SM221 Practice Test I (Melles)

Only basic "Nav" calculators allowed. Graphing calculators are not allowed.

1. A projectile is fired with an initial speed of v_0 ft/sec, at an angle of elevation θ, from an initial height of h_0 feet. Which of the following is the height of the object, in feet, after t seconds?
a. $v_0\cos(\theta)$ b. $v_0\cos(\theta)t$ c. $-32t + v_0\sin(\theta)$ d. $-16t^2 + v_0\sin(\theta)t$ e. $-16t^2 + v_0\sin(\theta)\,t + h_0$

2. Suppose that the position of an object at time t is given by $\vec{r}(t) = \langle\sqrt{t}, 1+t\rangle$, for $t \geq 0$.
a. Find the velocity of the object at time t.
b. Sketch the path of the object in the xy-plane. Mark the point where $t = 1$ and draw the velocity vector for $t = 1$ at this point.
c. Set up an integral to evaluate the arclength of the path for $0 \leq t \leq 1$.

3. Suppose that $z = f(x,y) = \frac{x^2}{4} + \frac{y^2}{9}$.
a. Sketch the contour curves where $z = 1$ and $z = 2$.
b. Find the gradient at (2,3), i.e. find $\nabla f(2,3)$, and draw it on your graph of part (a).
c. Find the directional derivative of f at $(2,3)$ in the direction of the vector $\vec{a} = \langle 1, -1\rangle$.
d. In what direction does z increase fastest at the point $(2,3)$?
e. Sketch the graph of $z = \frac{x^2}{4} + \frac{y^2}{9}$.
f. Find an equation of the tangent plane to the graph of $z = \frac{x^2}{4} + \frac{y^2}{9}$ at the point $(2,3,2)$.

4. Suppose that the acceleration of an object at time t seconds is given by $\vec{a}(t) = \langle -4\cos(t), -4\sin(t)\rangle$ cm/sec^2, and the initial velocity and position are given by $\vec{v}(0) = \langle 0,4\rangle$ cm/sec and $\vec{r}(0) = \langle 5,3\rangle$ cm. Find a formula for the position $\vec{r}(t)$ at time t.

5. a. Use the table of values of f(x,y) to estimate the values of $f_x(1,2)$ and $f_y(1,2)$.
b. Find the linear approximation of f(x,y) at (1,2).

x \ y	1	2	3
0.5	7	6	4
1	9	7	5
1.5	12	10	7

6. The volume of a cylinder of radius r and height h is given by $V = \pi r^2 h$. Suppose that $r = f(t)$ and $h = g(t)$, where r and h are measured in centimeters and t in seconds.
a. Use the chain rule to write a formula for $\frac{dV}{dt}$.
b. Suppose that at a certain time, $r = 10$ cm, $h = 6$ cm, $\frac{dr}{dt} = -1$ cm/sec, and $\frac{dh}{dt} = 2$ cm/sec. Find $\frac{dV}{dt}$ at this moment.

7. Find all local maxima, local minima, and saddle points of the function
$$f(x,y) = 3xy + 4 - x^3 - y^3.$$
Be sure to show all your work, including the second derivative test.