I. Practice Problem 1: R-C AC Circuit

Work on the questions for the given circuit; indicated links give (partial) solutions.

An R-C circuit consists of a $60 \cos(4t)$ volt AC generator connected in series with a 5 ohm resistor and a 0.05 farad capacitor.

Questions:

[a] Sketch the circuit diagram.

[b] Use Kirchhoff’s law to write the Initial Value Problem; assume current starts to flow and that there is no charge on the capacitor when the open switch is closed.

[c] Verify that $Q(t) = 1.5 \left( \cos(4t) + \sin(4t) - e^{-4t} \right)$ is the charge on the capacitor in this circuit for $t \geq 0$.

[d] Find the current $I(t)$ for the circuit.

[e] Graph $Q(t)$ and $I(t)$. 
[a] Sketch the circuit diagram for the circuit with $R = 5 \, \Omega$, $C = 0.05 \, \text{F}$, and $E(t) = 60 \cos(4t) \, \text{V}$. 

\[ EMF = 60 \cos(4t) \]
[b] Use Kirchhoff’s law to write the Initial Value Problem; assume current starts to flow and that there is no charge on the capacitor when the open switch is closed.

\[ E_R + E_C = E, \text{ with } E_R = R \cdot Q'(t) \text{ and } E_C = Q(t)/C, \text{ translates into} \]

\[ 5Q'(t) + \frac{Q(t)}{0.05} = 60 \cos(4t) \]

which simplifies to

\[ Q'(t) + 4Q(t) = 12 \cos(4t), \quad Q(t) = 0 \quad \text{at} \quad t = 0 \]
Verify that $Q(t) = 1.5 \left( \cos(4t) + \sin(4t) - e^{-4t} \right)$ is the charge on the capacitor in this circuit for $t \geq 0$.

If $Q(t) = 1.5 \left( \cos(4t) + \sin(4t) - e^{-4t} \right)$ then

$$Q'(t) = 1.5 \left( -4 \sin(4t) + 4 \cos(4t) + 4e^{-4t} \right)$$
$$= -6 \sin(4t) + 6 \cos(t) + 6e^{-4t}$$

and so

$$Q'(t) + 4Q(t) = -6 \sin(4t) + 6 \cos(t) + 6e^{-4t} + 4 \left( 1.5 \left( \cos(4t) + \sin(4t) - e^{-4t} \right) \right)$$
$$= -6 \sin(4t) + 6 \cos(4t) + 6e^{-4t} + 6 \cos(4t) + 6 \sin(4t) - 6e^{-4t}$$
$$= 12 \cos(4t)$$

and therefore $Q(t)$ does satisfy the ODE. Also, $Q(0) = 1.5 \left( \cos(0) + \sin(0) - e^0 \right) = 1.5 \left( 1 + 0 - 1 \right) = 0$ and $Q(t)$ thus satisfies the IC.
[d] Find the current $I(t)$ for the circuit.

Current is the time derivative of charge, or

$$I(t) = Q'(t) = 1.5 \left( \cos(4t) + \sin(4t) - e^{-4t} \right)' = -6 \sin(4t) + 6 \cos(t) + 6e^{-4t}$$
[e] Graph $Q(t)$ and $I(t)$. 

![Graph Q(t) and I(t)](image-url)