Calculus II Test II March 4, 2011

Name _____ Prof. D'Archangelo

1. [30 pts] Evaluate the following integrals by hand. Show all steps.

(a) $\int x \cos(5x)\, dx$

(b) $\int x \ln(x)\, dx$

(c) $\int \frac{3x^2 + 2}{(x)(x^2 + 1)}\, dx$

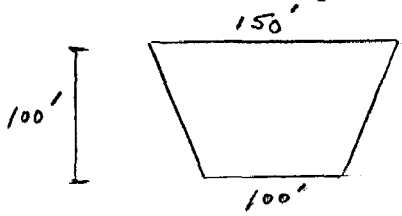
2. [10 pts] Prove the integration by parts formula $\int u\,dv = uv - \int v\,du$ by first differentiating $u(x)v(x)$ and then integrating the resulting formula.

3. [10 pts] Use S_4 (Simpson's rule with 4 subdivisions) to approximate the area under one hump of the function $\sin(x)$ over the interval $[\,0\,,\,\pi\,]$. Give your answer to 3 decimal places. How much error is in your approximation?

4. [10 pts] (a) Use a graph to explain why $\int_5^9 \frac{1}{\sqrt{x-5}}\,dx$ is an improper integral.

(b) Evaluate the integral by first writing it as a limit of a proper integral. Show all steps.

5. [10 pts] A dam with water to its top is in the shape of the trapezoid shown below. **Set up, but do not evaluate** an integral to find the hydrostatic force pushing against the dam. Use the fact that water pressure at a depth of x feet is $62.5x$ lbs/ft^2.



6. [10 pts] If $y(t)$ is the temperature of an object in a room of constant temperature R, then the differential equation for Newton's Law of Cooling is $\frac{dy}{dt} = k(y - R)$.

a) Explain what this differential equation means in plain English.

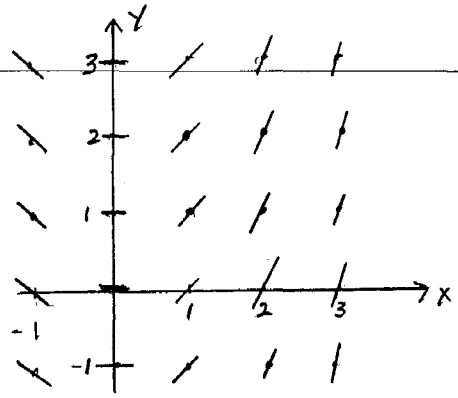
b) A cup of coffee has temperature $150^\circ F$ when taken from a microwave and placed in a room of $70^\circ F$. Ten minutes later the coffee is $110^\circ F$. Solve a differential equation to find $y(t)$, the temperature of the coffee at time t. When will the temperature of the coffee be $90^\circ F$?

Calculus II Test II take home questions March 4, 2011 P1 of 2

Name _____ Prof. D'Archangelo

I have not used assistance from any other person.

1. A portion of a direction field for a differential equation $y' = f(x, y)$ is drawn below.
If $y(0) = 1$, use two steps of Euler's method to approximate $y(2)$.
Hint: You can solve the problem with a ruler.



2. A simple series circuit consists of a 1.0 henry inductor, a 2.0 ohm resistor, and a constant EMF $E(t) = 10$ volts. If the initial current in the circuit is 20 amperes, set up and solve an initial value problem to determine the current $I(t)$ for $t > 0$.

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3. Sketch the region bounded by the curves $y = 1/x$, $x = 1$, and $x = 2$. Find the exact centroid of the region and plot it on your graph.