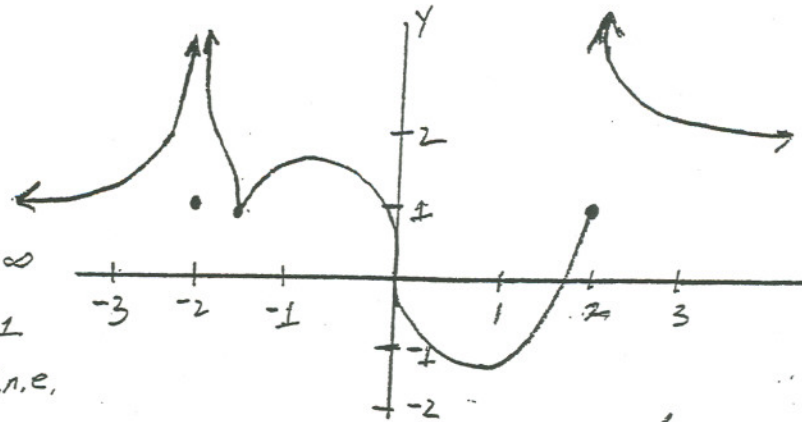
SOLUTIONS

CALCULUS I   CHAPTER 2   PRACTICE PROBLEMS



1. The graph of a function f is drawn on the right.

   Find the following:

   $1 =$ (a) $\lim_{x\to-\infty} f(x)$; (b) $\lim_{x\to-2} f(x)$; $= \infty$

   $0 =$ (c) $\lim_{x\to 0} f(x)$; (d) $\lim_{x\to 2^-} f(x)$; $= 1$

   $\infty =$ (e) $\lim_{x\to 2^+} f(x)$; (f) $\lim_{x\to 2} f(x)$; $=$ d.n.e.

   $2 =$ (g) $\lim_{x\to\infty} f(x)$. (h) What are the horizontal and vertical asymptotes? H: $y=1$ and $y=2$ V: $x=-2$, $x=2$

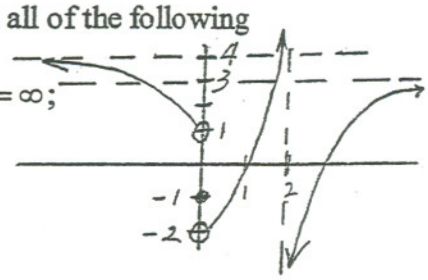
   (i) Where is f discontinuous? (j) Where is f not differentiable?

   $x=-2, 2$ $x = -2, -1.5, 0, 2$

2. Sketch the graph of an example of a single function which satisfies all of the following conditions:

   (a) $\lim_{x\to 0^+} f(x) = -2$; (b) $\lim_{x\to 0^-} f(x) = 1$; (c) $f(0) = -1$; (d) $\lim_{x\to 2^-} f(x) = \infty$;

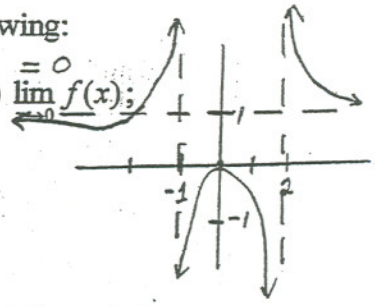
   (e) $\lim_{x\to 2^+} f(x) = -\infty$; (f) $\lim_{x\to\infty} f(x) = 3$; (g) $\lim_{x\to-\infty} f(x) = 4$.



3. Do not use your calculator. Let $f(x) = \dfrac{x^2}{(x-2)(x+1)}$. Find the following:

   (a) $\lim_{x\to 2^+} f(x) = \infty$; (b) $\lim_{x\to 2^-} f(x) = -\infty$; (c) $\lim_{x\to -1^+} f(x) = -\infty$; (d) $\lim_{x\to -1^-} f(x) = \infty$; (e) $\lim_{x\to 0} f(x) = 0$;

   (f) $\lim_{x\to\infty} f(x) = 1$; (g) $\lim_{x\to-\infty} f(x) = 1$. (h) Sketch the graph of $y = f(x)$.



4. Use the Intermediate Value Theorem to show that there is a solution to the equation $e^{-x^2} = x$ somewhere in the interval $(0, 1)$. $e^{-x^2} = x \Leftrightarrow e^{-x^2} - x = 0$.

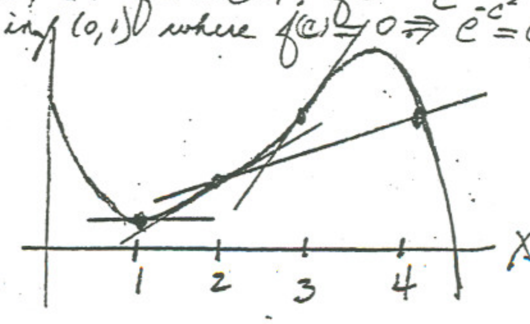
   Let $f(x) = e^{-x^2} - x$ which is continuous on $[0,1]$. $f(0) = e^0 = 1$. $f(1) = \frac{1}{e} - 1 < 0$.

   IVT $\Rightarrow$ if $N=0$, there is a number c in $(0,1)$ where $f(c) = 0 \Rightarrow e^{-c^2} = c$.

5. For the graph on the right, arrange the following numbers in increasing order:

   $f'(1)$; $f'(2)$; $f'(3)$; $\dfrac{f(4)-f(2)}{2}$.

