Solutions to Chapter 3 Practice Problems — Calculus I

1. $y' = 8(x+2)^7(x+3)^6 + (x+2)^8\, 6(x+3)^5$
$= 2(x+2)^7(x+3)^5[4(x+3)+3(x+2)]$
$= 2(x+2)^7(x+3)^5[7x+18]$

2. $y' = (x^{1/3} + x^{-1/3})' = \frac{1}{3}x^{-2/3} - \frac{1}{3}x^{-4/3}$
$= \frac{1}{3}x^{-4/3}(x^{2/3}-1) = \frac{1}{3}\,\frac{\sqrt[3]{x^2}-1}{\sqrt[3]{x^4}}$

3. $y' = \dfrac{e^x(1+x^2) - e^x(2x)}{(1+x^2)^2} = \dfrac{e^x(x^2-2x+1)}{(1+x^2)^2}$

4. $y' = \cos(\cos(x))\cdot(-\sin(x))$

5. $y' = \dfrac{1}{\sqrt{1-(e^x)^2}}\cdot e^x = \dfrac{e^x}{\sqrt{1-e^{2x}}}$

6. $y' = 1e^{-4/x} + xe^{-4/x}\cdot(-4/x)'$
$= e^{-4/x} + xe^{-4/x}\left(\frac{4}{x^2}\right)$
$= e^{-4/x}\left(1+\frac{4}{x}\right)$

7. $y' = \sec^2\left((1-x)^{1/2}\right)\cdot \frac{1}{2}(1-x)^{-1/2}(-1)$
$= \dfrac{-\sec^2\sqrt{1-x}}{2\sqrt{1-x}}$

8. $y' = \dfrac{1}{\csc(5x)}\cdot\left(-\csc(5x)\cot(5x)\right)\cdot 5$
$= -5\cot(5x)$

9. $y' = e^{e^x}\cdot e^x = e^{e^x+x}$

10. $y' = \dfrac{1}{1+(\arcsin\sqrt{x})^2}\cdot\dfrac{1}{\sqrt{1-\sqrt{x}^2}}\cdot\frac{1}{2}x^{-1/2}$
$= \dfrac{1}{1+(\arcsin\sqrt{x})^2}\cdot\dfrac{1}{\sqrt{1-x}}\cdot\dfrac{1}{2\sqrt{x}}$

11. $y' = 5^{x\tan x}(\ln 5)\cdot(1\tan x + x\sec^2 x)$

12. $\ln y = x^2\ln x$
$\frac{1}{y}\frac{dy}{dx} = 2x\ln x + x^2\cdot\frac{1}{x}$
$\frac{dy}{dx} = x^{x^2}\cdot[2\ln x+1]x$

13. $\ln y = \frac{1}{2}\ln(x+1) + 5\ln(2-x) - 7\ln(x+3)$
$\frac{1}{y}y' = \frac{1/2}{x+1} + \frac{5}{2-x}(-1) - \frac{7}{x+3}$
$y' = \dfrac{\sqrt{x+1}\,(2-x)^5}{(x+3)^7}\left[\frac{1/2}{x+1} - \frac{5}{2-x} - \frac{7}{x+3}\right]$

14. $\frac{d}{dx}(x^{1/2}+y^{1/2}) = \frac{d}{dx}(3) \Rightarrow \frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$
$y' = \dfrac{-x^{-1/2}}{y^{-1/2}} = -\dfrac{\sqrt{y}}{\sqrt{x}}\Big|_{(4,1)} = -\frac{1}{2}$
tang line: $y-1 = -\frac{1}{2}(x-4)$

15. $P'(x) = f'(x)g(x) + f(x)g'(x)$
$P'(2) = (5)(1) + (3)(4) = 5+12 = 17$
$Q'(x) = (f'(x)g(x) - f(x)g'(x))/g^2(x)$
$Q'(2) = ((5)(1) - (3)(4))/1^2 = -7$
$C'(x) = f'(g(x))\cdot g'(x)$
$C'(2) = f'(g(2))\cdot g'(2)$
$= f'(1)\cdot 4$
$= (-2)(4) = -8$

16. $[f(x)/g(x)]' = [f(x)\cdot(g(x))^{-1}]'$
$\overset{\text{prod rule}}{=} f'(x)\cdot(g(x))^{-1} + f(x)\cdot\underbrace{((g(x))^{-1})'}_{\text{chain rule}}$
$= \dfrac{f'(x)}{g(x)} + f(x)(-1)(g(x))^{-2}\cdot g'(x)$
$= \dfrac{f'(x)}{g(x)} - \dfrac{f(x)g'(x)}{(g(x))^2} = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
done

