SM121 Test I 14 SEP 2010

Name KEY

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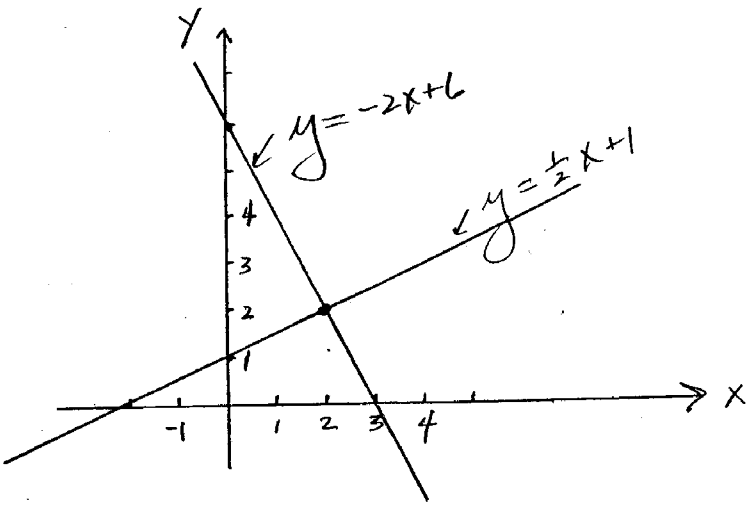
1. a) Find an equation for the line going through the point (2 , 2) and perpendicular to the line $2y - x = 2$.

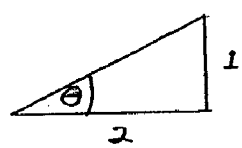
$2y - x = 2 \Rightarrow 2y = x + 2 \Rightarrow y = \frac{1}{2}x + 1$. $m = \frac{1}{2}$

$\perp$ line : $y - 2 = -2(x-2) \Rightarrow y - 2 = -2x + 4 \Rightarrow \boxed{y = -2x + 6}$

b) Graph and label both lines in part a) on the given set of axes.



2. a) For the right triangle shown, which trigonometric function applied to θ gives a result of $\frac{\sqrt{5}}{2}$?



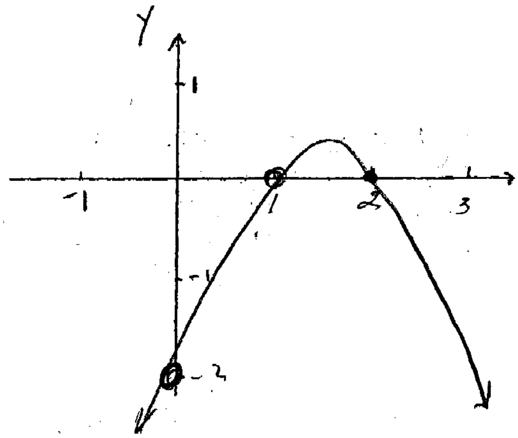
$\boxed{\sec(\theta)} = \frac{hyp}{adj} = \frac{\sqrt{5}}{2}$

b) For the triangle in part a), find the angle θ (in radians) to two decimal places.

$\tan(\theta) = \frac{1}{2} \Rightarrow \theta = \tan^{-1}(.5) \doteq \boxed{.46 \text{ rads}}$

3. Let f(x) be the second degree polynomial satisfying $f(1) = f(2) = 0$; $f(0) = -2$.

a) Sketch the graph of $y = f(x)$
on the axes to the right.



b) Find a formula for $y = f(x)$.

$y = k(x-1)(x-2)$

$f(0) = -2 \Rightarrow \quad -2 = k(0-1)(0-2) = 2k \Rightarrow k = -1$ so

$f(x) = \boxed{-1(x-1)(x-2)}$

$= -1[x^2 - 3x + 2]$

$= \boxed{-x^2 + 3x - 2}$

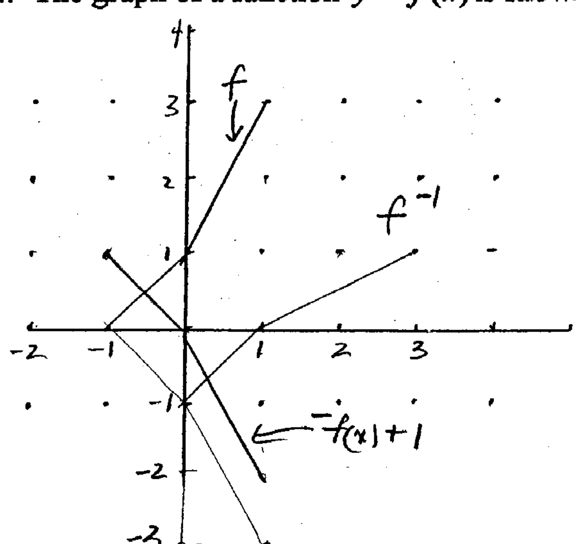
c) Find the range for the function $y = f(x)$.

