SM221 Practice Test I Solutions (Melles)

Only basic "Nav" calculators allowed. Graphing calculators are not allowed.

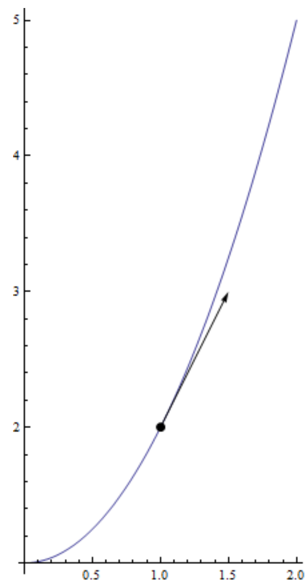
1. A projectile is fired with an initial speed of v_0 ft/sec, at an angle of elevation θ, from an initial height of h_0 feet. Which of the following is the height of the object, in feet, after t seconds?
a. $v_0\cos(\theta)$ b. $v_0\cos(\theta)t$ c. $-32t + v_0\sin(\theta)$ d. $-16t^2 + v_0\sin(\theta)t$ e. $-16t^2 + v_0\sin(\theta)\,t + h_0$

2. Suppose that the position of an object at time t is given by $\vec{r}(t) = \langle\sqrt{t}, 1+t\rangle$, for $t \geq 0$.
a. Find the velocity of the object at time t.

$\vec{v}(t) = \langle\frac{1}{2\sqrt{t}}, 1\rangle$ for $t > 0$.

b. Sketch the path of the object in the xy-plane. Mark the point where $t = 1$ and draw the velocity vector for $t = 1$ at this point.

Note that $x^2 = t$ so that $y = 1 + x^2$. $\vec{r}(t) = \langle 1,2\rangle$ and $\vec{v}(t) = \langle\frac{1}{2}, 1\rangle$.



c. Set up an integral to evaluate the arclength of the path for $0 \leq t \leq 1$.

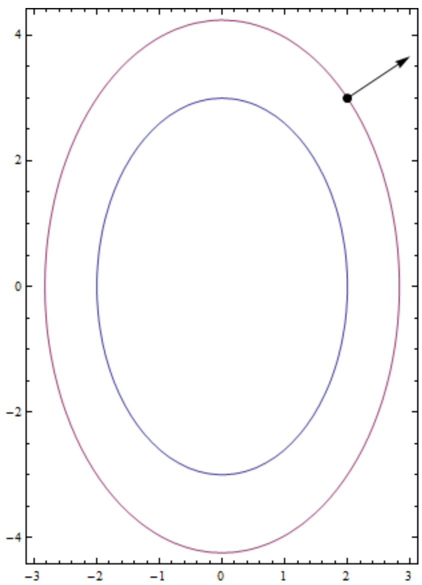
$$\int_0^1 \sqrt{\frac{1}{4t} + 1}\ dt$$

3. Suppose that $z = f(x,y) = \frac{x^2}{4} + \frac{y^2}{9}$.
a. Sketch the contour curves where $z = 1$ and $z = 2$.

See below

b. Find the gradient at (2,3), i.e. find $\nabla f(2,3)$, and draw it on your graph of part (a).

$\nabla f(2,3) = \langle 1, \frac{2}{3} \rangle$



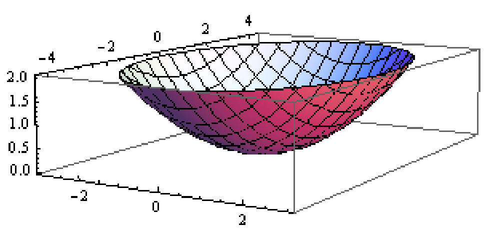
c. Find the directional derivative of f at $(2,3)$ in the direction of the vector $\vec{a} = \langle 1, -1 \rangle$.

$D_{\boldsymbol{u}} f(2,3) = \nabla f(2,3) \cdot \frac{1}{|\vec{a}|} \vec{a} = \langle 1, \frac{2}{3} \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle = \frac{1}{3\sqrt{2}}$

d. In what direction does z increase fastest at the point $(2,3)$?

In the direction of $\nabla f(2,3)$

e. Sketch the graph of $z = \frac{x^2}{4} + \frac{y^2}{9}$.



f. Find an equation of the tangent plane to the graph of $z = \frac{x^2}{4} + \frac{y^2}{9}$ at the point $(2,3,2)$.

$z - 2 = 1(x-2) + \frac{2}{3}(y-3)$ or $z = x + \frac{2}{3}y - 2$

4. Suppose that the acceleration of an object at time t seconds is given by $\vec{a}(t) = \langle -4\cos(t), -4\sin(t)\rangle$ cm/sec^2, and the initial velocity and position are given by $\vec{v}(0) = \langle 0,4\rangle$ cm/sec and $\vec{r}(0) = \langle 5,3\rangle$ cm. Find a formula for the position $\vec{r}(t)$ at time t.

$\vec{r}(t) = \langle 4\cos t + 1, 4\sin t + 3\rangle$

5. a. Use the table of values of f(x,y) to estimate the values of $f_x(1,2)$ and $f_y(1,2)$.

$f_x(1,2) \cong 4$ and $f_y \cong -2$

b. Find the linear approximation of f(x,y) at (1,2).

x \ y	1	2	3
0.5	7	6	4
1	9	7	5
1.5	12	10	7

$7 + 4(x-1) - 2(y-2)$ or $4x - 2y + 7$

6. The volume of a cylinder of radius r and height h is given by $V = \pi r^2 h$. Suppose that $r = f(t)$ and $h = g(t)$, where r and h are measured in centimeters and t in seconds.

a. Use the chain rule to write a formula for $\frac{dV}{dt}$.

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt} = 2\pi r h\frac{dr}{dt} + \pi r^2\frac{dh}{dt}$$

b. Suppose that at a certain time, $r = 10$ cm, $h = 6$ cm, $\frac{dr}{dt} = -1$ cm/sec, and $\frac{dh}{dt} = 2$ cm/sec. Find $\frac{dV}{dt}$ at this moment.

$80\pi\ \text{cm}^3/\text{sec}$

7. Find all local maxima, local minima, and saddle points of the function

$$f(x,y) = 3xy + 4 - x^3 - y^3.$$

Be sure to show all your work, including the second derivative test.

Set $f_x = 3y - 3x^2 = 0$
and $f_y = 3x - 3y^2 = 0$.
Critical points are (0,0) and (1,1).
$D(x,y) = f_{xx}f_{yy} - f_{xy}^2 = (-6x)(-6y) - 9$
$D(0,0) = -9 < 0$ so (0,0) is a saddle point.
$D(1,1) = 27 > 0$ and $f_{xx}(1,1) = -6 < 0$ so f has a local maximum at (1,1).