V. Solving Electric Circuit ODE

(The quantities R, L, C, and E below do not necessarily represent realistic physical electric circuits.)

1. A simple series circuit consists of a 50 ohm resistor, a 25 henry inductor, and a constant EMF $E(t) = 100$. If the initial current is 0 amps when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's laws;
[**b**] determine the current $I(t)$ for any time $t > 0$;
[**c**] compute the current at $t = 0.01$;
[**d**] compute the current at $t = 1.0$;
[**e**] find the "steady-state" current $\lim_{t\to\infty} I(t)$;
[**f**] approximate the time(s) when the current is at 90% of steady-state;
[**g**] compute the time unit τ for this circuit;
[**h**] compute the average current over the time interval $t = 0$ to $t = 5\tau$.

2. Repeat the preceding problem for a simple series circuit that consists of a 100 ohm resistor, a 25 henry inductor, a constant EMF $E(t) = 50$, and an initial current of 0 amps.

3. A simple series circuit consists of a 30 ohm resistor, a 0.01 farad capacitor, and a constant EMF $E(t) = 50$. If the initial charge on the capacitor is 0 coulombs when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's laws;
[**b**] determine the charge $Q(t)$ for any time $t > 0$;
[**c**] compute the charge at $t = 0.01$;
[**d**] compute the charge at $t = 1.0$;
[**e**] find the "steady-state" charge $\lim_{t\to\infty} Q(t)$;
[**f**] approximate the time(s) when the charge is at 90% of steady-state;
[**g**] compute the time unit τ for this circuit;
[**h**] compute the average charge over the time interval $t = 0$ to $t = 5\tau$.

4. Repeat the preceding problem for a simple series circuit that consists of a 50 ohm resistor, a 0.005 farad capacitor, a constant EMF $E(t) = 100$, and an initial charge of 0 coulombs.

5. A simple series circuit consists of a 25 ohm resistor, a 25 henry inductor, and a decaying EMF $E(t) = 50\,e^{-2t}$. If the initial current is 0 amps when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's laws;
[**b**] determine the current $I(t)$ for any time $t > 0$;
[**c**] compute the current at $t = 0.01$;
[**d**] compute the current at $t = 1.0$;
[**e**] approximate what the maximum current is and the time when it is achieved.

6. Repeat the preceding problem for a simple series circuit that consists of a 40 ohm resistor, a 10 henry inductor, a decaying EMF $E(t) = 50\,e^{-4t}$, and an initial current of 0 amps.

7. A simple series circuit consists of a 25 ohm resistor, a 0.01 farad capacitor, and a decaying EMF $E(t) = 50e^{-2t}$. If the initial charge on the capacitor is 0 coulombs when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's laws;
[**b**] determine the charge $Q(t)$ for any time $t > 0$;
[**c**] compute the charge at $t = 0.01$;
[**d**] compute the charge at $t = 1.0$;
[**e**] approximate what the maximum charge is and the time when it is achieved.

8. Repeat the preceding problem for a simple series circuit that consists of a 25 ohm resistor, a 0.01 farad capacitor, a decaying EMF $E(t) = 50e^{-2t}$, and an initial charge of 0.2 coulombs.

9. Repeat the preceding problem for a simple series circuit that consists of a 25 ohm resistor, a 0.01 farad capacitor, a decaying EMF $E(t) = 50e^{-4t}$, and an initial charge of 0 coulombs.

10. A simple series circuit consists of a 40 ohm resistor, a 15 henry inductor, and an oscillatory EMF $E(t) = 50\cos(6t)$. If the initial current is 0 amps when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's laws;
[**b**] determine the current $I(t)$ for any time $t > 0$;
[**c**] compute the current at $t = 0.01$;
[**d**] compute the current at $t = 1.0$;
[**e**] identify the "steady-state" current as an oscillatory function;
[**f**] compute the root-mean-square steady-state current for this circuit.

