SM121 Test IV 6 Dec 2010

Name KEY Prof. J. D'Archangelo

1. Use the sign of f, f', and f'' to sketch a graph of $f(x) = x^4 - 2x^3$.
Label all relative maximums and minimums and inflection points. (#29 DEC2008 final exam)

$f(x) = x^4 - 2x^3 = x^3(x-2)$

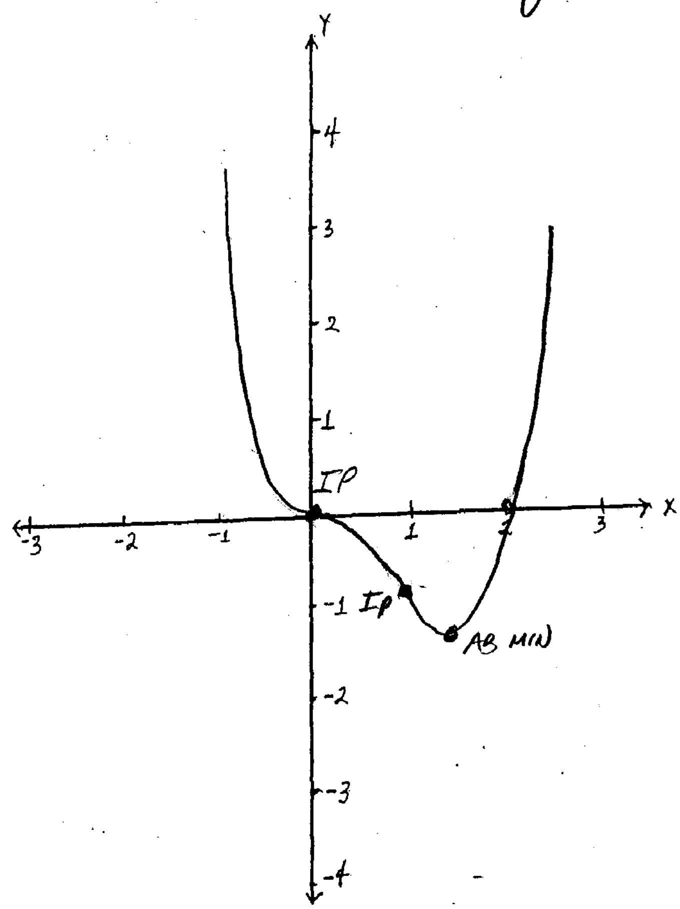
$f'(x) = 4x^3 - 6x^2 = 2x^2(2x-3)$

$f''(x) = 12x^2 - 12x = 12x(x-1)$

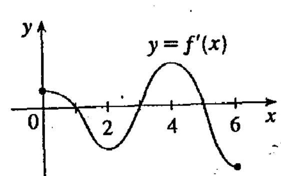
sign of f: + + 0 − − − − 0 + + (−1, 0, 1, 2)

sign of f': − − 0 − − 0 + + (−1, 0, 1, 1.5, 2)

sign of f'': + + 0 − 0 + + (−1, 0, 1, 2)



2. The graph of the derivative f' of a function f is shown.

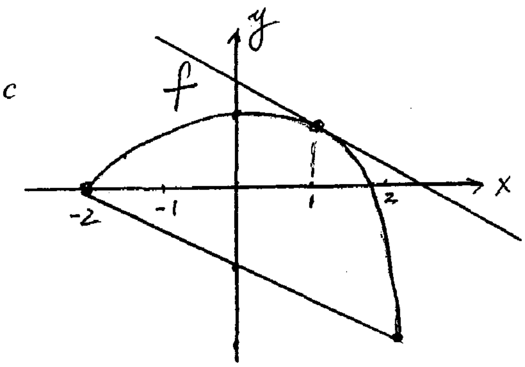


(hw problem)

a) On what intervals is f increasing? $(0,1) \cup (3,5)$ where $f' > 0$

b) On what intervals is f concave up? where $f'' > 0$ or f' is increasing $\Rightarrow (2,4)$

