Calculus II  
Test III  
April 5, 2011  
Prof. D'Archangelo  

Name _____________________

1. (a) Sketch the point whose polar coordinates are $(r, \theta) = (2, 3\pi/4)$ and find the point’s Cartesian coordinates $(x, y)$.

(b) Four sets of polar coordinates $(r, \theta)$ are given. Three of them represent the same point. Which set of polar coordinates represents a different point from the other three? (Circle your answer.)

(i) $(4, \pi/3)$  
(ii) $(4, -5\pi/3)$  
(iii) $(-4, 4\pi/3)$  
(iv) $(-4, -5\pi/3)$

(c) Find a polar equation for the curve represented by the Cartesian equation $x^2 + y^2 = -5y$.

2. The curve below shows the graph of $r$ as a function of $\theta$ in the $\theta - r$ plane. Use it to sketch the corresponding polar curve in the $x - y$ plane.
3. (a) Sketch the 3-petal curve \( r = \sin(3\theta) \) in the \( x - y \) plane (you may use your calculator) and
(b) find the area in one petal by setting up and evaluating an integral (you may use your calculator).

\[ \text{Area} = \]

4. Find the sum of the geometric series \( 2 + \frac{4}{3} + \frac{8}{9} + \ldots \).

5. Write out the first three terms of each of the following series and tell whether each of the series converges or diverges (you must justify your answers.)

(a) \( \sum_{n=1}^{\infty} \frac{-4n}{5n+1} \)

(b) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{3n-1} \)

(c) \( \sum_{n=1}^{\infty} \frac{1}{n} \)
6. Find the open interval of convergence (no endpts) and radius of convergence for the series \( \sum_{n=1}^{\infty} \frac{n(x-1)^n}{7^n} \).

7. Show that the Maclaurin series for \( f(x) = \sin(x) \) is \( x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... \) by finding the Maclaurin coefficients directly.
8. (a) Use the Maclaurin series in problem 7 to find the Maclaurin series for $f(x) = x^2 \sin(x^2)$. Show the first four non-zero terms of your answer. Extra credit for writing the answer in summation notation.

(b) Find $\int_0^{0.5} x^2 \sin(x^2) \, dx$ to within .001 using your series from part (a).

9. Let $f(x) = \cos(x)$. Find the first four non-zero terms of the Taylor series for $f(x)$ centered at $a = \pi/4$. 