17. $y(t)=t^3-6t^2+8t=t(t^2-6t+8)$

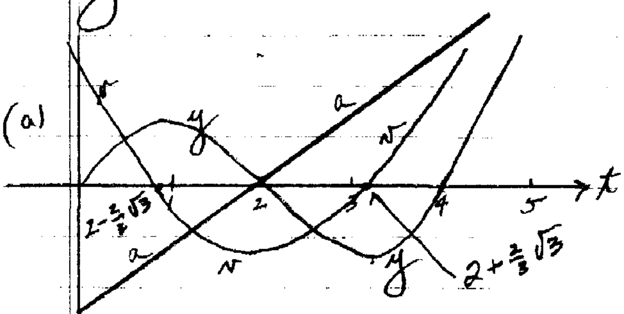
$\quad = t(t-2)(t-4)$

$v(t)=y'(t)=3t^2-12t+8$

roots $2\pm\frac{2}{3}\sqrt{3}$

$a(t)=y''(t)=6t-12=6(t-2)$

(a)



(b) moving up $0<t<2-\frac{2}{3}\sqrt{3}$ and $2+\frac{2}{3}\sqrt{3}<t<5$

moving down $2-\frac{2}{3}\sqrt{3}<t<2+\frac{2}{3}\sqrt{3}$

(c) dist: $\left|y(2-\frac{2}{3}\sqrt{3})-y(0)\right|$
$+\left|y(2+\frac{2}{3}\sqrt{3})-y(2-\frac{2}{3}\sqrt{3})\right|$
$+\left|y(5)-y(2+\frac{2}{3}\sqrt{3})\right|$

$= \frac{16\sqrt{3}}{9}+\frac{32\sqrt{3}}{9}+15+\frac{16\sqrt{3}}{9}$

$= \frac{64\sqrt{3}}{9}+15 \doteq 27.3$

18. $V=\pi r^2 h$

(a) $\frac{dV}{dh}=\pi r^2\cdot 1 \quad \left(\frac{m^3}{m}=m^2\right)$

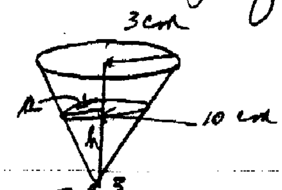
(b) $\frac{dV}{dr}=2\pi r h \quad \left(\frac{m^3}{m}=m^2\right)$

(c) $V(t)=\pi(r(t))^2 h(t)$

$\frac{dV}{dt}=\pi\left(2(r(t))\frac{dr}{dt}h(t)+(r(t))^2\frac{dh}{dt}\right)$

$=\pi(2rhr'+r^2h') \quad \left(\frac{m^3}{s}\right)$

19. $V=\frac{1}{3}\pi r^2 h$



$\frac{r}{h}=\frac{3}{10}$

$\Rightarrow r=.3h$

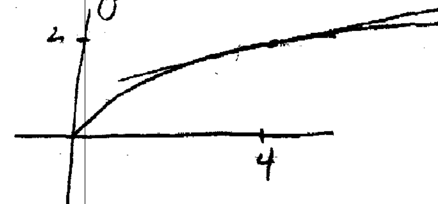
$V=\frac{1}{3}\pi(.3h)^2h=.03\pi h^3$

$\frac{dV}{dt}=.09\pi h^2\cdot dh/dt$

$2\,cm^3/s=.09\pi(5\,cm)^2\cdot\frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt}=\frac{2\,cm^3/s}{.09\pi\, 25\,cm^2}=\frac{8}{9\pi}\,cm/s$

20. $f(x)=x^{1/2} \quad a=4 \quad (4,2)$



$y-2=m(x-4) \quad m=f'(4)$

$m=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}\Big|_4=\frac{1}{4}$

linearization: $y=\frac{1}{4}(x-4)+2$

$y=\frac{1}{4}x+1$

$\sqrt{4.03}\doteq\frac{1}{4}(4.03)+1=2.0075$

too high because curve is concave down.

21. $y=Ae^{-x}+Bxe^{-x}$

$2[y'=(-A+B)e^{-x}-Bxe^{-x}]\,2$

$y''=(A-2B)e^{-x}+Bxe^{-x}$

$y''+2y'+y=0$