I. Practice Problem 2: R-C DC Circuit

Work on the questions for the given circuit; indicated links give (partial) solutions.

An R-C circuit consists of a 2 V DC battery connected in series with a 1 μF capacitor and a 5 kΩ resistor. (This problem is related to the example on p. 175 of Hambley: **Electrical Engineering—Principles & Applications**, 1997. The symbol k stands for "kilo", so 5 kΩ is 5000 ohms. The symbol μ stands for "micro", so 1 μF stands for 1×10^{-6} farads.)

Questions:

[a] Sketch the circuit diagram.

[b] Use Kirchhoff's law to write the Initial Value Problem; assume current starts to flow and that there is a charge of 1 μC (or micro coulombs) on the capacitor when the open switch is closed.

[c] Verify that $Q(t) = 2 - e^{-200t}$ μC is the charge on the capacitor in this circuit for $t \geq 0$.

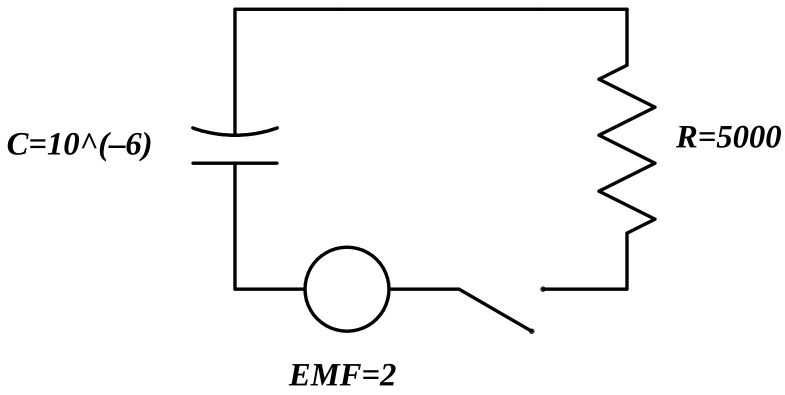
[d] Find the current $I(t)$ for the circuit.

[e] Graph $Q(t)$ and $I(t)$.

[f] What is the steady-state charge Q_∞ for this circuit by formula? Does your answer agree with the pictures in part [e]?

[g] What is the time constant τ for this circuit? Do the pictures in part [e] suggest that $Q(5\tau) \approx Q_\infty$?

[a] Sketch the circuit diagram for the circuit with $R = 5\ \mathrm{k}\Omega = 5000\ \Omega$, $C = 1\ \mu\mathrm{F} = 1 \times 10^{-6}\ \mathrm{F}$, and $E(t) = 2\ \mathrm{V}$.



[b] Use Kirchhoff's law to write the Initial Value Problem; assume current starts to flow and that there is no charge on the capacitor when the open switch is closed.

$E_R + E_C = E$, with $E_R = R \cdot Q'(t)$ and $E_C = Q(t)/C$, translates into

$$5000\, Q'(t) + \frac{Q(t)}{10^{-6}} = 2$$

which simplifies to

$$Q'(t) + 200\, Q(t) = 4 \times 10^{-4}, \qquad Q(t) = 10^{-6} \quad \text{at} \quad t = 0$$

[c] Verify that $Q(t) = 2 - e^{-200t}$ μC is the charge on the capacitor in this circuit for $t \geq 0$.

