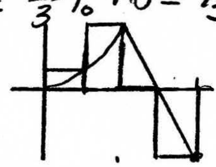
1(a) $\int_0^2 f(x)dx = \int_0^1 x^2 dx + \int_1^2 f(x)dx$

$= \frac{x^3}{3}\Big|_0^1 + 0 = 1/3$

(b)



$R_4 = [f(.5) + f(1) + f(1.5) + f(2)]\cdot(.5)$

$= [.25 + 1 + 0 - 1](.5) = .125$

(c) $M_2 = [f(.5) + f(1.5)][1] = .25$

2(a) $\int \frac{x^3}{\sqrt{x}}dx = \int x^{5/2}dx = \frac{2}{7}x^{7/2} + C$

(b) $\int e^x \cos(e^x)dx$; let $u = e^x \Rightarrow du = e^x dx$

$= \int \cos(u)\,du = \sin(u) + C = \sin(e^x) + C$

(c) $\int_0^1 (3x^2+1)^{1/2} x\,dx$; let $u = 3x^2+1$, $du = 6x\,dx$, $\frac{1}{6}du = x\,dx$, $0 \xrightarrow{x} 1$, $1 \xrightarrow{u} 4$

$= \frac{1}{6}\int_1^4 u^{1/2}du$

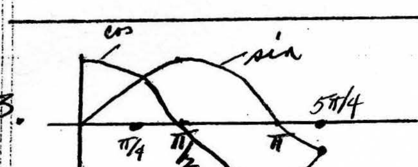
$= \frac{1}{6}\cdot\frac{2}{3}u^{3/2}\Big|_1^4$

$= \frac{1}{9}(\sqrt{4})^3 - \frac{1}{9}(\sqrt{1})^3$

$= \frac{8}{9} - \frac{1}{9} = 7/9$

(d) $\frac{d}{dx}\int_2^x t\sin(t^3)dt \overset{FTC}{=} x\sin(x^3)$.

3.



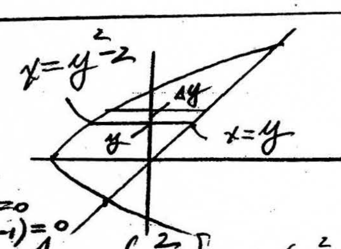
$A = \int_0^{5\pi/4}(y_T - y_B)\,dx$

$= \int_0^{\pi/4}[\cos(x) - \sin(x)]dx + \int_{\pi/4}^{5\pi/4}[\sin(x) - \cos(x)]dx$

$= [\sin(x) + \cos(x)]_0^{\pi/4} + [-\cos(x) - \sin(x)]_{\pi/4}^{5\pi/4}$

$= [\sqrt{2} - 1] + [\sqrt{2} - (-\sqrt{2})] = 3\sqrt{2} - 1$

4.



$y^2 - 2 = y$
$y^2 - y - 2 = 0$
$(y-2)(y+1) = 0$
$y = 2, -1$

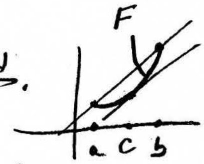
$\Delta A = $ area of rect $= (x_R - x_L)\Delta y$
$= (y - (y^2-2))\Delta y$

$A = \int_{-1}^2 [y - (y^2-2)]dy = \frac{y^2}{2} - \frac{y^3}{3} + 2y\Big|_{-1}^2$

$= \left(2 - \frac{8}{3} + 4\right) - \left(\frac{1}{2} + \frac{1}{3} - 2\right) = 9/2$

5(a) MVT for Integrals: If f is cont on $[a,b]$ then there exists at least one number c in $[a,b]$ where $f(c) = f_{ave} = \frac{1}{(b-a)}\int_a^b f(x)dx$.

Proof: Let $F(x) = \int_a^x f(t)dt$.
By the MVT for derivatives, there is at least one c in $[a,b]$ where $F'(c) = \frac{F(b) - F(a)}{b-a}$.



