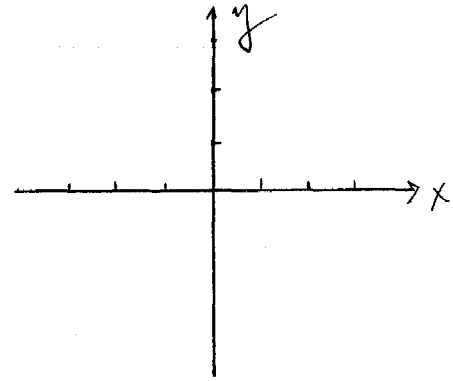
Calculus II Test III April 5, 2011

Name ____________________________ Prof. D'Archangelo

1. (a) Sketch the point whose polar coordinates are $(r,\theta) = (2, 3\pi/4)$ and find the point's Cartesian coordinates (x , y).

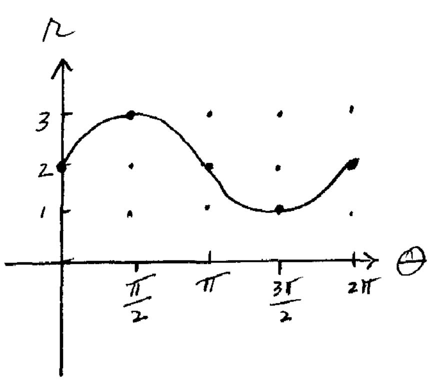


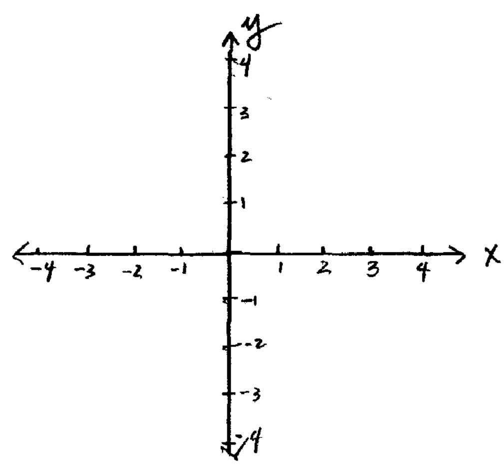
(b) Four sets of polar coorinates (r,θ) are given. Three of them represent the same point. Which set of polar coordinates represents a different point from the other three? (Circle your answer.)

(i) $(4, \pi/3)$ (ii) $(4, -5\pi/3)$ (iii) $(-4, 4\pi/3)$ (iv) $(-4, -5\pi/3)$

(c) Find a polar equation for the curve represented by the Cartesian equation $x^2 + y^2 = -5y$.

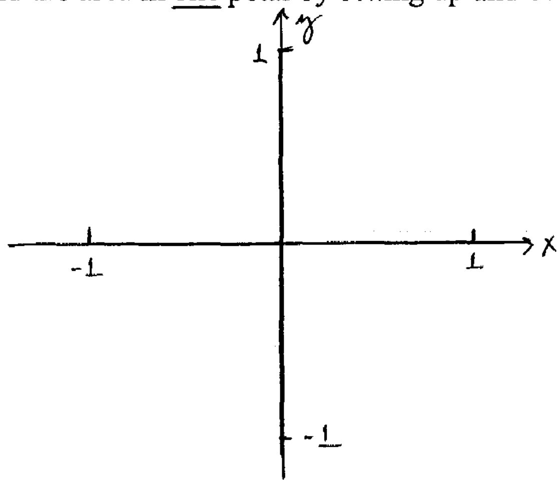
2. The curve below shows the graph of r as a function of θ in the $\theta - r$ plane. Use it to sketch the corresponding polar curve in the $x - y$ plane.





3. (a) Sketch the 3-petal curve $r = \sin(3\theta)$ in the x – y plane (you may use your calculator) and
(b) find the area in <u>one</u> petal by setting up and evaluating an integral (you may use your calculator).

(a)



(b) AREA

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4. Find the sum of the geometric series $2 + 4/3 + 8/9 + \ldots$.

5. Write out the first three terms of each of the following series and tell whether each of the series converges or diverges (you must justify your answers.)

(a) $\sum_{n=1}^{\infty} \frac{-4n}{5n+1}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n-1}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n}$

6. Find the open interval of convergence (no endpts) and radius of convergence for the series $\sum_{n=1}^{\infty} \frac{n(x-1)^n}{7^n}$.

7. Show that the Maclaurin series for $f(x) = \sin(x)$ is $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$ by finding the Maclaurin coefficients directly.

8. (a) Use the Maclaurin series in problem 7 to find the Maclaurin series for $f(x) = x^2 \sin(x^2)$. Show the first four non-zero terms of your answer. Extra credit for writing the answer in summation notation.

(b) Find $\int_0^{0.5} x^2 \sin(x^2)\,dx$ to within .001 using your series from part (a).

9. Let $f(x) = \cos(x)$. Find the first four non-zero terms of the Taylor series for $f(x)$ centered at $a = \pi/4$.