III. Practice Problem 2: R-L AC Circuit

**Task:** Work on solving for the current for the given circuit; indicated links give (partial) solutions.

An R-L circuit consists of a $60\sin(50t)$ volt AC generator connected in series with a 0.4 henry inductor and a 1.2 ohm resistor; no current flows until the switch is closed. The associated Initial Value Problem is

$$0.4I'(t) + 1.2I(t) = 60\sin(50t), \quad I(0) = 0$$

**Method of Integrating Factor:**

[a] Put the ODE in **standard form**.

[b] Determine the **integrating factor** $\mu$.

[c] **Multiply** the standard form ODE by the integrating factor.

[d] **Use the product rule for derivatives** to simplify the preceding equation so that $[\mu I]'$ is on one side of it.

[e] **Integrate** both sides of the preceding equation with respect to $t$.

[f] **Solve** the preceding equation for the general solution $I$.

[g] Use the initial condition to **determine** the current $I(t)$ for this circuit and graph $I(t)$.

[h] **Determine** the steady-state current and the transient current for this circuit and graph both.
[a] Put the ODE in standard form.

After dividing through by the coefficient of $I'(t)$

$$0.4 I'(t) + 1.2 I(t) = 60 \sin(50t)$$

becomes

$$I'(t) + 3 I(t) = 150 \sin(50t)$$
[b] Determine the integrating factor $\mu$.

From the general standard form ODE $y' + p(x) y = q(x)$ we recognize that

$$I'(t) + 3I(t) = 150\sin(50t)$$

has $p(t) = 3$ and so

$$\mu = e^{\int p(t) \, dt} = e^{\int 3 \, dt} = e^{3t}$$
[c] Multiply the standard form ODE by the integrating factor.

Standard form:

\[ I'(t) + 3I(t) = 150\sin(50t) \]

Integrating factor:

\[ \mu = e^{3t} \]

Product:

\[ e^{3t}I' + 3e^{3t}I = 150e^{3t}\sin(50t) \]
[d] Use the product rule for derivatives to simplify the preceding equation so that $[\mu I]'$ is on one side of it.

By the product rule

$$e^{3t}I' + 3e^{3t}I = [e^{3t}I]'$$

So the preceding ODE

$$e^{3t}I' + 3e^{3t}I = 150e^{3t}\sin(50t)$$

is equivalent to

$$[e^{3t}I]' = 150e^{3t}\sin(50t)$$
Integrate both sides of the preceding equation with respect to \( t \).

Integrating both sides of

\[
\left[ e^{3t} I \right]' = 150 e^{3t} \sin(50t)
\]

yields

\[
e^{3t} I = \int 150 e^{3t} \sin(50t) \, dt
\]
Solve the preceding equation for the general solution \( I \).

Using integration by parts on the right hand side of

\[
e^{3t} I = \int 150 e^{3t} \sin(50t) \, dt
\]

we get (after rounding numbers to 3 decimal digits)

\[
e^{3t} I = e^{3t} \left[ -2.989 \cos(50t) + 0.179 \sin(50t) \right] + C
\]

which, after dividing both sides by \( e^{3t} \), becomes

\[
I(t) = -2.989 \cos(50t) + 0.179 \sin(50t) + C e^{-3t}
\]
[g] Use the initial condition to determine the current $I(t)$ for this circuit and graph $I(t)$.

From the general solution

$$I(t) = -2.989 \cos(50t) + 0.179 \sin(50t) + Ce^{-3t}$$

in the preceding part we have at $t = 0$

$$I(0) = -2.989 \cos(0) + 0.179 \sin(0) + Ce^0 = -2.989 + 0 + C = -2.989 + C$$

whereas by the initial condition from the IVP we have

$$I(0) = 0$$

So these two equations together imply that

$$-2.989 + C = 0 \quad \text{or} \quad C = 2.989$$

Putting this $C$ into the general solution yields the circuit current

$$I(t) = -2.989 \cos(50t) + 0.179 \sin(50t) + 2.989 e^{-3t}$$

graphed below.
Determine the steady-state current and the transient current for this circuit and graph both.

The current

\[ I(t) = -2.989 \cos(50t) + 0.179 \sin(50t) + 2.989 e^{-3t} \]

has a steady-state part \(-2.989 \cos(50t) + 0.179 \sin(50t)\) (graphed below left) and a transient part \(2.989 e^{-3t}\) (graphed below right).