{v} \times \boldsymbol{w}$.

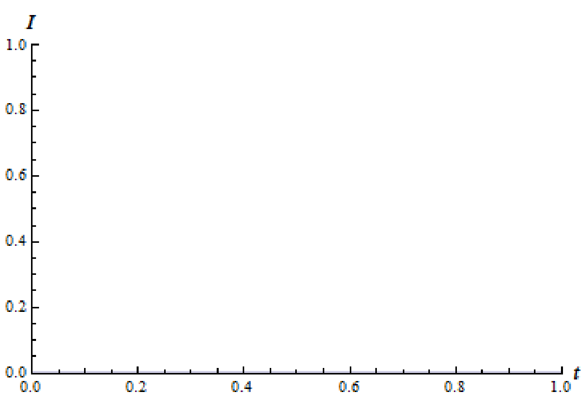
26 (b) Find an equation of the plane which contains the point $(2,0,-1)$ and is perpendicular to the vector $4\boldsymbol{i}+5\boldsymbol{j}+7\boldsymbol{k}$.

27. Recall that in a simple series circuit with and inductor of L Henries, a resistor of R ohms, and an electromotive force of E volts, the current I in amps at time t seconds satisfies the differential equation

$$L\frac{dI}{dt}+RI=E.$$

Suppose that the inductance is 20 Henries, the resistance is 80 ohms, and the electromotive force is constant at 40 volts. Suppose also that the current satisfies $I(0)=0$.

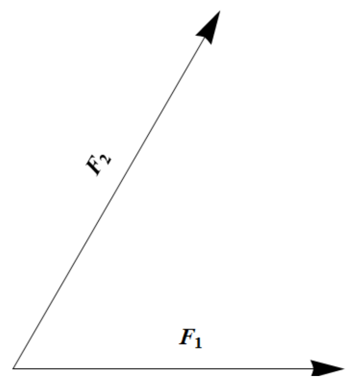
(a) Sketch the direction field of the differential equation. (Suggestion: Show the direction at the following points: (0,0), (0,0.2), (0,0.5), (0,0.8), (1,0), (1,0.2), (1,0.5), and (1,0.8).) On your sketch, draw the solution that satisfies the initial condition $I(0)=0$.



(b) Use two steps of Euler's method, with step size $h=0.1$, to approximate the current at time $t=0.2$ seconds.

28. Solve the circuit initial value problem from the previous question exactly, using separation of variables. To receive full credit, you must show your work. Find the current at time $t = 0.2$ seconds and compare to your estimate from the previous question.

29. Two forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ act on an object. The force $\boldsymbol{F}_1$ has a magnitude of 8 lb and is in the direction of the positive x-axis. The force $\boldsymbol{F}_2$ has a magnitude of 10 lb and makes an angle of 60° with the positive x-axis. Find the magnitude and direction of the resultant force.



30. (a) Use the Maclaurin series for e^x given in the table on the last page of this exam to find the first four nonconstant terms (i.e., four terms not including the $+C$ term) of a Maclaurin series for

$$\int e^{-x^2}\,dx.$$

(b) Estimate the value of the integral

$$\int_0^{\frac{1}{2}} e^{-x^2}\,dx$$

using the four terms from your answer to part (a). Give your answer to 6 decimal places.

Power Series

P1. $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \quad -\infty < x < \infty$$

P2. $$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots, \quad -\infty < x < \infty$$

P3. $$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots, \quad -\infty < x < \infty$$

P4. $$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdots, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

P5. $$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots, \quad -1 < x < 1$$

P6. $$\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots, \quad -\infty < x < \infty$$

P7. $$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots, \quad -\infty < x < \infty$$

P8. $$\tanh x = x - \frac{x^3}{3} + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \cdots, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

P9. $$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

P10. Taylor Series with remainder:

$$\begin{aligned} f(x) &= \textstyle\sum_{n=0}^{N} \frac{f^{(n)}(a)}{n!}(x-a)^n + R_{N+1}(x), \quad \text{where} \\ R_{N+1}(x) &= \frac{f^{(N+1)}(\xi)}{(N+1)!}(x-a)^{N+1} \qquad \text{for some } \xi \text{ between } a \text{ and } x. \end{aligned}$$