I. Practice Problem 1: R-C AC Circuit

Work on the questions for the given circuit; indicated links give (partial) solutions.

An R-C circuit consists of a $60\cos(4t)$ volt AC generator connected in series with a 5 ohm resistor and a 0.05 farad capacitor.

Questions:

[a] Sketch the circuit diagram.

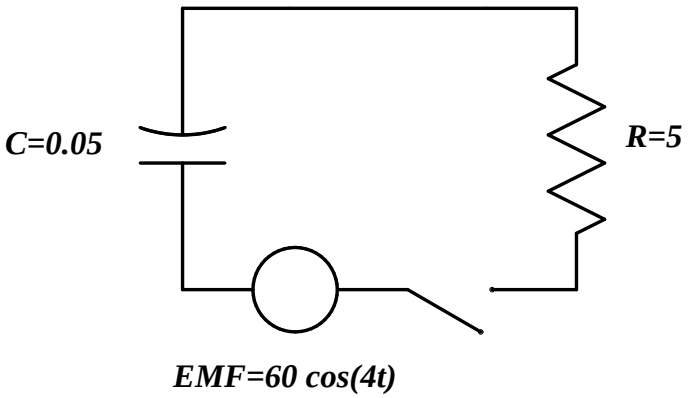
[b] Use Kirchhoff's law to write the Initial Value Problem; assume current starts to flow and that there is no charge on the capacitor when the open switch is closed.

[c] Verify that $Q(t) = 1.5\left(\cos(4t) + \sin(4t) - e^{-4t}\right)$ is the charge on the capacitor in this circuit for $t \geq 0$.

[d] Find the current $I(t)$ for the circuit.

[e] Graph $Q(t)$ and $I(t)$.

[a] Sketch the circuit diagram for the circuit with $R = 5\ \Omega$, $C = 0.05$ F, and $E(t) = 60\cos(4t)$ V.



[b] Use Kirchhoff's law to write the Initial Value Problem; assume current starts to flow and that there is no charge on the capacitor when the open switch is closed.

$E_R + E_C = E$, with $E_R = R \cdot Q'(t)$ and $E_C = Q(t)/C$, translates into

$$5\,Q'(t) + \frac{Q(t)}{0.05} = 60\cos(4t)$$

which simplifies to

$$Q'(t) + 4\,Q(t) = 12\cos(4t)\,, \qquad Q(t) = 0 \quad \text{at} \quad t = 0$$

[c] Verify that $Q(t) = 1.5\left(\cos(4t) + \sin(4t) - e^{-4t}\right)$ is the charge on the capacitor in this circuit for $t \geq 0$.

If $Q(t) = 1.5\left(\cos(4t) + \sin(4t) - e^{-4t}\right)$ then

$$\begin{aligned} Q'(t) &= 1.5\left(-4\sin(4t) + 4\cos(4t) + 4e^{-4t}\right) \\ &= -6\sin(4t) + 6\cos(t) + 6e^{-4t} \end{aligned}$$

and so

$$\begin{aligned} Q'(t) + 4\,Q(t) &= -6\sin(4t) + 6\cos(4t) + 6e^{-4t} + 4\left(1.5\left(\cos(4t) + \sin(4t) - e^{-4t}\right)\right) \\ &= -6\sin(4t) + 6\cos(4t) + 6e^{-4t} + 6\cos(4t) + 6\sin(4t) - 6e^{-4t} \\ &= 12\cos(4t) \end{aligned}$$

and therefore $Q(t)$ does satisfy the ODE. Also, $Q(0) = 1.5\left(\cos(0) + \sin(0) - e^{0}\right) = 1.5\,(1 + 0 - 1) = 0$ and $Q(t)$ thus satisfies the IC.

[d] Find the current $I(t)$ for the circuit.

Current is the time derivative of charge, or

$$I(t) = Q'(t) = 1.5\left(\cos(4t) + \sin(4t) - e^{-4t}\right)' = -6\sin(4t) + 6\cos(t) + 6e^{-4t}$$

[e] Graph $Q(t)$ and $I(t)$.