max value $= f(1.5)$

$= -1(1.5-1)(1.5-2)$

$= -1(.5)(-.5) = \frac{1}{4} = .25 \Rightarrow \boxed{\text{range}(f) = (-\infty, \frac{1}{4}]}$

4. The graph of a function $y = f(x)$ is shown below.



On the same axes as $y = f(x)$,
a) Plot the graph of $y = f^{-1}(x)$ (inverse), and (label your graphs.)

b) plot the graph of $y = -f(x) + 1$.

7. a) If $\log_a(x) = 2$ and $\log_a(y) = 3$, find $\log_a(a^4x^3/y)$.

$$\log_a(a^4x^3/y) = \log_a(a^4) + \log_a(x^3) - \log_a(y)$$
$$= 4\cancel{\log_a(a)}^{1} + 3\log_a(x) - \log_a(y)$$
$$= 4 + 3(2) - 3 = \boxed{7}$$

b) Solve for x to two decimal places if $9 = 4 + 5(e^{2x} - 3)$.

$$9 - 4 = 5(e^{2x} - 3)$$
$$\Rightarrow \frac{5}{5} = e^{2x} - 3 \Rightarrow e^{2x} = 4 \Rightarrow 2x = \ln 4$$
$$\Rightarrow x = \tfrac{1}{2}\ln(4) \doteq .69$$

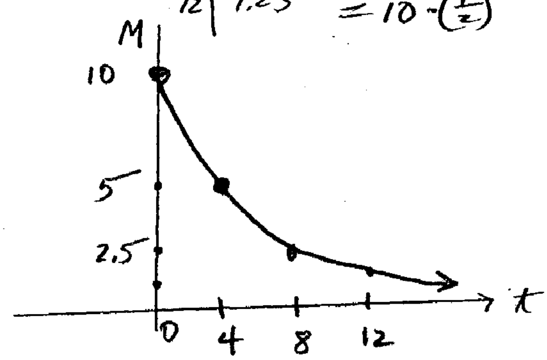
8. If we start with 10 grams of radio-active material, and it decreases by ½ every 4 days (half-life),

a) How much material is left after 8 days?

t	M	
0	10	
4	5	$= 10\cdot\frac{1}{2}$
8	2.5	$= 10\cdot(\frac{1}{2})^2$
12	1.25	$= 10\cdot(\frac{1}{2})^3$

$\boxed{\text{2.5 grams}}$

b) Sketch a graph of the amount of material M versus the time t.



c) Find a formula for the amount M as a function of t.

$$M = 10\cdot\left(\tfrac{1}{2}\right)^{t/4}$$

d) At what time will the material equal 0.1 grams?

$$0.1 = 10\cdot\left(\tfrac{1}{2}\right)^{t/4} \rightarrow \frac{.1}{10} = \frac{1}{2}^{t/4}$$
$$\Rightarrow \ln(.01) = \frac{t}{4}\ln(1/2)$$
$$\Rightarrow \frac{4\ln(.01)}{\ln(1/2)} = t \doteq 26.6 \text{ days.}$$

5. If $f(x)=\sqrt{x}$ and $g(x)=2-x$,

a) find $(g\circ f)(x)$ and give its domain and range,

$g(f(x)) = g(\sqrt{x}) = 2-\sqrt{x}$; dom $=[0,\infty)$; range $=(-\infty,2]$

b) find $(f\circ g)(x)$ and give its domain and range.

$f(g(x)) = f(2-x) = \sqrt{2-x}$: dom $=(-\infty,2]$; range $=[0,\infty)$

6. Use your calculator and graph $y=x^3+x$ and $y=\cos(2x)$ on the same axes below and determine any points of intersection accurate to 1 decimal place.