Name: ____________________________

Instructor: ____________________________

Section: ____________________________

Part 1: Multiple choice, calculators allowed.

1. Which of the following expressions gives a plane passing through the point $(-1, 1, 2)$?

   A. $2x + 3y + z = 3$
   B. $-x + y + 2z = 1$
   C. $-x + y + 2z = -2$
   D. $-x + y + 2z = 2$
   E. $x - y - 2z = -2$

   Solution: A

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2. The angle between the two vectors $\mathbf{u} = \langle 1, 1, 0\rangle$ and $\mathbf{v} = \langle 1, 0, 0\rangle$ is which of the following?

   A. $\pi/2$
   B. 0
   C. $\pi/4$
   D. $3\pi/4$
   E. Cannot be determined from the information given

   Solution: C

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3. The two vectors $\mathbf{u} = \langle 2, 4, -3\rangle$ and $\mathbf{v} = \langle -4, -8, 6\rangle$ have which property?

   A. $\mathbf{u} \cdot \mathbf{v} = 0$
   B. The angle between $\mathbf{u}$ and $\mathbf{v}$ is $90°$
   C. $\mathbf{u} \times \mathbf{v} =$ a unit vector
   D. $\mathbf{u} \cdot \mathbf{v} = \langle 1, 1, 1\rangle$
   E. $\mathbf{u} \times \mathbf{v} = \mathbf{0}$

   Solution: E

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4. Which of the following lines is perpendicular to $x = 1 + 2t, y = -1 - 2t, z = 3 + 3t$?

   A. $x = 1 + 4t, y = -1 - 4t, z = 3 + 6t$

   B. $x = 2t, y = -2t, z = 3t$

   C. $x = 1 + t, y = -1 + t, z = 3$

   D. $x = 1 + (1/2)t, y = -1 - (1/2)t, z = 3 + (1/3)t$

   E. $x = 1 - 2t, y = -1 + 2t, z = 3 - 3t$

   Solution: C

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5. The equation $z = x^2 + y^2$ represents which quadric surface?

   A. circle

   B. cone

   C. paraboloid

   D. sphere

   E. ellipsoid

   Solution: C

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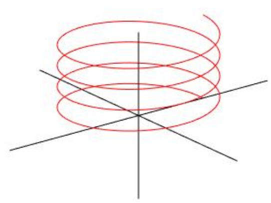
6. The following space curve



   is described by which of these position functions?

   A. $\mathbf{r}(t) = t\mathbf{i} + \sin 3t\mathbf{j} + \cos 3t\mathbf{k}$

   B. $\mathbf{r}(t) = \cos 3t\mathbf{i} + \sin 3t\mathbf{j} + 3\mathbf{k}$

   C. $\mathbf{r}(t) = \sin t\mathbf{i} + t\mathbf{j} + \cos t\mathbf{k}$

   D. $\mathbf{r}(t) = t\cos 3t\mathbf{i} + t\sin 3t\mathbf{j} + t\mathbf{k}$

   E. $\mathbf{r}(t) = \cos 3t\mathbf{i} + \sin 3t\mathbf{j} + t\mathbf{k}$

   Solution: E

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7. Let $\mathbf{r}(t) = e^t\mathbf{i} + e^{-t}\mathbf{j} + t\mathbf{k}$. The parametric equations for the tangent line at point $(1, 1, 0)$ are which of the following?

   A. $x = 1 + t, y = 1 - t, z = t$

   B. $x = 1 + t, y = 1 + t, z = t$

   C. $x = 1 + e^t t, y = 1 + -e^{-t}t, z = t^2$

   D. $x = 1 + e^t, y = 1 + -e^{-t}, z = t$

   E. $x = e + t, y = 1/e - t, z = t$

   Solution: A

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8. Let $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + 2t\mathbf{k}$. The arclength along the curve from $t = 0$ to $t = 2\pi$ is which of the following?

