SM121 Test III 2 November 2010

Name Key Prof. J. D'Archangelo

Find $\frac{dy}{dx}$ for functions 1. - 5. (Do not use your calculator and do not simplify.)

1. $y=(x-4)^7\cos(3x)$

$y' = 7(x-4)^6\cos(3x) + (x-4)^7(-\sin(3x))3$

2. $y=\dfrac{e^{2x}}{\tan(x)}$

$y' = \left[e^{2x}\cdot 2\tan(x) - e^{2x}\sec^2(x)\right]/(\tan(x))^2$

3. $y=\arctan(x^3))$

$y' = \dfrac{1}{1+(x^3)^2}\cdot 3x^2$

4. $y=(\sin(5x)^x$

$\ln y = x\cdot\ln(\sin(5x))$

$\Rightarrow \frac{1}{y}y' = 1\ln(\sin(5x)) + x\cdot\frac{1}{\sin(5x)}\cdot\cos(5x)\cdot 5$

$\Rightarrow y' = (\sin(5x))^x\left[\ln(\sin(5x)) + 5x\cot(5x)\right]$

5. $y=x\ln(x)\sec(x)$

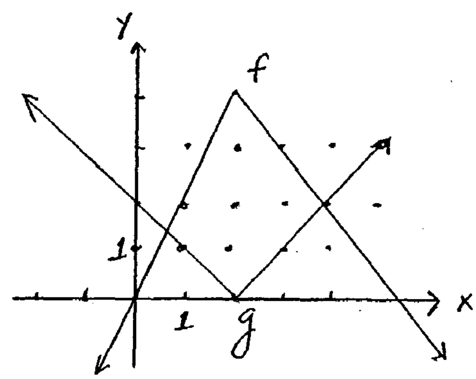
$y' = [x\ln(x)]'\sec(x) + x\ln(x)[\sec(x)]'$

$= \left[1\cdot\ln(x) + x\cdot\frac{1}{x}\right]\sec(x) + x\ln(x)\sec(x)\tan(x)$

$= 1\cdot\ln x\sec(x) + 1\sec(x) + x\ln(x)\sec(x)\tan(x)$

Over

6. If f and g are functions whose graphs are shown on the right,
let $p(x) = f(x)g(x)$
and $c(x) = f(g(x))$.



a) Find $p'(3)$.

$$p'(3) = f'(3)g(3) + f(3)g'(3)$$
$$= (-1)(1) + (3)(1)$$
$$= -1 + 3$$
$$= 2$$

b) Find $c'(3)$.

$$c'(3) = f'(g(3)) \cdot g'(3)$$
$$= f'(1) \cdot (1)$$
$$= (2)(1)$$
$$= 2$$

7. Prove the Quotient Rule by first writing $f(x)/g(x) = f(x)[g(x)]^{-1}$ and then using the Product Rule and the Chain Rule.

$$\left[\frac{f(x)}{g(x)}\right]' = \left(f(x) \cdot [g(x)]^{-1}\right)' \overset{\text{prod}}{=} f'(x) \cdot [g(x)]^{-1} + f(x) \cdot \left([g(x)]^{-1}\right)'$$

$$\overset{\text{chain rule}}{=} \frac{f'(x)}{g(x)} + f(x) \cdot (-1)[g(x)]^{-2} \cdot g'(x)$$

$$\overset{\text{alg}}{=} \frac{f'(x)}{g(x)} \cdot \frac{g(x)}{g(x)} - \frac{f(x)\,g'(x)}{(g(x))^2}$$

$$\overset{\text{alg}}{=} \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

✓ quotient rule!

