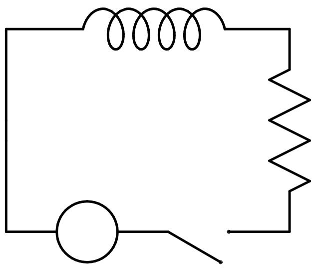
III. Practice Problem 2: R-L AC Circuit

Task: Work on solving for the current for the given circuit; indicated links give (partial) solutions.



An R-L circuit consists of a $60\sin(50t)$ volt AC generator connected in series with a 0.4 henry inductor and a 1.2 ohm resistor; no current flows until the switch is closed. The associated Initial Value Problem is

$$0.4\,I'(t) + 1.2\,I(t) = 60\sin(50t), \quad I(0) = 0$$

Method of Integrating Factor:

[a] Put the ODE in standard form.

[b] Determine the integrating factor μ.

[c] Multiply the standard form ODE by the integrating factor.

[d] Use the product rule for derivatives to simplify the preceding equation so that $[\mu\, I]'$ is on one side of it.

[e] Integrate both sides of the preceding equation with respect to t.

[f] Solve the preceding equation for the general solution I.

[g] Use the initial condition to determine the current $I(t)$ for this circuit and graph $I(t)$.

[h] Determine the steady-state current and the transient current for this circuit and graph both.

[a] Put the ODE in standard form.

After dividing through by the coefficient of $I'(t)$

$$0.4\,I'(t) + 1.2\,I(t) = 60\sin(50t)$$

becomes

$$I'(t) + 3\,I(t) = 150\sin(50t)$$

[b] Determine the integrating factor μ.

From the general standard form ODE $y' + p(x)\,y = q(x)$ we recognize that

$$I'(t) + 3\,I(t) = 150\sin(50t)$$

has $p(t) = 3$ and so

$$\mu = e^{\int p(t)\,dt} = e^{\int 3\,dt} = e^{3t}$$

[c] Multiply the standard form ODE by the integrating factor.

Standard form:

$$I'(t) + 3\,I(t) = 150\sin(50t)$$

Integrating factor:

$$\mu = e^{3t}$$

Product:

$$e^{3t}I' + 3\,e^{3t}I = 150\,e^{3t}\sin(50t)$$

[d] Use the product rule for derivatives to simplify the preceding equation so that $[\mu\, I]'$ is on one side of it.

By the product rule

$$e^{3t} I' + 3\, e^{3t} I = \left[e^{3t}\, I\right]'$$

So the preceding ODE

$$e^{3t} I' + 3\, e^{3t} I = 150\, e^{3t} \sin(50t)$$

is equivalent to

$$\left[e^{3t}\, I\right]' = 150\, e^{3t} \sin(50t)$$

[e] Integrate both sides of the preceding equation with respect to t.

Integrating both sides of

$$\left[e^{3t}\, I\right]' = 150\, e^{3t} \sin(50t)$$

yields

$$e^{3t} I = \int 150\, e^{3t} \sin(50t)\, dt$$

[f] Solve the preceding equation for the general solution I.

Using integration by parts on the right hand side of

$$e^{3t} I = \int 150\, e^{3t} \sin(50t)\, dt$$

we get (after rounding numbers to 3 decimal digits)

$$e^{3t} I = e^{3t} \left[-2.989 \cos(50t) + 0.179 \sin(50t)\right] + C$$

which, after dividing both sides by e^{3t}, becomes

$$I(t) = -2.989 \cos(50t) + 0.179 \sin(50t) + C\, e^{-3t}$$

[g] Use the initial condition to determine the current $I(t)$ for this circuit and graph $I(t)$.

From the general solution

$$I(t) = -2.989\cos(50t) + 0.179\sin(50t) + C\,e^{-3t}$$

in the preceding part we have at $t = 0$

$$I(0) = -2.989\cos(0) + 0.179\sin(0) + C\,e^{0} = -2.989 + 0 + C = -2.989 + C$$

whereas by the initial condition from the IVP we have

$$I(0) = 0$$

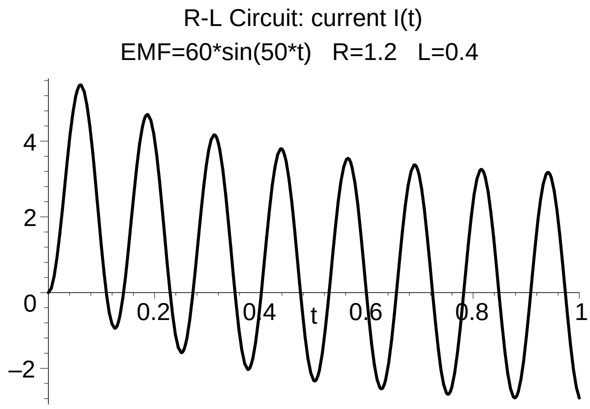
So these two equations together imply that

$$-2.989 + C = 0 \qquad \text{or} \qquad C = 2.989$$

Putting this C into the general solution yields the circuit current

$$I(t) = -2.989\cos(50t) + 0.179\sin(50t) + 2.989\,e^{-3t}$$

graphed below.



[h] Determine the steady-state current and the transient current for this circuit and graph both.

The current

$$I(t) = -2.989\cos(50t) + 0.179\sin(50t) + 2.989\,e^{-3t}$$

has a steady-state part $-2.989\cos(50t) + 0.179\sin(50t)$ (graphed below left) and a transient part $2.989\,e^{-3t}$ (graphed below right).