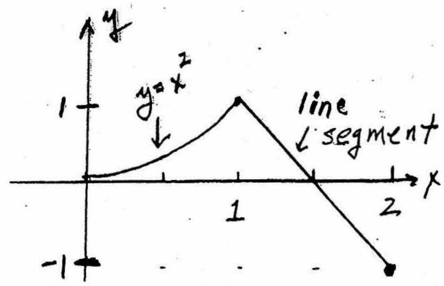
Calculus II Chapters 5 & 6 Practice Problems

Name ______________________________



1. Consider the function f graphed on the right.

(a) Find the exact value for $\int_0^2 f(x)dx$.

(b) Approximate $\int_0^2 f(x)dx$ using R_4 (the right endpoint rule using 4 subdivisions) and show the rectangles you are using on the graph.

(c) Approximate $\int_0^2 f(x)dx$ using M_2 (the midpoint rule using 2 subdivisions).

Solve problems 2 - 5 by hand. Show every step. Use your calculator only to check your answers.

2. (a) $\int \frac{x^3}{\sqrt{x}}dx$, (b) $\int e^x \cos(e^x)dx$, (c) $\int_0^1 x\sqrt{3x^2+1}dx$, (d) $\frac{d}{dx}\int_2^x t\sin(t^3)\,dt$.

3. Sketch both $y=\sin(x)$ and $y=\cos(x)$ over the interval $[0, \frac{5\pi}{4}]$. Find the area bounded between the curves over that interval.

4. Sketch and find the area of the region bounded by the curves $x=y^2-2$ and $y=x$.

5. (a) State the Mean Value Theorem for Integrals and prove by applying the Mean Value Theorem for derivatives to $F(x)=\int_a^x f(t)dt$. See problem #23 p.445 of our text book.

(b) For $f(x)=x^2$ on [-1 , 2], find the number c where $f_{ave}=f(c)$.

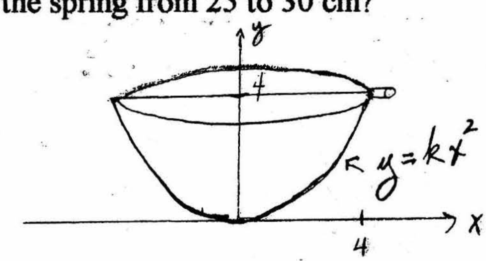
(c) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

For the remaining problems, set up the necessary integrals and use your calculator to evaluate them.

6. Let R be the region under the curve $y=\sin(x)$ over the interval $[0, \pi]$.
Find the volume generated by revolving R about the axis y = -1.

7. A force of 30 N is required to hold a spring that has been stretched from its natural length of 20 cm to a length of 25 cm. How much work is done in stretching the spring from 25 to 30 cm?

8. A tank full of water has the shape of a paraboloid of revolution (see figure). Find the work required to pump the water out of the top of the tank. Use the fact that water weights 62.5 lbs/ft^3.



Short answers: (1a) 1/3, (1b) .125, (1c) .25; (2a) $\frac{2}{7}x^{7/2}+C$, (2b) $\sin(e^x)+C$; (2c) 7/9; (2d) $x\sin(x^3)$; (3) $3\sqrt{2}-1$; (4) 9/2; (5b) $c=$ -1 or 1; (6a) $\pi(\pi+8)/2$, (6b) $2\pi(8-\pi)$; (7) 2.25 J; (8) $8000\pi/3\ ft-lb$