Now substitute for F. We know that $F'(x) = f(x)$ from the Fund. Thm. of Calculus so $F'(c) = f(c)$ and

$\frac{F(b) - F(a)}{b-a} = \left(\int_a^b f(t)dt - \cancel{\int_a^a f(t)dt}\right)/(b-a) = \frac{\int_a^b f(x)dx}{b-a}$

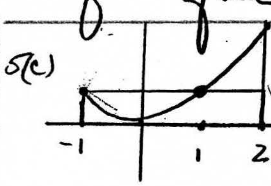
So $F'(c) = \frac{F(b) - F(a)}{b-a}$ turns into

$f(c) = \frac{1}{(b-a)}\int_a^b f(x)dx$. done

5(b) $f_{ave} = \frac{\int_a^b f(x)dx}{[b-a]} = \frac{\int_{-1}^2 x^2 dx}{2-(-1)} = \frac{x^3/3\Big|_{-1}^2}{3} = \frac{9/3}{3} = 1$

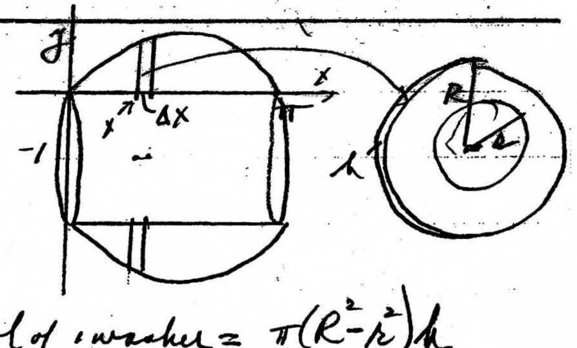
$f(c) = f_{ave} \Rightarrow c^2 = 1 \Rightarrow c = -1$ or 1.

5(c)



area of rectangle = area under curve = 3.

6.

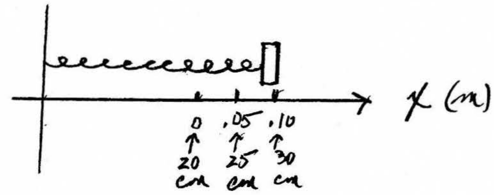


$\Delta V = $ vol of washer $= \pi(R^2 - r^2)h$
$= \pi\left[(1+\sin(x))^2 - 1^2\right]\Delta x \Rightarrow$
$V = \pi\int_0^\pi [(1+\sin(x))^2 - 1]dx = \frac{\pi(\pi+8)}{2}$.

7. Hooke's law $f(x) = kx$

$\Rightarrow 30\,N = k(.05\,m)$

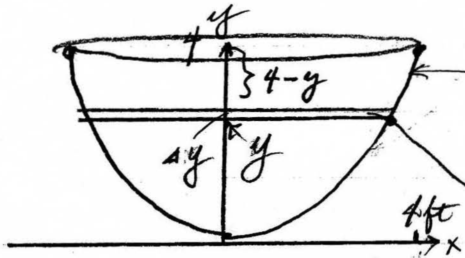
$\Rightarrow k = 600\ N/m$



$W = \int_{.05}^{.10} 600x\,dx = 2.25\ N\text{-}m$

or $2.25\ J$

8.



$y = kx^2$, when $x=4, y=4 \Rightarrow 4 = k4^2 \Rightarrow k = \frac{1}{4}$

so $y = \frac{1}{4}x^2$ or $x = \sqrt{4y}$

1 disk of water

$\Delta W =$ work needed to lift 1 disk of water to top

$= F \cdot d$

$= \text{weight} \cdot (4-y)\,ft$

$= \left(62.5 \frac{lbs}{ft^3}\right)(\text{Vol}) \cdot (4-y)\,ft$

$= \left(62.5 \frac{lbs}{ft^3}\right)(\pi r^2 \Delta y) \cdot (4-y)\,ft$

$r = x = \sqrt{4y}$

$= \left(62.5 \frac{lbs}{\cancel{ft^3}}\right)\left[\pi\left(\sqrt{4y}\right)^2 \Delta y\ \cancel{ft^3}\right](4-y)\,ft$

$= 62.5\pi\ 4y(4-y)\,\Delta y\ ft\text{-}lbs$

$W = \int_0^4 62.5\pi\ 4y(4-y)\,dy\ ft\text{-}lbs = \frac{8000\pi}{3}\ ft\text{-}lbs$

$\doteq 8{,}378\ ft\text{-}lbs.$