If $Q(t) = 2 - e^{-200t}$ μC then

$$\begin{aligned} Q'(t) + 200\,Q(t) &= \left(2 - e^{-200t}\right)' + 200\left(2 - e^{-200t}\right) \\ &= \left(0 + 200\,e^{-200t}\right) + 400 - 200e^{-200t} \\ &= 200\,e^{-200t} + 400 - 200\,e^{-200t} \\ &= 400 \end{aligned}$$

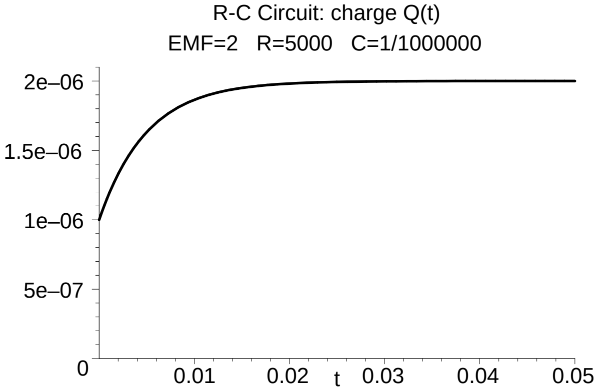
and therefore $Q(t) = \left(2 - e^{-200t}\right) \times 10^{-6}$ does satisfy the ODE. Also, $Q(0) = 2 - e^0 = 2 - 1 = 1$ μC and $Q(t)$ thus satisfies the IC.

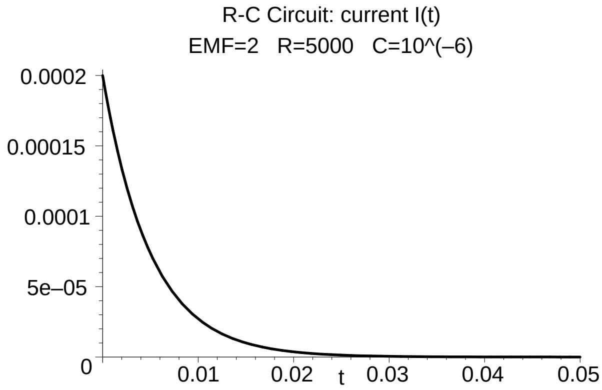
[d] Find the current $I(t)$ for the circuit.

Current is the time derivative of charge; for $t > 0$:

$$I(t) = Q'(t) = \left(2 - e^{-200t}\right)' \times 10^{-6} = 200\, e^{-200t} \text{ micro amps}$$

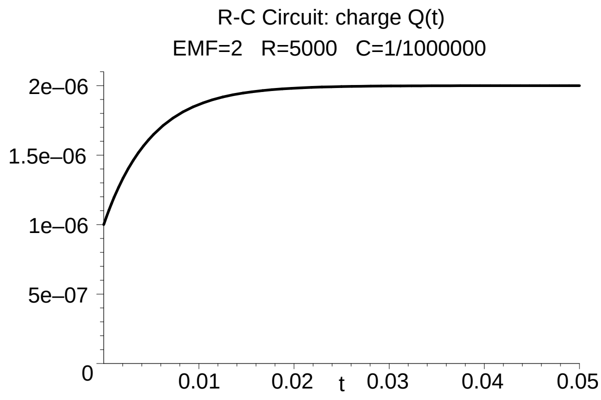
[e] Graph $Q(t)$ and $I(t)$.





[f] What is the steady-state charge Q_∞ for this circuit by formula? Does your answer agree with the pictures in part [e]?

$Q_\infty = E \cdot C = 2 \cdot 10^{-6}$ which does indeed look like the horizontal asymptote for the graph of $Q(t)$.



[g] What is the time constant τ for this circuit? Do the pictures in part [e] suggest that $Q(5\tau) \approx Q_\infty$.

For an R-C DC circuit

$$\tau = C \cdot R = 10^{-6}\left(5 \times 10^3\right) = 5 \times 10^{-3}$$

Hence $5\tau = 25 \times 10^{-3} = 0.025$ and from the solution given in part [c]:

$$Q(5\tau) = \left(2 - e^{-200\cdot(0.025)}\right) \times 10^{-6} = \left(2 - e^{-5}\right) \times 10^{-6} \approx 1.993 \times 10^{-6}$$

which is very near $Q_\infty = 2 \times 10^{-6}$ as shown in the following graph.