11. Repeat the preceding problem for a simple series circuit that consists of a 25 ohm resistor, a 15 henry inductor, an oscillatory EMF $E(t) = 50\cos(3t)$, and an initial current of 0 amps.

12. A simple series circuit consists of a 25 ohm resistor, a 0.005 farad capacitor, and an oscillatory EMF $E(t) = 50\cos(6t)$. If the initial charge on the capacitor is 0 coulombs when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's laws;
[**b**] determine the charge $Q(t)$ for any time $t > 0$;
[**c**] compute the charge at $t = 0.01$;
[**d**] compute the charge at $t = 1.0$;
[**e**] identify the "steady-state" charge as an oscillatory function;
[**f**] compute the root-mean-square steady-state charge for this circuit.

13. Repeat the preceding problem for a simple series circuit that consists of a 25 ohm resistor, a 0.005 farad capacitor, an oscillatory EMF $E(t) = 50\cos(6t)$, and initial charge 3 coulombs.

14. Repeat the preceding problem for a simple series circuit that consists of a 25 ohm resistor, a 0.005 farad capacitor, an oscillatory EMF $E(t) = 50\cos(3t)$, and initial charge 0 coulombs.

15. A simple series circuit consists of a 25 ohm resistor, a 25 henry inductor, and an EMF $E(t) = 20e^{-3t} + 10e^{-6t}$. If the initial current is 0 amps when the switch is closed:

[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's laws;
[**b**] determine the current $I(t)$ for any time $t > 0$;
[**c**] compute the current at $t = 0.01$;
[**d**] compute the current at $t = 1.0$;
[**e**] approximate what the maximum current is and the time when it is achieved.

16. Repeat the preceding problem for a simple series circuit that consists of a 20 ohm resistor, a 5 henry inductor, an EMF $E(t) = 20e^{-t} + 10e^{-4t}$, and an initial current of 0 amps.

17. A simple series circuit consists of a 20 ohm resistor, a 0.01 farad capacitor, and an EMF $E(t) = 20e^{-3t} + 10e^{-6t}$. If the initial charge on the capacitor is 0 coulombs when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's laws;
[**b**] determine the charge $Q(t)$ for any time $t > 0$;
[**c**] compute the charge at $t = 0.01$;
[**d**] compute the charge at $t = 1.0$;
[**e**] approximate what the maximum charge is and the time when it is achieved.

18. Repeat the preceding problem for a simple series circuit that consists of a 20 ohm resistor, a 0.01 farad capacitor, an EMF $E(t) = 20e^{-3t} + 10e^{-6t}$, and an initial charge of 0.2 coulombs when the switch is closed.

19. Repeat the preceding problem for a simple series circuit that consists of a 20 ohm resistor, a 0.01 farad capacitor, an EMF $E(t) = 20e^{-t} + 10e^{-4t}$, and an initial charge of 0 coulombs when the switch is closed.

20. A simple series circuit consists of a 25 ohm resistor, a 25 henry inductor, and a decaying oscillatory EMF $E(t) = 50e^{-3t}\cos(4t)$. If the initial current is 0 amps when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's laws;
[**b**] determine the current $I(t)$ for any time $t > 0$;
[**c**] compute the current at $t = 0.01$;
[**d**] compute the current at $t = 1.0$;
[**e**] approximate what the maximum current is and the time when it is achieved.

21. Repeat the preceding problem for a simple series circuit that consists of a 15 ohm resistor, a 30 henry inductor, a decaying EMF $E(t) = 50e^{-2t}\cos(6t)$ and an initial current of 0 amps.

22. A simple series circuit consists of a 25 ohm resistor, a 0.01 farad capacitor, and a decaying oscillatory EMF $E(t) = 50e^{-2t}\cos(6t)$. If the initial charge on the capacitor is 0 coulombs when the switch is closed:
[**a**] write down the ODE and initial condition mathematics model from Kirchhoff's laws;
[**b**] determine the charge $Q(t)$ for any time $t > 0$;
[**c**] compute the charge at $t = 0.01$;

[**d**] compute the charge at $t = 1.0$;
[**e**] approximate what the maximum charge is and the time when it is achieved.

