No calculators allowed.

1. Match the vectors fields with the correct graphs.

(a) $i + j$  (b) $-yi + xj$  (c) $xj$  (d) $xi + yj$  (e) $-xi - yj$  (f) $\frac{-y}{\sqrt{x^2+y^2}}i + \frac{x}{\sqrt{x^2+y^2}}j$
2. Find the work done by a force field \( \mathbf{F}(x, y) = (x + 2y)\mathbf{i} + (x^2 + 1)\mathbf{j} \) in moving an object from the point \((1, -2)\) to the point \((4, -2)\), i.e., evaluate the following line integral, where \( \mathcal{C} \) is the line segment from \((1, -2)\) to \((4, -2)\).

\[
\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}
\]

Answer: \(-\frac{9}{2}\)

3. a. Given \( \mathbf{F}(x, y, z) = (2x + 2y)\mathbf{i} + (2x + 3z)\mathbf{j} + (3y + 4)\mathbf{k} \), find a function \( f \) such that \( \mathbf{F} = \nabla f \).

Answer: \( f(x, y, z) = x^2 + 2xy + 3yz + 4z \)

b. Use your answer to part (a) to evaluate the following integral, where \( \mathcal{C} \) is the line segment from the point \((0,0,0)\) to the point \((-1,2,1)\) (i.e. use the Fundamental Theorem for line integrals).

\[
\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}
\]

Answer: 7

4. Let \( \mathcal{C} \) be the circle given by the equation \( x^2 + y^2 = 9 \) travelled in the counterclockwise direction. Let \( \mathbf{F}(x, y) = x\mathbf{j} \). A graph of \( \mathcal{C} \) and \( \mathbf{F} \) is shown below.

\[\text{Graph of } \mathcal{C} \text{ and } \mathbf{F} \]

a. Find \( \text{curl } \mathbf{F} \).

Answer: \( \mathbf{k} \)

b. Find \( \text{div } \mathbf{F} \).

Answer: 0

c. Evaluate the following integral using Green’s Theorem.

\[
\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r} = \int_{\mathcal{C}} x \, dy
\]

Answer: \(9\pi\)

d. Check your answer to part (c) by evaluating the line integral directly, i.e., by using a parametrization of \( \mathcal{C} \) and calculating the line integral. Hint: One of both of the following half-angle formulas may be useful:

\[
\sin^2 x = \frac{1 - \cos 2x}{2}
\]

\[
\cos^2 x = \frac{1 + \cos 2x}{2}
\]

Answer: \(9\pi\)

5. Suppose that \( f(x, y, z) = 2xy + 3yz \) and \( \mathbf{F} = \nabla f \). What is \( \text{curl } \mathbf{F} \)?

Answer: 0 since \( \mathbf{F} \) is conservative (or by direct computation)