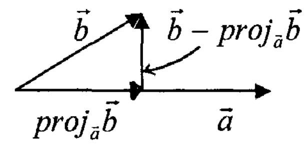
Calculus II Test IV April 27, 2011

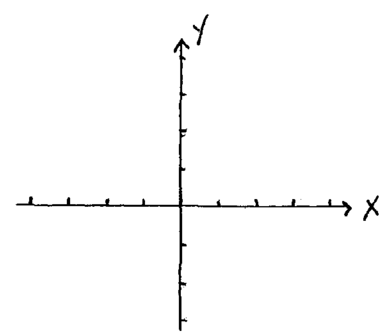
Name ____________________ Prof. D'Archangelo

(10) 1. Prove that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{b} - proj_{\vec{a}}\vec{b}$ is perpendicular to $\vec{a}$.
Show all steps and give reasons.



(20) 2. Consider the two vectors $\vec{a} =< -3,4 >$ and $\vec{b} =< 1,3 >$.

a) Sketch both vectors on the axes to the right.
(with tails at the origin)



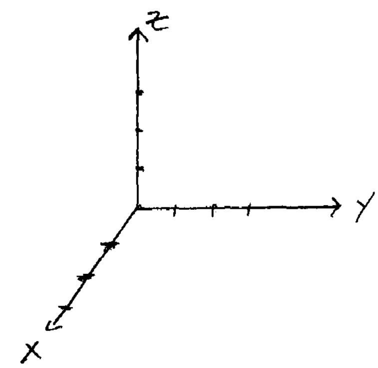
(b) Find the angle (degrees) between $\vec{a}$ and $\vec{b}$.

(c) Find the scalar component of $\vec{b}$ onto $\vec{a}$, ($comp_{\vec{a}}\vec{b}$).

(d) Find the vector projection of $\vec{b}$ onto $\vec{a}$, ($proj_{\vec{a}}\vec{b}$) <u>and</u> sketch it in your graph above.

(15) 3. Consider the three points in space: P(1, 0, 0), Q(0, 2, 0), and R(0, 0, 3).

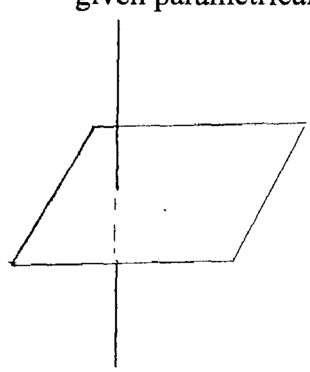
a) Sketch the triangle in 3-d space on the axes to the right.



b) Find the area of the triangle ΔPQR.

c) Find parametric equations for the line going through the points P and R.

(10) 4. Find an equation for the plane going through the origin and perpendicular to the line given parametrically by $x = 1 + 2t,\ y = 1 + 3t,\ z = -2 + 4t$.



(15) 5. State whether each of the following is a vector, a scalar, or makes no sense:

(a) $(\vec{a} \cdot \vec{b})\vec{c}$

(b) $(\vec{a} \times \vec{b}) \cdot |\, \vec{c} \,|$

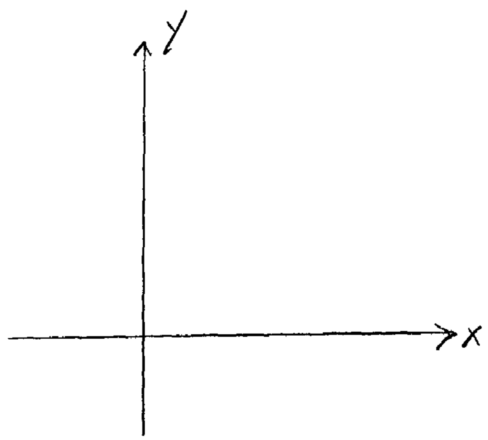
(c) $(\vec{a} \times \vec{b}) \times \vec{c}$

(d) $\vec{a} \cdot (\vec{b} \times \vec{c})$

(e) $(\vec{a} \cdot \vec{b}) \times \vec{c}$

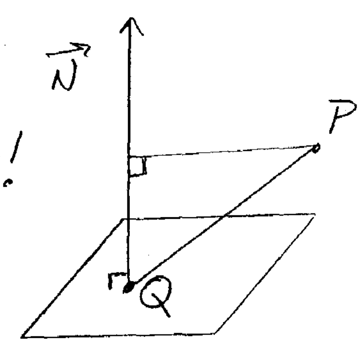
(f) $(\vec{a} \cdot \vec{b}) + |\vec{c}|$

(10) 6. Two forces $\vec{F}_1$ and $\vec{F}_2$ each with magnitude 10 Newtons make angles of 30° and 60° respectively with the positive x –axis in the 2-d plane. If $\vec{F} = \vec{F}_1 + \vec{F}_2$, find the magnitude and the angle $\vec{F}$ makes with the positive x – axis.



(10) 7. Find the distance between the point P(1, 1, 1) and the plane $x + 2y + 3z = 1$.

Hint!



(10) 8. Where does the line given parametrically by $x = 3 + t$, $y = 2 + 3t$, $z = -2 + 2t$ intersect the plane $x + y - z = 3$?