I. Practice Problem 1: R-L DC Circuit

Work on the questions for the given circuit; indicated links give (partial) solutions.

An R-L circuit consists of a 100 volt DC battery connected in series with a 2 henry inductor and a 6 ohm resistor.

Questions:
[a] Sketch the circuit diagram.
[b] Use Kirchhoff’s law to write the Initial Value Problem; assume current starts to flow when the open switch is closed.
[c] Verify that $I(t) = \frac{50}{3} \left(1 - e^{-3t}\right), \ t \geq 0$ is the solution to the IVP in part [b].
[d] Graph $I(t)$.
[e] What is the steady-state current $I_\infty$ for this circuit by formula? Does your answer agree with the picture in part [d]?
[f] What is the time constant $\tau$ for this circuit? Does the picture in part [d] suggest that $I(5\tau) \approx I_\infty$?
[a] Sketch the circuit diagram for the circuit with $L = 2$, $R = 6$, and $E(t) = 100$. 

\[ L=2 \]
\[ R=6 \]

\[ E_{MF}=100 \]
[b] Use Kirchhoff’s law to write the Initial Value Problem; assume current starts to flow when the open switch is closed.

\[ E_L + E_R = E, \text{ with } E_L = L \cdot I'(t) \text{ and } E_R = R \cdot I, \text{ translates into} \]

\[ 2I'(t) + 6I(t) = 100, \quad I(t) = 0 \quad \text{at} \quad t = 0 \]
[c] Verify that \( I(t) = \frac{50}{3} (1 - e^{-3t}) \), \( t \geq 0 \) is the solution to the IVP in part [b].

If \( I(t) = \frac{50}{3} (1 - e^{-3t}) \) then

\[
2 I'(t) + 6 I(t) = 2 \left( \frac{50}{3} (1 - e^{-3t}) \right)' + 6 \left( \frac{50}{3} (1 - e^{-3t}) \right)
\]

\[
= 2 \left( \frac{50}{3} (3e^{-3t}) \right) + 100 (1 - e^{-3t})
\]

\[
= 100e^{-3t} + 100 - 100e^{-3t}
\]

\[
= 100
\]

and therefore \( I(t) \) does satisfy the ODE. Also, \( I(0) = \frac{50}{3} (1 - e^0) = \frac{50}{3}(1 - 1) = 0 \) and \( I(t) \) thus satisfies the IC.
[d] Graph $I(t)$.
What is the steady-state current $I_\infty$ for this circuit by formula? Does your answer agree with the picture in part [d]?

$I_\infty = E/R = 100/6 = 50/3 \approx 16.67$ which does indeed look like the horizontal asymptote.
[f] What is the time constant $\tau$ for this circuit? Does the picture in part [d] suggest that $I(5\tau) \approx I_\infty$?

For an R-L DC circuit

$$\tau = R/L = 2/6 = 1/3$$

Hence $5\tau = 5/3$ and from the solution given in part [c]

$$I(5\tau) = \frac{50}{3} \left(1 - e^{-3(5/3)}\right) = \frac{50}{3} \left(1 - e^{-5}\right) \approx 16.55$$

which is very close to $I_\infty = 50/3 \approx 16.67$ as shown in the following graph.