   A. $2\pi\sqrt{-\sin^2 t + \cos^2 t + 4}$

   B. $2\pi\big(-\sin t\mathbf{i} + \cos t\mathbf{j} + 2\mathbf{k}\big)$

   C. $-\sin t\mathbf{i} + \cos t\mathbf{j} + 2\mathbf{k}$

   D. $2\pi\sqrt{5}$

   E. $2\pi\sqrt{3}$

   Solution: D

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9. Let $\mathbf{r}(t) = -\frac{1}{2}t^2\mathbf{i} + \frac{1}{2}t^2\mathbf{j} + t\mathbf{k}$ be the position function of a particle. What is the speed of the particle at $t = 2$?

   A. $-t\mathbf{i} + t\mathbf{j} + \mathbf{k}$

   B. 3

   C. $-2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

   D. $\sqrt{5}$

   E. 1

   Solution: B

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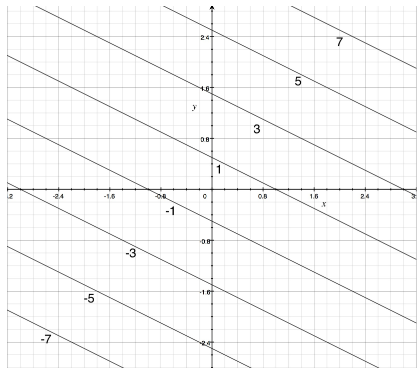
10. The following contour map



applies to which of the following functions?

A. $-x + 2y + z = 3$
B. $z = x^2 + y^2$
C. $1 + z = x^2 + y^2$
D. $z^2 = x^2 + y^2$
E. $z = x + 2y$

Solution: E

11. Given the function $f(x, y) = x\cos(xy)$, the gradient ∇f is is

A. $\nabla f = \langle \cos(xy), -xy\sin(xy) \rangle$
B. $\nabla f = \langle \frac{x^2}{2}\cos(xy), -xy\sin(xy) \rangle$
C. $\nabla f = \langle -xy\sin(xy) + y\cos(xy), -x^2\sin(xy) \rangle$
D. $\nabla f = \langle -xy\sin(xy) + \cos(xy), -x^2\sin(xy) \rangle$
E. $\nabla f = \langle -xy\sin(xy), -x^2\sin(xy) \rangle$

Solution: D

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12. Given the function $f(x, y) = \ln(2x + y)$, the equation for the tangent plane at point $(-1, 3)$ is

A. $2x + y + z = -1$
B. $-2x - y + z = -1$
C. $x + y + z = 1$
D. $(1/3)x + (1/7)y + z = 1$
E. $-2x - y + z = -5$

Solution: B

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13. Which of the following is a linear approximation $L(x,y)$ of the function $f(x,y) = 2x^2y + 3y^2$ near the point $(1,2)$?

A. $L(x,y) = 4xy(x-1) + (2x^2 + 6y)(y-2) - 20$

B. $L(x,y) = 4xy(x-1) + (2x^2 + 6y)(y-2)$

C. $L(x,y) = x + 2y$

D. $L(x,y) = 8x + 14y$

E. $L(x,y) = 8x + 14y - 20$

Solution: E

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14. Suppose that the profit P in dollars of a company is a function of the budgeted capital K and the labor L, i.e. $f(K,L) = P$. Suppose that $\frac{\partial f}{\partial K}(3,000, 10,000) = 2$ and $\frac{\partial f}{\partial L}(3,000, 10,000) = -3$. Suppose that capital is falling at a rate of \$700 per year and labor is rising at \$200 per year. The rate at which the profit is changing is given by

A. $-2,500$ \$/year

B. $-8,000$ \$/year

C. $-2,000$ \$/year

D. $10,000$ \$/year

E. $12,000$ \$/year

Solution: C

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15. Let $f(x,y,z) = x^2y + y^2z$. The direction derivative at the point $(1,1,1)$ in the direction of $\mathbf{v} = \langle 1,2,2 \rangle$ is which of the following?

A. 5

B. 6

C. 10/3

D. $\mathbf{i} + \mathbf{j} + \mathbf{k}$

E. $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

Solution: C

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16. Let $f(x,y) = xe^{-y} + 3y$. At the point, $(1,0)$, which of the following is the maximum rate of change?

