Calculus I Chapter 4.5 Practice Problem Solutions

1. $f(x) = x^2 + \frac{1}{x}$ is continuous on $[.5, 2]$

$f'(x) = 2x - 1/x^2 = 0 \Rightarrow 2x^3 = 1$

$\Rightarrow x = \sqrt[3]{1/2} = \frac{1}{\sqrt[3]{2}}$ (only crit pt)

	x	$f(x)$
endpts	.5	$\frac{1}{4} + 2 = 2.25$
	2	$4 + \frac{1}{2} = 4.5$
crit pt	$1/\sqrt[3]{2}$	$\left(\frac{1}{\sqrt[3]{2}}\right)^2 + \sqrt[3]{2} \doteq 1.89$

$\Rightarrow$ abs min at $1/\sqrt[3]{2}$, abs max at 2.

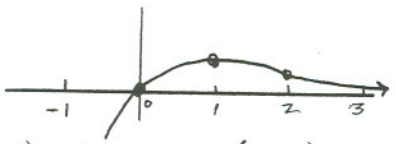
2. $f(x) = xe^{-x}$; $f'(x) = (1-x)e^{-x}$; $f''(x) = (x-2)e^{-x}$

sign of f: $- - - 0 + + +$ (at $-1, 0, 1, 2$)

sign of f': $+ + + + 0 - - -$ (at $-1, 0, 1, 2$)

sign of f'': $- - - - 0 + +$ (at $3, 0, 1, 2$)

(a) abs max $(1, \frac{1}{e})$
infl pt $(2, 2/e^2)$



(b) f increases on $(-\infty, 1)$, decreases on $(1, \infty)$

(c) f concave up on $(2, \infty)$, down on $(-\infty, 2)$

3. (a) $\lim_{x\to 1} \frac{x^2+2x+1}{x+2} = \frac{4}{3}$; (b) $\lim_{x\to 1} \frac{\ln(x)}{x} = \frac{0}{1} = 0$

(c) $\lim_{x\to 1} \frac{x-1}{\ln(x)} \overset{\text{L'}}{=} \lim_{x\to 1} \frac{1}{1/x} = 1$

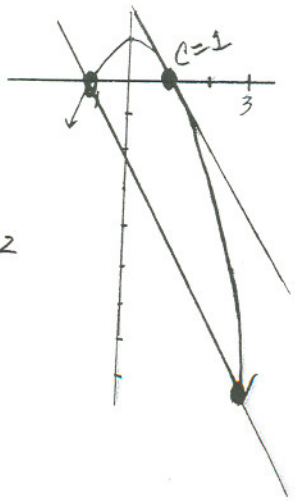
4. $f(x) = 1 - x^2$; $f'(x) = -2x$

$a = -1$, $b = 3$.

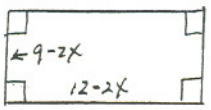
$f'(c) = \frac{f(b) - f(a)}{b - a}$

$\Rightarrow -2c = (-8 - 0)/(3 - (-1)) = -2$

$\Rightarrow c = 1$



5. (a) $\lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{2}{-1} = -2$; (b) $\lim_{x\to 1} \frac{f(x)}{g(x)} \overset{\text{L'}}{=} \lim_{x\to 1} \frac{f'(x)}{g'(x)} = \frac{0}{1} = 0$

6. 

$V = (9-2x)(12-2x)x$; $x \in [0, \frac{9}{2}]$

$V(x) = 4x^3 - 42x^2 + 108x$

$V'(x) = 12x^2 - 84x + 108 = 12(x^2 - 7x + 9) = 0$

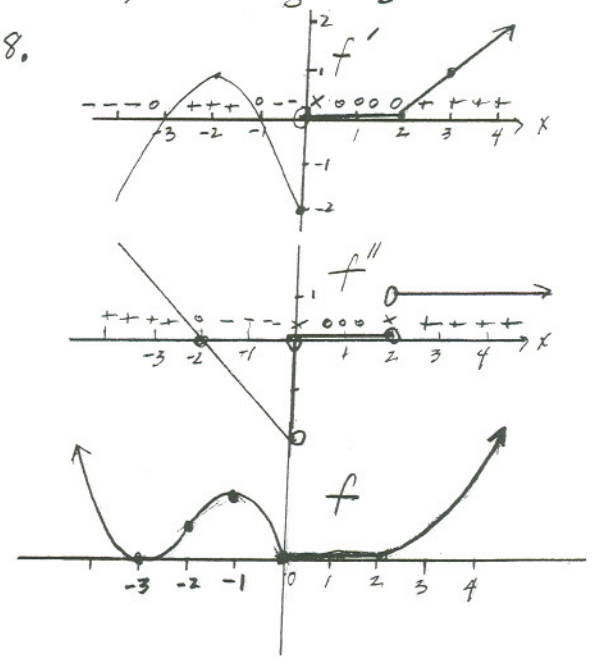
$\Rightarrow x = (7 - \sqrt{13})/2 \doteq 1.70$ or $x = (7 + \sqrt{13})/2 \doteq 5.30$ (too big)

$V''(x) = 24x - 84 \Rightarrow V''(1.70) < 0 \Rightarrow$ max

7. $a(t) = 4t^2 \xrightarrow{\text{antideriv}} s(t) = \frac{4t^3}{3} + C$

$s(1) = 5 \Rightarrow \frac{4(1)^3}{3} + C = 5 \Rightarrow C = 5 - \frac{4}{3} = \frac{11}{3}$

$\Rightarrow s(t) = \frac{4}{3}t^3 + \frac{11}{3}$,

8. 

9. (a) $\int_0^2 f(x)\,dx = \int_0^1 x\,dx + \int_1^2 f(x)\,dx$

$= \frac{x^2}{2}\Big|_0^1 + 0 = 1/2$

(b) 

$R_4 = [f(.5) + f(1) + f(1.5) + f(2)] \cdot (.5)$

$= [.25 + 1 + 0 - 1](.5) = .125$

(c) $M_2 = [f(.5) + f(1.5)][1] = .25$