CALCULUS I   CHAPTER 2   PRACTICE PROBLEMS
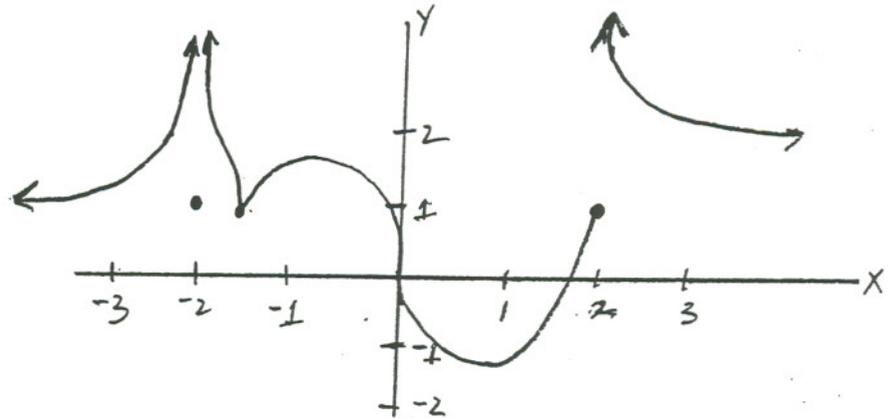
1. The graph of a function
   f is drawn on the right.

   Find the following:
   (a) $\lim\limits_{x\to-\infty} f(x)$; (b) $\lim\limits_{x\to-2} f(x)$;
   (c) $\lim\limits_{x\to 0} f(x)$; (d) $\lim\limits_{x\to 2^-} f(x)$;
   (e) $\lim\limits_{x\to 2^+} f(x)$; (f) $\lim\limits_{x\to 2} f(x)$;
   (g) $\lim\limits_{x\to\infty} f(x)$. (h) What are the horizontal and vertical asymptotes?
   (i) Where is f discontinuous?   (j) Where is f not differentiable?

2. Sketch the graph of an example of a <u>single</u> function which satisfies all of the following
   conditions:
   (a) $\lim\limits_{x\to 0^+} f(x) = -2$;   (b) $\lim\limits_{x\to 0^-} f(x) = 1$;   (c) $f(0) = -1$;   (d) $\lim\limits_{x\to 2^-} f(x) = \infty$;
   (e) $\lim\limits_{x\to 2^+} f(x) = -\infty$;   (f) $\lim\limits_{x\to\infty} f(x) = 3$;   (g) $\lim\limits_{x\to-\infty} f(x) = 4$.
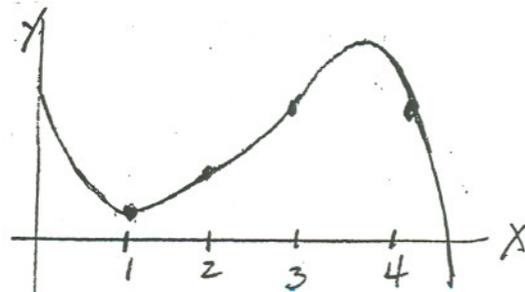
3. <u>Do not use your calculator</u>. Let $f(x) = \dfrac{x^2}{(x-2)(x+1)}$. Find the following:
   (a) $\lim\limits_{x\to 2^+} f(x)$;   (b) $\lim\limits_{x\to 2^-} f(x)$;   (c) $\lim\limits_{x\to-1^+} f(x)$;   (d) $\lim\limits_{x\to-1^-} f(x)$;   (e) $\lim\limits_{x\to 0} f(x)$;
   (f) $\lim\limits_{x\to\infty} f(x)$;   (g) $\lim\limits_{x\to-\infty} f(x)$.   (h) Sketch the graph of $y = f(x)$.

4. Use the Intermediate Value Theorem to show that there is a solution to the equation
 $e^{-x^2} = x$ somewhere in the interval $(0, 1)$.

5. For the graph on the right, arrange the
   following numbers in increasing order:

   $f'(1); f'(2); f'(3);$ $\dfrac{f(4) - f(2)}{2}$.

6. If you take a 5 year $20,000 loan to buy a car, the total cost (C) of the loan is a function of (r), the annual interest rate that you are charged. I.e., $C = f(r)$.
(a) What does $f(6) = 26{,}000$ mean? What are the units?
(b) What does $f'(6) = 1200$ mean? What are the units?
(c) Use (a) and (b) to approximate $f(6.25)$.
(d) What is $f(0)$?

7. The graph of a function f is drawn on the right. Draw the graph of f'.
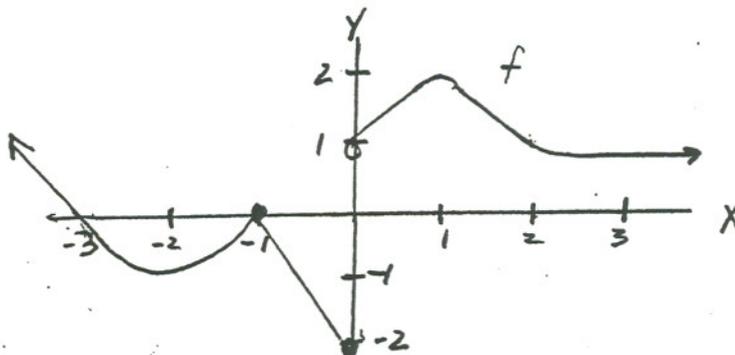
8. (a) If the tangent line to the curve $y = f(x)$ at the point $(3, 2)$ also goes through the point $(4, 4)$, find $f(3)$ and $f'(3)$.
(b) Make a careful sketch of the graph for $f(x) = \cos(x)$. Below it sketch a graph for $y = f'(x)$. Guess a formula for $f'(x)$.

9. (a) Label, on the graph to the right, $f(x); f(x+h); f(x+h) - f(x); h$.

(b) What is $\dfrac{f(x+h) - f(x)}{h}$?

(c) Give the definition for $f'(x)$.
(d) Use your definition in (c) to find $f'(x)$ if $f(x) = \dfrac{1}{x} + x$.
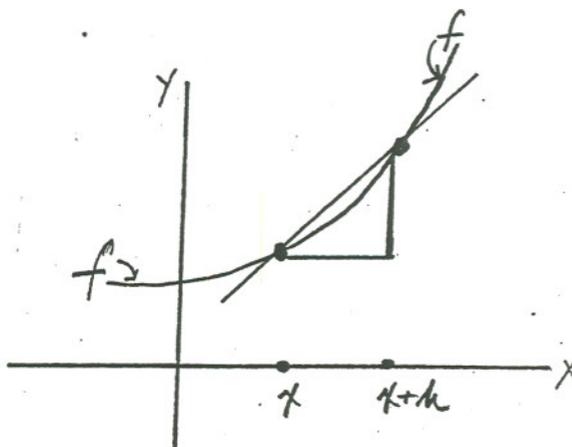
10. The table below gives $C(t)$, the total value of U.S. currency in circulation (in billions of dollars) as a function of time t (years).
(a) Find the average rate of change of the currency over the time interval $[1980, 2000]$.
(b) Give an estimate for the value of $C'(1990)$.

t	1980	1985	1990	1995	2000
C(t)	130	187	272	409	568