A. $\langle 1,2 \rangle$

B. 1

C. $\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$

D. $\sqrt{5}$

E. 4

Solution: D

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17. Let $f(x, y) = \cos y + e^{xy}$. At the point, $(0, \frac{\pi}{2})$, which of the following vectors points in the direction of max ascent?

A. $\mathbf{i} + \mathbf{j}$

B. $\mathbf{i} + \pi\mathbf{j}$

C. $-\mathbf{i} + \left(\frac{\pi}{2}\right)\mathbf{j}$

D. $-\mathbf{i} - \left(\frac{\pi}{2}\right)\mathbf{j}$

E. $\left(\frac{\pi}{2}\right)\mathbf{i} - \mathbf{j}$

Solution: E

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18. Suppose that f is a real-valued function with continuous second partial derivatives such that $f_x(1, 2) = 0 = f_y(1, 2)$ and $f_{xx}(1, 2) = -3, f_{yy} = -5$ and $f_{xy}(1, 2) = 1$. Which of the following as true?

A. $(1, 2)$ is a critical point of f and is a saddle point.

B. $(1, 2)$ is a critical point of f and is a local minimum.

C. $(1, 2)$ is a critical point of f and is a local maximum.

D. $(-3, 5)$ is a critical point of f and is a local minimum.

E. $(-3, 5)$ is a critical point of f and is a local maximum.

Solution: C

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19. Suppose you wish to maximize the function $f(x, y) = xy$ subject to the constraint that the points are on a circle of radius 2. According to the *Lagrange multiplier* method, what are the three equations?

A. $y = \lambda x,\ x = \lambda y,\ x^2 + y^2 = 4$

B. $y = \lambda x,\ x = \lambda y,\ x^2 + y^2 = 2$

C. $y = \lambda x^2,\ x = \lambda y^2,\ x^2 + y^2 = 4$

D. $y = 2\lambda x,\ x = 2\lambda y,\ x^2 + y^2 = 4$

E. $y = 2\lambda x,\ x = 2\lambda y,\ x^2 + y^2 = 2$

Solution: D

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20. Suppose you have \$200 and wish to put together a care package for your friend containing peppermint candies (\$6 a piece), peanut butter cups (\$4 a piece), and Hershey's kisses (\$1 a piece). If x, y and z represent the number of peppermint candies, peanut butter cups and Hershey's kisses respectively, which of the following represents maximizing the number of pieces of candy you can send to your friend as a *Lagrange multiplier* problem?

    A. **maximize** xyz , **subject to** $11xyz = 200$
    B. **maximize** $x + y + z$, **subject to** $6x + 4y + z = 200$
    C. **minimize** $6x + 4y + z$, **subject to** $x + y + z = 200$
    D. **minimize** $11xyz$, **subject to** $x + y + z = 200$
    E. **minimize** xyz , **subject to** $6x + 4y + z = 200$

    Solution: B

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21. Let rectangle $R = [0, 4] \times [2, 6]$. Breaking this region up into four equal sub-rectangles, and using the mid-point rule, which of the following represents a Reimann sum estimate of the volume **under** $f(x, y) = xy$ and **above** the rectangle R?

    A. 32
    B. 64
    C. 128
    D. 512
    E. 2048

    Solution: C

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22. Let rectangle $R = [0, 1] \times [1, 3]$. Which of the following double integrals represents the volume under $f(x, y) = e^{xy}$ and above the rectangle R?

    A. $\int_1^3 \int_0^1 e^{xy}\, dy\, dx$
    B. $\int_0^1 \int_0^{e^{xy}} dy\, dx$
    C. $\int_1^3 \int_0^1 e^{xy}\, dx\, dy$
    D. $\int_0^{e^{xy}} \int_0^3 dy\, dx$
    E. $\int_0^1 \int_1^3 e^{xy}\, dx\, dy$

    Solution: C

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23. Let D be the region in the 1st quadrant bounded by the parabola $y = x^2$ and the line $y = 4$. The double integral

$$\int_0^2 \int_{x^2}^4 x\cos(xy)\,dy\,dx$$

is equivalent to which of the following when the order is changed?

