Setting Up Electric Circuit ODE and Verifying Solution

In each of the following problems you are to: [a] use Kirchhoff's law to write down the Initial Value Problem — ODE and initial condition(s) — for the given circuit ; [b] show that the given function is the solution to the IVP . (The quantities R, L, C, and E below do not necessarily represent realistic physical electric circuits.)

1. **[a]** $L = 25$ henries, $R = 50$ ohms, no capacitor, constant EMF $E(t) = 100$ volts, initial current is 0 amps when the switch is closed; **[b]** $I(t) = 2 - 2e^{-2t}$

2. **[a]** No inductor, $R = 30$ ohms, $C = 0.01$ farads, constant EMF $E(t) = 50$ volts, initial charge is 0 coulombs when the switch is closed; **[b]** $Q(t) = \frac{1}{2} - \frac{1}{2}e^{-10t/3}$

3. **[a]** $L = 25$ henries, $R = 25$ ohms, no capacitor, decaying EMF $E(t) = 50e^{-2t}$ volts, initial current is 0 amps when the switch is closed; **[b]** $I(t) = 2e^{-t} - 2e^{-2t}$

4. **[a]** No inductor, $R = 25$ ohms, $C = 0.01$ farads, decaying EMF $E(t) = 50e^{-2t}$ volts, initial charge is 0 coulombs when the switch is closed; **[b]** $Q(t) = e^{-2t} - e^{-4t}$

5. **[a]** $L = 15$ henries, $R = 40$ ohms, no capacitor, oscillatory EMF $E(t) = 50\cos(6t)$ volts, initial current is 0 amps; **[b]** $I(t) = \frac{1}{97}\left(20\cos(6t) + 45\sin(6t) - 20e^{-8t/3}\right)$

6. **[a]** No inductor, $R = 25$ ohms, $C = 0.005$ farads, oscillatory EMF $E(t) = 50\cos(6t)$ volts, initial charge is 0 coulombs; **[b]** $Q(t) = \frac{1}{25}\left(4\cos(6t) + 3\sin(6t) - 4e^{-8t}\right)$

7. **[a]** A simple series circuit consists of 25 ohm resistor, a 25 henry inductor, and an EMF $E(t) = 20e^{-3t} + 10e^{-6t}$ volts; the initial current is 0 amps when the switch is closed; **[b]** $I(t) = \frac{1}{25}\left(12e^{-t} - 10e^{-3t} - 2e^{-6t}\right)$

8. **[a]** A simple series circuit consists of a 20 ohm resistor, a 0.01 farad capacitor, and an EMF $E(t) = 20e^{-3t} + 10e^{-6t}$ volts; the initial charge on the capacitor is 0 coulombs when the switch is closed; **[b]** $Q(t) = \frac{1}{2}\left(e^{-3t} - e^{-6t}\right)$

9. **[a]** A simple series circuit consists of 25 ohm resistor, a 25 henry inductor, and a decaying oscillatory EMF $E(t) = 50e^{-3t}\cos(4t)$ volts; the initial current is 0 amps when the switch is closed; **[b]** $I(t) = \frac{1}{5}\left(e^{-t} - e^{-3t}\cos(4t) + 2e^{-3t}\sin(4t)\right)$

10. **[a]** A simple series circuit consists of a 25 ohm resistor, a 0.01 farad capacitor, and a decaying oscillatory EMF $E(t) = 50e^{-2t}\cos(6t)$ volts; the initial charge on the capacitor is 0 coulombs when the switch is closed; **[b]** $Q(t) = \frac{1}{10}e^{-2t}\left(\cos(6t) + 3\sin(6t) - e^{-2t}\right)$

11. **[a]** A simple series circuit consists of a 1 henry inductor, 12 ohm resistor, a 0.01 farad capacitor, and a constant EMF $E(t) = 100$ volts; the initial charge on the capacitor is 0 coulombs and the current is 0 amps when the switch is closed; **[b]** $Q(t) = 1 - e^{-6t}\left(\cos(8t) + (3/4)\sin(8t)\right)$

12. **[a]** A simple series circuit consists of a 2 henry inductor, 60 ohm resistor, a 0.004 farad capacitor, and a constant EMF $E(t) = 50$ volts; the initial charge on the capacitor is 0 coulombs and the current is 0 amps when the switch is closed; **[b]** $Q(t) = \frac{1}{5} - \frac{1}{4}e^{-5t} + \frac{1}{20}e^{-25t}$