SM221 Practice Test II Solutions (Melles)

No calculators allowed.

1. Which of the following correctly reverses the order of integration for

$$\int_0^9 \int_{\sqrt{x}}^3 \sin(y^3)\,dy\,dx \text{ ?}$$

a. $\int_{\sqrt{x}}^3 \int_0^9 \sin(y^3)\,dx\,dy$ b. $\int_0^3 \int_{y^2}^9 \sin(y^3)\,dx\,dy$ **[c.]** $\int_0^3 \int_0^{y^2} \sin(y^3)\,dx\,dy$
d. $\int_0^9 \int_{\sqrt{y}}^3 \sin(y^3)\,dx\,dy$ e. none of these

2. Which of the following do we obtain when we convert the integral

$$\int_0^3 \int_0^{\sqrt{9-x^2}} x^2 + y^2\,dy\,dx$$

to an integral in polar coordinates?

a. $\int_0^3 \int_0^{\sqrt{9-r^2\cos^2\theta}} r^2\,dr\,d\theta$ **[b.]** $\int_0^{\frac{\pi}{2}} \int_0^3 r^3\,dr\,d\theta$ c. $\int_0^{2\pi} \int_0^3 r^3\,dr\,d\theta$
d. $\int_0^{2\pi} \int_0^3 r^2\,dr\,d\theta$ e. $\int_0^{\frac{\pi}{2}} \int_0^3 r^2\,dr\,d\theta$

3. Set up an integral for the volume of the region bounded by the graphs of the equations $x = 0$, $y = 0$, $z = 0$, and $4x + 2y + z = 8$. Do not evaluate the integral.

$$\int_0^2 \int_0^{4-2x} \int_0^{8-4x-2y} dz\,dy\,dx$$

4. A lamina has the shape of the part of the disk $x^2 + y^2 = 9$ in the first quadrant. The density at any point is equal to the distance from the origin. Set up an integral for the mass of the lamina. You do not have to evaluate the integral.

$$m = \int_0^{\frac{\pi}{2}} \int_0^3 r^2\,dr\,d\theta \qquad \text{(density = } r\text{)}$$

5. Set up integrals for the mass of the region inside the sphere $x^2 + y^2 + z^2 = 4$ and in the first octant (where $x \geq 0$, $y \geq 0$, and $z \geq 0$) if the density at any point is the square of the distance from the origin:
a. in rectangular coordinates $(x, y, z)$.

$$m = \int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} x^2 + y^2 + z^2\,dz\,dy\,dx \qquad \text{(density = } x^2 + y^2 + z^2\text{)}$$

b. in cylindrical coordinates $(r, \theta, z)$.

$$m = \int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{\sqrt{4-r^2}} (r^2 + z^2)\ r\, dz\, dr\, d\theta \qquad \text{(density} = r^2 + z^2\text{)}$$

c. in spherical coordinates $(\rho, \theta, \varphi)$.

$$m = \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^2 \rho^4 \sin\varphi\ d\rho\, d\varphi\, d\theta \qquad \text{(density} = \rho^2\text{)}$$

Do not evaluate the integrals.

6. Evaluate the following integral: $\int_0^4 \int_0^4 xy - x\ dy\ dx$. Answer: 32

7. A contour map for a function $f$ is shown. Use the midpoint rule with $m = n = 2$ subdivisions to approximate $\int_0^4 \int_0^4 f(x, y) dy\ dx$. (Remark: $f$ is actually the function of problem #6.)

$\Delta A\big(f(1,1) + f(3,1) + f(1,3) + f(3,3)\big) = 4(0 + 0 + 2 + 6) = 32$