23. Repeat the preceding problem for a simple series circuit that consists of a 25 ohm resistor, a 0.01 farad capacitor, a decaying oscillatory EMF $E(t) = 50e^{-2t}\cos(6t)$, and a charge of 0.1 coulombs when the switch is closed.

24. Repeat the preceding problem for a simple series circuit that consists of a 25 ohm resistor, a 0.01 farad capacitor, a decaying oscillatory EMF $E(t) = 50e^{-4t}\cos(3t)$, and a charge of 0 coulombs when the switch is closed.

25. A simple series circuit consists of a 50 ohm resistor, a 25 henry inductor, and an EMF $E(t)$ that is a constant 100 volt battery that drives the circuit for one second and then is disconnected. If the initial current is 0 amps when the switch is closed, then determine the current $I(t)$ for any time $t > 0$. [Suggestions: [a] Represent the EMF as a piecewise defined function and use the method of integrating factors. [b] Or, first solve the problem for an EMF that is constantly 100 volts, then determine the current $I(1)$ and use this value as the initial value for the same circuit with an EMF of 0 volts. Piece these two solutions together to get the current for the given circuit.]

26. A simple series circuit consists of a 50 ohm resistor, a 0.01 farad capacitor, and an EMF $E(t)$ that is a constant 25 volt battery that drives the circuit for one second and then is disconnected. If the initial charge on the capacitor is 0 coulombs when the switch is closed, then determine the charge $Q(t)$ for any time $t > 0$. [Suggestions: [a] Represent the EMF as a piecewise defined function and use the method of integrating factors. [b] Or, first solve the problem for an EMF that is constantly 25 volts, then determine the charge $Q(1)$ and use this value as the initial value for the same circuit with an EMF of 0 volts. Piece these two solutions together to get the charge for the given circuit.]

27. A simple series circuit consists of a 100 ohm resistor, a 25 henry inductor, and an EMF $E(t)$ that is a constant 50 volt battery that drives the circuit for two seconds and then is disconnected. If the initial current is 0 amps when the switch is closed, then determine the current $I(t)$ for any time $t > 0$.

28. Repeat the previous problem for the case when the initial current is 1 amp.

29. A simple series circuit consists of a 20 ohm resistor, a 0.01 farad capacitor, and an EMF $E(t)$ that is a constant 50 volt battery that drives the circuit for two seconds and then is disconnected. If the initial charge on the capacitor is 0 coulombs when the switch is closed, then determine the charge $Q(t)$ for any time $t > 0$.

30. Repeat the previous problem for the case when the initial charge on the capacitor is 1 coulomb when the switch is closed.

31. Use an appropriate integrating factor to solve the following ODE:

[**a**] $\dfrac{dy}{dx} - y = 2e^{3x}$

[**b**] $x\,y' + 2\,y = x\,\cos\left(x^3\right)$

[**c**] $L\,I'(t) + R\,I(t) = V$ where L, R, and V are positive constants.

32. A simple series circuit consists of a varying resistance of $50/(1+0.5t)$ ohms, a 25 henry inductor, and a constant EMF $E(t) = 100$. If the initial current is 0 amps when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's laws;
[**b**] determine the current $I(t)$ for any time $t > 0$.

33. A simple series circuit consists of a 50 ohm resistor, a varying capacitance of $0.005 + 0.01t$ farads, and a constant EMF $E(t) = 100$. If the initial charge on the capacitor is 0 coulombs when the switch is closed:
[**a**] write down the (DE and initial condition) mathematics model from Kirchhoff's laws;
[**b**] determine the charge $Q(t)$ for any time $t > 0$;