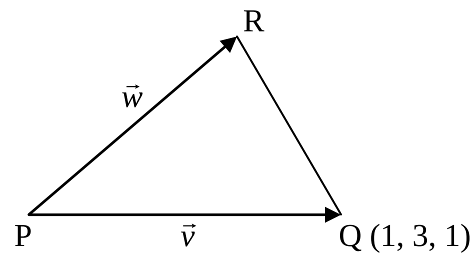
Calculus II Chapters 12 Practice Problems

Name ___________________________



1. P, Q, and R are three points in space.
The coordinates for Q are (1, 3, 1).
Let $\vec{v} = \overrightarrow{PQ} = 1\vec{i} + 2\vec{j} - 1\vec{k} = <1,2,-1>$
and $\vec{w} = \overrightarrow{PR} = 4\vec{i} + 1\vec{j} + 1\vec{k} = <4,1,1>$.

a) Find the coordinates for the points P and R.

b) Find $\vec{v} \cdot \vec{w}$ and the angle θ between the two vectors.

c) Find a vector of length 5 in the same direction as $\vec{v}$.

d) Find the scalar component of $\vec{v}$ *onto* $\vec{w}$: $comp_{\vec{w}}\vec{v}$.

e) Find the vector projection of $\vec{v}$ *onto* $\vec{w}$: $proj_{\vec{w}}\vec{v}$.

f) Find the vector projection of $\vec{v}$ *orthogonal to* $\vec{w}$ = $orth_{\vec{w}}\vec{v} = \vec{v} - proj_{\vec{w}}\vec{v}$.

g) Verify that $orth_{\vec{w}}\vec{v}$ is perpendicular to $\vec{w}$.

h) Find $\vec{v} \times \vec{w}$, and the area of the triangle ΔPQR.

i) Find parametric equations for the line going through the points P and R.

j) Find an equation for the plane containing the points P, Q, and R.

k) Find the distance between the origin (0, 0, 0) and the line through the points P and R.

l) Find the distance between the origin (0, 0, 0) and the plane containing P, Q, and R.

2. State whether each of the following is a vector, a scalar, or makes no sense:

(a) $(\vec{a} \cdot \vec{b})\vec{c}$ (b) $\dfrac{\vec{c}}{\vec{a} \cdot \vec{b}}$ (c) $\dfrac{\vec{a} \cdot \vec{b}}{\vec{c}}$ (d) $(\vec{a} \times \vec{b}) \cdot |\vec{c}|$

(e) $(\vec{a} \times \vec{b}) \times \vec{c}$ (f) $\vec{a} \cdot (\vec{b} \times \vec{c})$ (g) $(\vec{a} \cdot \vec{b}) \times \vec{c}$ (h) $(\vec{a} \cdot \vec{b}) + \vec{c}$.

3. Prove that for any two non-zero vectors $\vec{a}$ and $\vec{b}$,
$\vec{b} - proj_{\vec{a}}\vec{b}$ is perpendicular to $\vec{a}$.