I. Practice Problem 1: R-L DC Circuit

Work on the questions for the given circuit; indicated links give (partial) solutions.

An R-L circuit consists of a 100 volt DC battery connected in series with a 2 henry inductor and a 6 ohm resistor.

Questions:

[a] Sketch the circuit diagram.

[b] Use Kirchhoff's law to write the Initial Value Problem; assume current starts to flow when the open switch is closed.

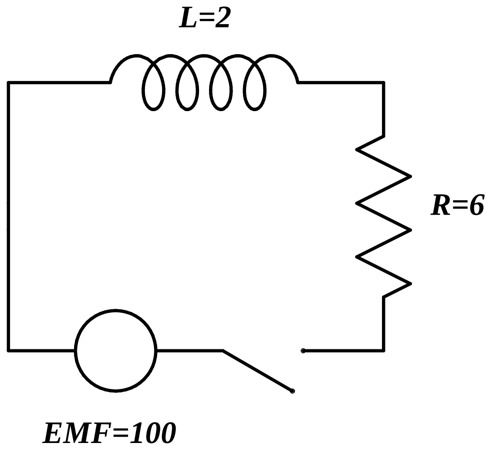
[c] Verify that $I(t) = \frac{50}{3}\left(1 - e^{-3t}\right)$, $t \geq 0$ is the solution to the IVP in part [b].

[d] Graph $I(t)$.

[e] What is the steady-state current I_∞ for this circuit by formula? Does your answer agree with the picture in part [d]?

[f] What is the time constant τ for this circuit? Does the picture in part [d] suggest that $I(5\tau) \approx I_\infty$?

[a] Sketch the circuit diagram for the circuit with $L = 2$, $R = 6$, and $E(t) = 100$.



[b] Use Kirchhoff's law to write the Initial Value Problem; assume current starts to flow when the open switch is closed.

$E_L + E_R = E$, with $E_L = L \cdot I'(t)$ and $E_R = R \cdot I$, translates into

$$2\,I'(t) + 6\,I(t) = 100, \qquad I(t) = 0 \quad \text{at} \quad t = 0$$

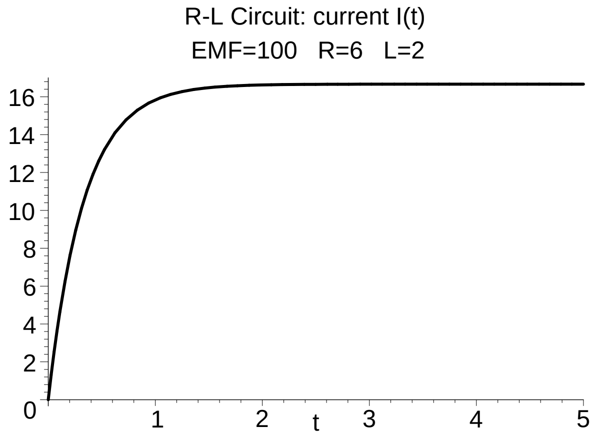
[c] Verify that $I(t) = \frac{50}{3}\left(1 - e^{-3t}\right),\ t \geq 0$ is the solution to the IVP in part [b].

If $I(t) = \frac{50}{3}\left(1 - e^{-3t}\right)$ then

$$\begin{aligned} 2\,I'(t) + 6\,I(t) &= 2\left(\frac{50}{3}\left(1 - e^{-3t}\right)\right)' + 6\left(\frac{50}{3}\left(1 - e^{-3t}\right)\right) \\ &= 2\left(\frac{50}{3}\left(3\,e^{-3t}\right)\right) + 100\left(1 - e^{-3t}\right) \\ &= 100\,e^{-3t} + 100 - 100\,e^{-3t} \\ &= 100 \end{aligned}$$

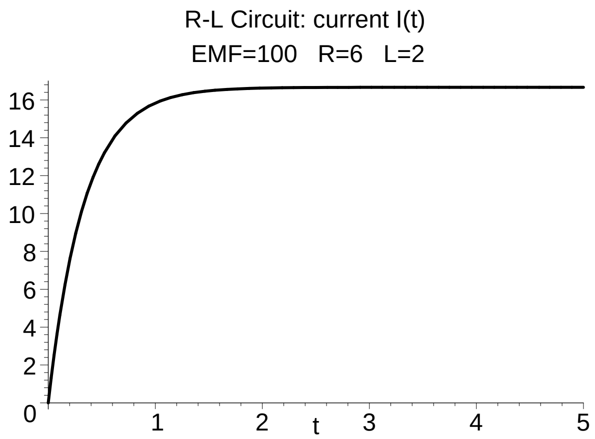
and therefore $I(t)$ does satisfy the ODE. Also, $I(0) = \frac{50}{3}\left(1 - e^{0}\right) = \frac{50}{3}(1 - 1) = 0$ and $I(t)$ thus satisfies the IC.

[d] Graph $I(t)$.



[e] What is the steady-state current I_∞ for this circuit by formula? Does your answer agree with the picture in part [d]?

$I_\infty = E/R = 100/6 = 50/3 \approx 16.67$ which does indeed look like the horizontal asymptote.



[f] What is the time constant τ for this circuit? Does the picture in part [d] suggest that $I(5\tau) \approx I_\infty$?

For an R-L DC circuit

$$\tau = R/L = 2/6 = 1/3$$

Hence $5\tau = 5/3$ and from the solution given in part [c]

$$I(5\tau) = \frac{50}{3}\left(1 - e^{-3(5/3)}\right) = \frac{50}{3}\left(1 - e^{-5}\right) \approx 16.55$$

which is very close to $I_\infty = 50/3 \approx 16.67$ as shown in the following graph.