A. $\int_0^4 \int_{\sqrt{x}}^2 x\cos(xy)\,dx\,dy$

B. $\int_0^4 \int_0^{\sqrt{y}} x\cos(xy)\,dx\,dy$

C. $\int_0^2 \int_0^{\sqrt{y}} x\cos(xy)\,dx\,dy$

D. $\int_{x^2}^4 \int_0^2 x\cos(xy)\,dx\,dy$

E. $\int_0^2 \int_{\sqrt{y}}^4 x\cos(xy)\,dx\,dy$

Solution: B

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24. Which of the following equations corresponds to the right half of a circle of radius 2?

A. $x = -\sqrt{4-y^2}$

B. $y = \sqrt{4-x^2}$

C. $x^2 + y^2 = 4$

D. $x = \sqrt{2-y^2}$

E. $x = \sqrt{4-y^2}$

Solution: E

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25. Let D be the region bounded on the **left** by the circle $x^2 + y^2 = 4$ and on the **right** by the two lines $y = -x + 2$ and $y = x - 2$. Which of the following double integrals represents the volume under the surface $f(x, y) = yx^2$?

A. $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} yx^2\,dy\,dx + \int_{-2}^2 \int_{y+2}^{2-y} yx^2\,dy\,dx$

B. $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{yx^2}\,dy\,dx$

C. $\int_{-2}^2 \int_0^{yx^2}\,dy\,dx$

D. $\int_{-2}^0 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} yx^2\,dy\,dx + \int_0^2 \int_{y+2}^{2-y} yx^2\,dy\,dx$

E. $\int_{-2}^0 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} yx^2\,dy\,dx + \int_0^2 \int_{x-2}^{-x+2} yx^2\,dy\,dx$

Solution: E

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26. The following double integral

$$\int_0^3 \int_{-\sqrt{9-y^2}}^0 xy\,dx\,dy$$

is equivalent to which of the following when changed to polar coordinates?

A. $\int_{\frac{\pi}{2}}^{\pi} \int_0^3 r^3 \cos\theta \sin\theta\,dr\,d\theta$

B. $\int_{\frac{\pi}{2}}^{\pi} \int_0^3 r^2 \cos\theta \sin\theta\,dr\,d\theta$

C. $\int_0^{2\pi} \int_0^3 xy\,dr\,d\theta$

D. $\int_0^{2\pi} \int_0^3 r^2 \cos\theta \sin\theta\,dr\,d\theta$

E. $\int_{\frac{\pi}{2}}^{\pi} \int_{-3}^3 r^3 \cos\theta \sin\theta\,dr\,d\theta$

Solution: A

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27. Evaluate the following integral by changing to polar coordinates

$$\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \sqrt{x^2+y^2}\,dx\,dy$$

A. 8π

B. $16\pi/3$

C. 16π

D. $8\pi\sqrt{x^2+y^2}$

E. $4\pi\sqrt{x^2+y^2}$

Solution: B

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28. If the density anywhere on a lamina D is proportional to the distance to the x axis, which of the following double integrals represents the mass of the lamina?

A. $\iint_D Kxy\,dA$

B. $\iint_D Ky\,dA$

C. $\iint_D Kx\,dA$

D. $\iint_D K\sqrt{x^2+y^2}\,dA$

E. $\iint_D \frac{K}{\sqrt{x^2+y^2}}\,dA$

Solution: B

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29. Evaluate the following triple integral

$$\int_0^3 \int_0^2 \int_1^3 \frac{e^x y}{z} \, dz \, dy \, dx$$

A. $-\frac{2e^3}{9}$

B. $-\frac{2e^9}{9}$

C. $2\ln 3(e^3 - 1)$

D. $4\ln 3(e^9 - 1)$

E. $\frac{2e^3}{9}$

Solution: C

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30. The following triple integral

$$\int_0^3 \int_0^{-2x+6} \int_0^{\frac{6-2x-y}{3}} f(x,y,z) \, dz \, dy \, dx$$

is equivalent to which of the following when the order is changed?

