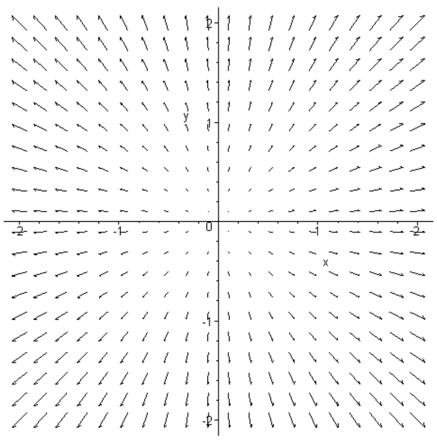
SM221 Practice Test III Solutions (Melles)

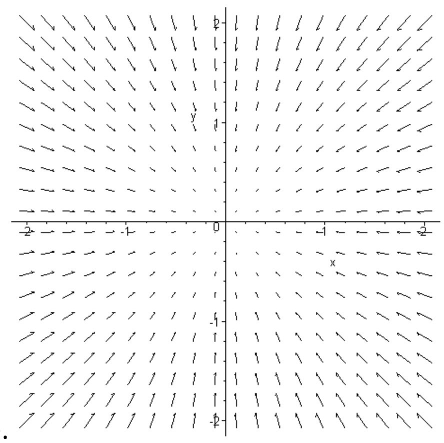
No calculators allowed.

1. Match the vectors fields with the correct graphs.

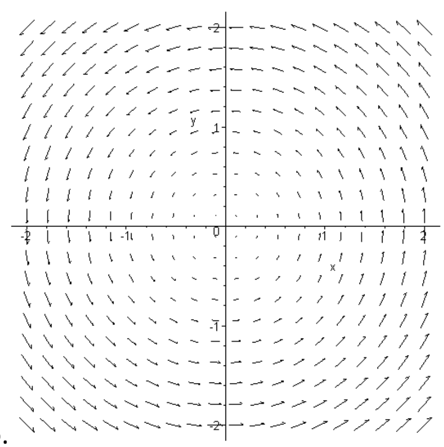
(a) $\boldsymbol{i}+\boldsymbol{j}$ (b) $-y\boldsymbol{i}+x\boldsymbol{j}$ (c) $x\boldsymbol{j}$ (d) $x\boldsymbol{i}+y\boldsymbol{j}$ (e) $-x\boldsymbol{i}-y\boldsymbol{j}$ (f) $\frac{-y}{\sqrt{x^2+y^2}}\boldsymbol{i}+\frac{x}{\sqrt{x^2+y^2}}\boldsymbol{j}$



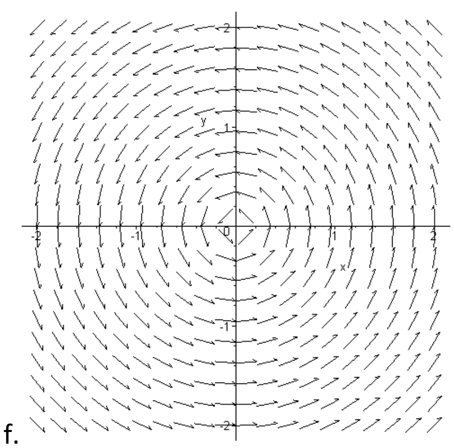
d.



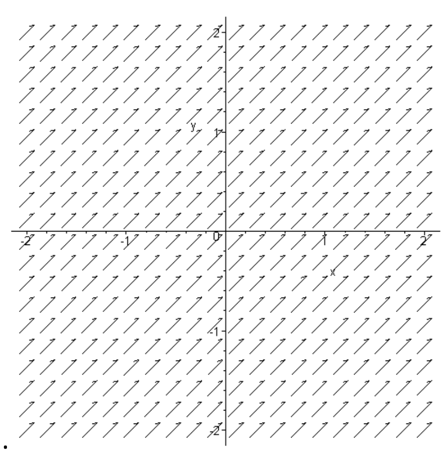
e.



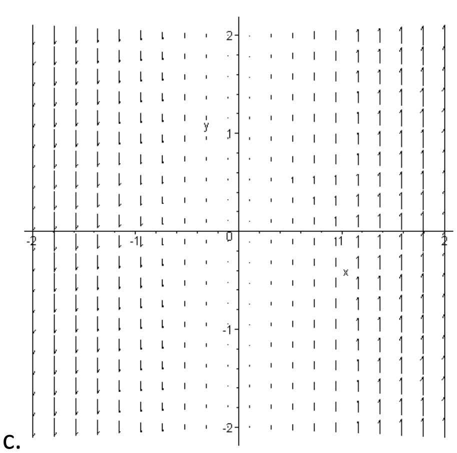
b.



f.



a.



c.

2. Find the work done by a force field $\boldsymbol{F}(x,y) = (x+2y)\boldsymbol{i} + (x^2+1)\boldsymbol{j}$ in moving an object from the point $(1,-2)$ to the point $(4,-2)$, i.e., evaluate the following line integral, where C is the line segment from $(1,-2)$ to $(4,-2)$.

$$\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$$

Answer: $-\frac{9}{2}$

3. a. Given $\boldsymbol{F}(x,y,z) = (2x+2y)\boldsymbol{i} + (2x+3z)\boldsymbol{j} + (3y+4)\boldsymbol{k}$, find a function f such that $\boldsymbol{F} = \nabla$f.

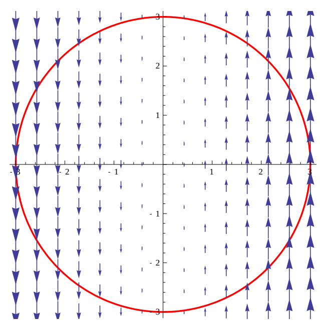
Answer: $f(x,y,z) = x^2 + 2xy + 3yz + 4z$

b. Use your answer to part (a) to evaluate the following integral, where C is the line segment from the point $(0,0,0)$ to the point $(-1,2,1)$ (i.e. use the Fundamental Theorem for line integrals).

$$\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$$

Answer: 7

4. Let C be the circle given by the equation $x^2 + y^2 = 9$ travelled in the counterclockwise direction. Let $\boldsymbol{F}(x,y) = x\boldsymbol{j}$. A graph of C and $\boldsymbol{F}$ is shown below.



a. Find curl $\boldsymbol{F}$.
Answer: $\boldsymbol{k}$
b. Find div $\boldsymbol{F}$.
Answer: 0
c. Evaluate the following integral using Green's Theorem.

$$\int_C \boldsymbol{F} \cdot d\boldsymbol{r} = \int_C x\, dy$$

Answer: 9π
d. Check your answer to part (c) by evaluating the line integral directly, i.e., by using a parametrization of C and calculating the line integral. Hint: One of both of the following half-angle formulas may be useful:

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

Answer: 9π

5. Suppose that $f(x,y,z)$ =2xy + 3yz and $\boldsymbol{F} = \nabla f$. What is curl $\boldsymbol{F}$?
Answer: $\boldsymbol{0}$ since $\boldsymbol{F}$ is conservative (or by direct computation)