SM121 Test II (no calculators) 4 OCT 2010

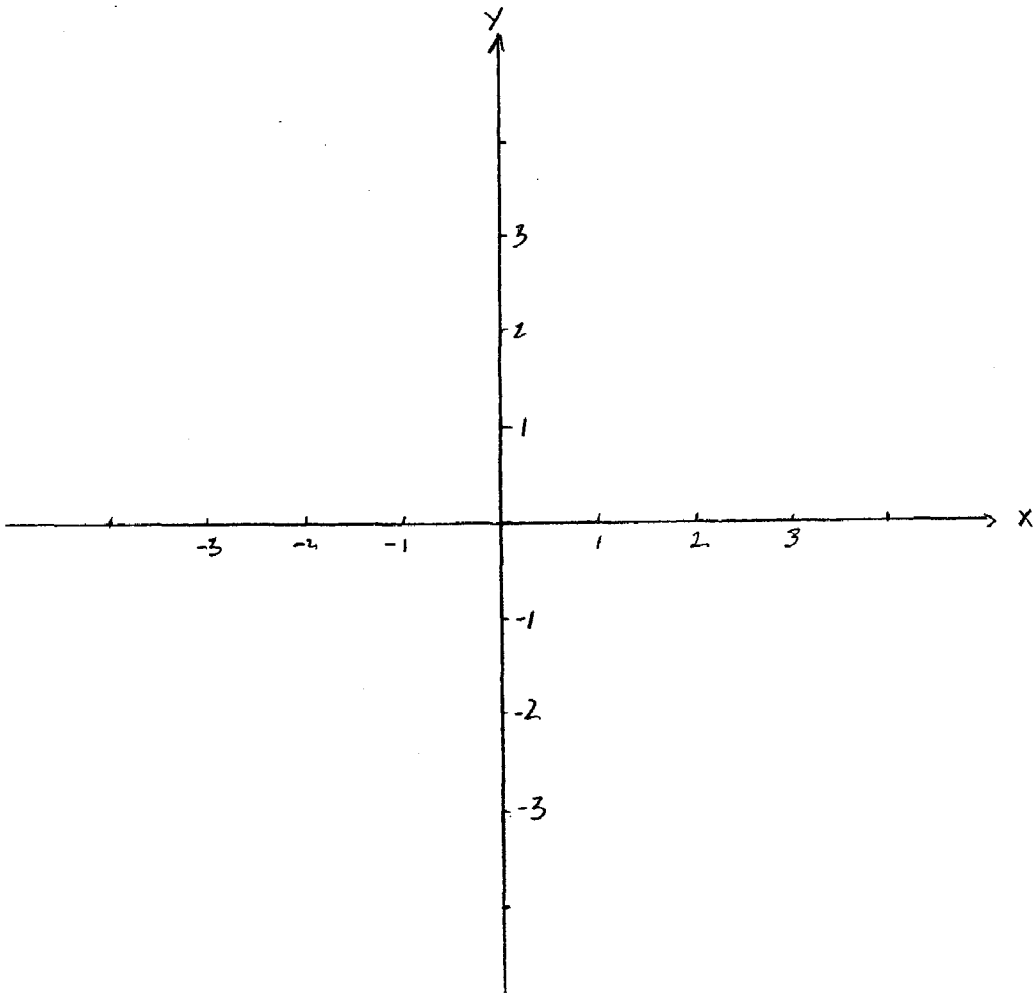
Name_____ Prof. J. D'Archangelo

1. Sketch the graph of a single function which satisfies all of the following:

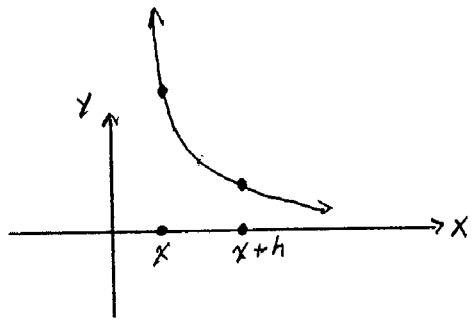
(a) $\lim_{x\to-\infty} f(x)=\infty$; (b) $\lim_{x\to-2} f(x)=-1$; (c) $f(-2)=0$;

(d) $f'(-1)=1$; (e) $\lim_{x\to0^-} f(x)=2$; (f) f is continuous from the right, but not from the left; at $x=0$.