A. $\int_0^6 \int_0^2 \int_0^3 f(x,y,z) \, dx \, dz \, dy$

B. $\int_0^3 \int_0^{6-2x} \int_0^{\frac{6-y-2x}{3}} f(x,y,z) \, dx \, dz \, dy$

C. $\int_0^6 \int_0^{-2y+6} \int_0^{\frac{6-2z-y}{3}} f(x,y,z) \, dx \, dz \, dy$

D. $\int_0^{-2x+6} \int_0^{\frac{6-2x-y}{3}} \int_0^3 f(x,y,z) \, dx \, dz \, dy$

E. $\int_0^6 \int_0^{-\frac{y}{3}+2} \int_0^{\frac{6-y-3z}{2}} f(x,y,z) \, dx \, dz \, dy$

Solution: E

Part 2: Short answer, calculators allowed.

1. Given points $P(1,-1,3)$, $Q(2,1,4)$, and $R(3,0,2)$:

   (a) Find the parametric equations of the line passing through the points P and Q.

   (b) Find the equation of the plane passing through the points P, Q and R.

   (c) Find the area of the triangle PQR. (Hint: the area of the triangle is half the area of the parallelogram spanned by the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$.

2. Captain Jack Sparrow fires a cannon from the *Black Pearl*. The cannon is fired from a position 10 m above sea level with an initial speed of 60 m/s and an angle of 60° above the horizontal. Ignoring air resistance and assuming a gravitational constant $g = 9.8\ m/s^2$:

   (a) Find the velocity vector function $\vec{v}(t)$.

   (b) Find the position vector function $\vec{r}(t)$.

   (c) Find the maximum height of the projectile. Round your answer to the nearest meter.

   (d) A fort is located 200 m away from the *Black Pearl*. The fort is 50 meters tall, and sits on top of a hill that is 100 meters tall. At what height will the cannon ball hit the fort? Round your answer to the nearest meter.

   (e) At what **speed** does the cannon ball hit the fort? Round your answer to the nearest meter.

3. The volume of a right circular cylinder is $\pi r^2 h$. The radius of the cylinder is increasing at a rate of $2m/s$, while the height is decreasing at a rate of $3m/s$. At what rate is the volume of the cylinder changing when the height is $2m$ and the radius is $3m$?

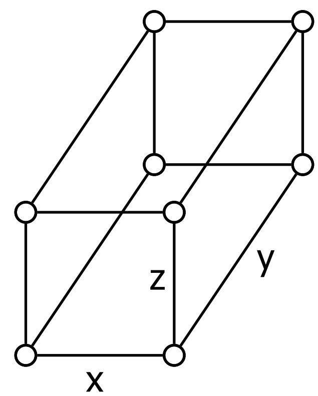
4. Let $f(x, y) = 6xy - x^2y - xy^2$.

(a) Find f_x and f_y.

(b) The function f has four critical points, two of which are $(x, y) = (0, 0)$ and $(x, y) = (2, 2)$. Find the other two critical points.

(c) Classify $(0, 0)$ and $(2, 2)$ as a relative maximum, relative minimum or saddle point.

5. A Christmas present box has four sides and a top and bottom. Extra shiny wrapping paper is used on the sides (costing \$4 per sq ft), plain wrapping paper is used on the bottom (costing \$2 per sq ft), and very fancy patterned paper with frolicking kittens and brightly colored balls of yarn is used on the top (costing \$14 per sq ft). Using *Lagrange multipliers*, maximize the volume of the present you can buy, subject to the constraint that you have exactly \$192 to spend on wrapping paper.



6. Given the double integral

$$\int_{-2}^{0} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \cos(x^2 + y^2)\, dy\, dx$$

- Sketch the region of integration.
- Convert the integral to polar coordinates.
- Evaluate the integral.