3.a) The Mean Value Theorem states that for the function f graphed on the right there is a number c in the interval [a , b] = [-2 , 2] where $f'(c) = \dfrac{f(b) - f(a)}{b - a}$. Based on the graph, which of the following numbers is the best estimate for c ?



a) -2 b) -1 c) 0 d) 1 e) 2

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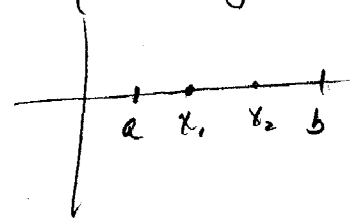
3.b) Use the Mean Value Theorem to prove the following theorem.
If $f' > 0$ on and interval (a , b), then f is increasing on (a , b). (homework problem)



Pick any two numbers $x_1 < x_2$ in (a, b).

By the MVT $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(c)$ for some number c between x_1 and x_2.

$\Rightarrow f(x_2) - f(x_1) = \underbrace{f'(c)}_{>0}\underbrace{(x_2 - x_1)}_{>0} > 0$

so $f(x_2) > f(x_1)$ which means f is increasing on (a, b).

given

4. A particle moves horizontally along a line with acceleration $a(t) = e^t - \sin(t) + t$. Find its velocity $v(t)$ and position $s(t)$ if $v(0) = 3$ and $s(0) = 4$.

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$v(t) = \int a(t)\,dt = \int \left(e^t - \sin(t) + t\right) dt$

$\Rightarrow v(t) = e^t + \cos(t) + \frac{t^2}{2} + C$

$v(0) = 3 = e^0 + \cos(0) + 0 + C \Rightarrow C = 1$

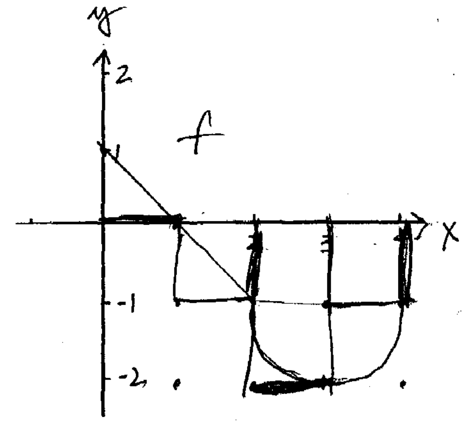
$\Rightarrow \boxed{v(t) = e^t + \cos(t) + t^2/2 + 1}$

$s(t) = \int v(t)\,dt = \int \left(e^t + \cos(t) + t^2/2 + 1\right) dt$

$= e^t + \sin(t) + t^3/6 + t + C$

$s(0) = 4 \Rightarrow C = 3 \Rightarrow \boxed{s(t) = e^t + \sin(t) + t^3/6 + t + 3}$

7. Consider the function f graphed on the right consisting of a semi-circle and a line segment.



homework problem

(a) Find the exact value for $\int_0^4 f(x)dx$.

$$-2-\frac{1}{2}\pi(1)^2=-2-\pi/2$$

(b) Approximate $\int_0^4 f(x)dx$ using R_4 (using right endpts and 4 subintervals) and show the rectangles you are using on the graph above.

$$R_4 = f(1)(\Delta x)+f(2)(\Delta x)+f(3)\Delta x+f(4)\Delta x$$
$$= 0(1) + (-1)(1) + (-2)(1) + (-1)(1)$$
$$= -4$$

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8. Evaluate the following:

(a) $\int_{-1}^{2}(6x^2+6x+6)\,dx = 2x^3+3x^2+6x\Big|_{-1}^{2}$

$$= [2(2)^3+3(2)^2+6(2)] - [2(-1)^3+3(-1)^2+6(-1)]$$
$$= [16+12+12] - [-2+3-6]$$
$$= 40+5 = 45$$

(b) $\int\left(e^{3x}+\sec(x)\tan(x)\right)dx = \frac{e^{3x}}{3}+\sec(x)+C$