(g) $\lim_{x\to1} f(x)=-\infty$; (h) f is continuous at x = 3, but not differentiable there.



2. (a) Label on the graph to the right, the pairs $(x, f(x))$ *and* $(x+h, f(x+h))$.

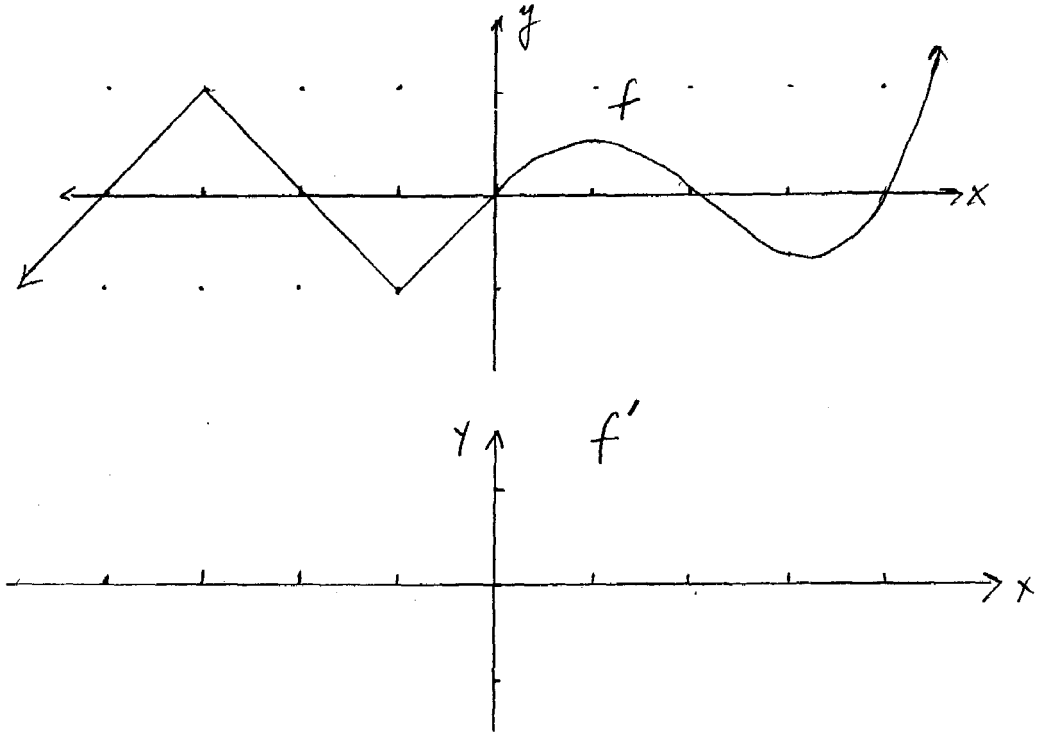


(b) What does $\dfrac{f(x+h)-f(x)}{h}$ measure ?
Show it on your graph.

(c) Give the definition for $f'(x)$.

(d) Use your definition in (c) to find $f'(x)$ for $f(x)=\dfrac{1}{x^2}$.

3. The graph of a function f is shown. Sketch the graph of f' below it.



4. The purchase price P for a house enclosing x square feet is given by the following table:

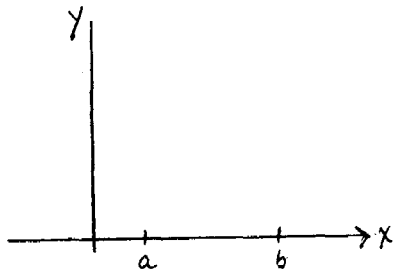
x (ft^2)	2,000	2,500	3,000	3,500
P (dollars)	200,000	260,000	310,000	350,000

a) Find the average rate of change of the price with respect to the square footage over the interval [2,000 , 3,000]. Use the correct units.

b) Approximate the instantaneous rate of change of the price at x = 3,000 ft^2.

c) Use your answer to part b) to approximate the price of a 3,100 square foot house.

5. (a) State the Intermediate Value Theorem and sketch an illustrative graph.



(b) Prove that there is at least one number x between 0 and $\frac{\pi}{2}$ where $\cos(x) - x = 0$.

6. Let $f(x) = \dfrac{x^3}{(x-1)(x+1)}$.

(a) Sketch the graph of $y = f(x)$.

(b) What are the horizontal and vertical asymptotes?