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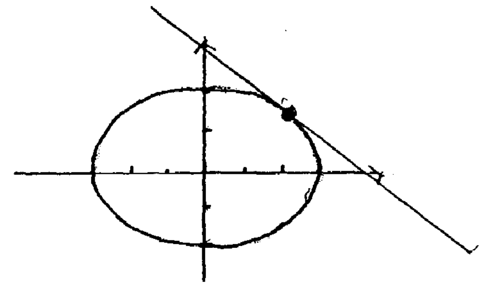


8. Find the line tangent to the ellipse defined implicitly by $4x^2+9y^2=36$ at the point $(2, \frac{2\sqrt{5}}{3})$.

tang line: $y - y_0 = m(x - x_0)$

$y - \frac{2\sqrt{5}}{3} = m(x-2)$

$m = \frac{dy}{dx}\Big|_{(2, \frac{2\sqrt{5}}{3})}$

implicit differentiation:

$\frac{d}{dx}[4x^2+9y^2] = \frac{d}{dx}[36]$

$8x + 18y\,y' = 0$

$y' = \frac{-8x}{18y}\Big|_{(2, 2\sqrt{5}/3)} = \frac{-4}{9} \cdot \frac{2}{2\sqrt{5}/3} = \frac{-4}{3\sqrt{5}} = \frac{-4\sqrt{5}}{15}$

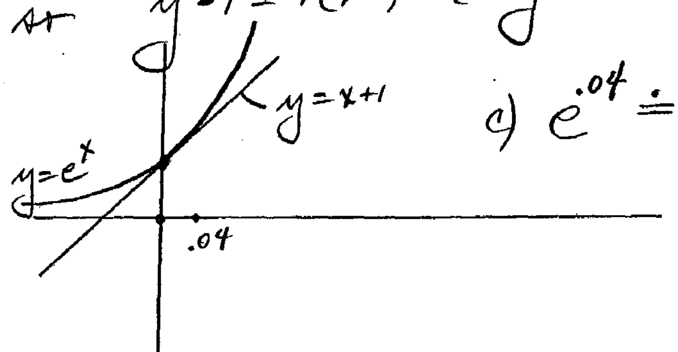
so $y - \frac{2\sqrt{5}}{3} = \frac{-4}{3\sqrt{5}}(x-2)$

9. a) Find the linearization (tangent line) for $f(x)=e^x$ at $x=0$.
   b) Graph the function and its tangent line on the same axes.
   c) Use your linearization to approximate $e^{.04}$.
   d) Is you approximation in c) too large or too small? Why?

(a) tangent line: at $(0,1)$ $y - 1 = m(x-0)$ where $m = f'(0) = e^0 = 1$

so $y - 1 = 1(x-0)$ or $y = x+1$ or $L(x) = x+1$.

(b)



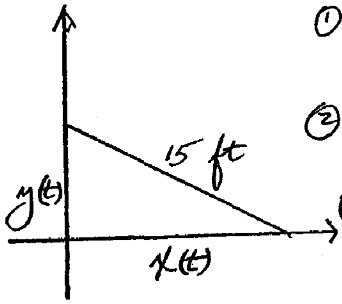
c) $e^{.04} \doteq L(.04) = x+1\big|_{x=.04} = .04+1 = 1.04$

(d) linear approx is too small because the tangent line lies below the curve.

calculator value $e^{.04} \doteq 1.04081$

Over

10. A ladder 15 ft long rests against a vertical wall. If the top of the ladder slides down the wall at a rate of 2 ft/s, how fast is the bottom of the ladder sliding away from the wall when the bottom is 9 ft from the wall? (Draw and label a diagram.)



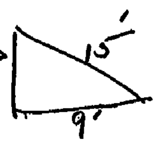
① Question: find $\frac{dx}{dt}$ when $x = 9$ ft.

② Given: $\frac{dy}{dt} = -2$ ft/s

③ relate quantities: Pythagorean thm.

$$x^2(t) + y^2(t) = (15)^2$$

④ relate rates: $\frac{d}{dt}\left[(x(t))^2 + (y(t))^2\right] = \frac{d}{dt}(225)$



$225-81$
$=\sqrt{144}$
$=12'$

$$2x\,x' + 2y\,y' = 0 \implies$$

$$x' = -\frac{\cancel{2}\,y\,y'}{\cancel{2}\,x} = -\frac{(12\text{ ft})(-2\text{ ft/s})}{(9\text{ ft})} = \frac{24}{9}\text{ ft/s} = \frac{8}{3}\text{ ft/s}$$