   $f'(1) < \dfrac{f(4)-f(2)}{2} < f'(2) < f'(3)$



6. If you take a 5 year \$20,000 loan to buy a car, the total cost (C) of the loan is a function of (r), the annual interest rate that you are charged. I.e., $C = f(r)$.

(a) What does $f(6) = 26{,}000$ mean? What are the units?

(b) What does $f'(6) = 1200$ mean? What are the units?

~~(c) Use~~ (a) and (b) to approximate $f(6.25)$:

(d) What is $f(0)$?

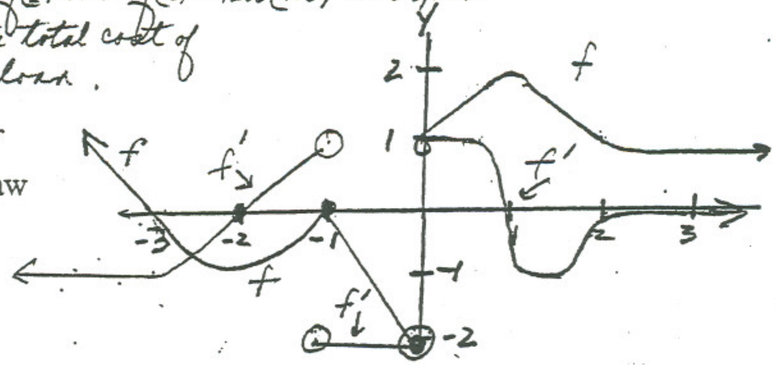
(a) A 6% loan has a total cost of \$26,000. The units are dollars.

(b) The rate of change of the total cost with respect to percent interest at 6% is 1200 $\frac{\text{dollars}}{\%\text{ interest}}$

(c) $f(6.25) \doteq f(6) + 1200(.25) = \$26{,}300$

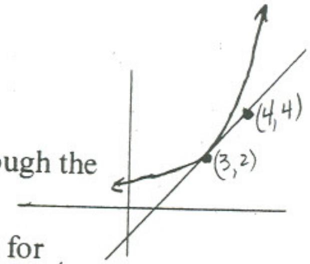
(d) $f(0) = \$20{,}000$ - the total cost of a no interest loan.

7. The graph of a function f is drawn on the right. Draw the graph of f'.

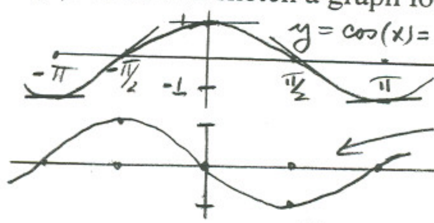


8. (a) If the tangent line to the curve $y = f(x)$ at the point (3 , 2) also goes through the point (4 , 4), find $f(3)$ and $f'(3)$.

$f(3) = 2$; $f'(3) = \frac{(4-2)}{(4-3)} = 2$



(b) Make a careful sketch of the graph for $f(x) = \cos(x)$. Below it sketch a graph for $y = f'(x)$. Guess a formula for $f'(x)$.



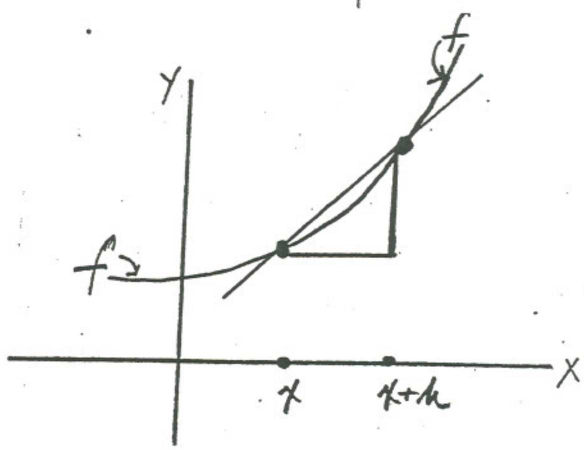
f'(x) looks like −sin(x)

9. (a) Label, on the graph to the right, $f(x)$; $f(x+h)$; $f(x+h) - f(x)$; h.

(b) What is $\dfrac{f(x+h) - f(x)}{h}$?

(c) Give the definition for $f'(x)$.

(d) Use your definition in (c) to find $f'(x)$ if $f(x) = \dfrac{1}{x} + x$.

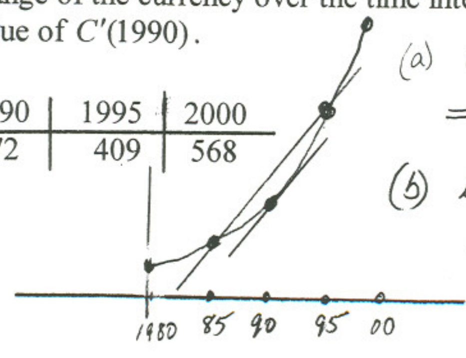


10. The table below gives $C(t)$, the total value of U.S. currency in circulation (in billions of dollars) as a function of time t (years).

(a) Find the average rate of change of the currency over the time interval [1980 , 2000].

(b) Give an estimate for the value of $C'(1990)$.

t	1980	1985	1990	1995	2000
$C(t)$	130	187	272	409	568



(a) ave rate of change $= \frac{\Delta C}{\Delta t} = \frac{568-130}{2000-1980} = \frac{438}{20} = 21.9\ \frac{\text{billion\$}}{\text{yr}}$

(b) inst rate of change at 1990 $\doteq$ ave over [1985, 1995]

$= \left[\frac{409-187}{1995-1985}\right] = 22.5\ \frac{\text{billion\$}}{\text{yr}}$