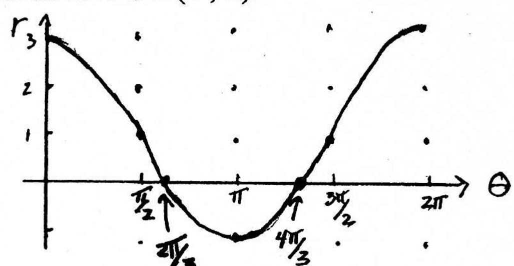
Calculus II Chapters 10 & 11 Practice Problems

Name ____________________________

1. (a) Find polar coordinates for the point whose Cartesian coordinates are (2 , 2).
(b) Find Cartesian coordinates for the point whose polar coordinates are (2 , 2).

2. The curve on the right shows the graph of r as a function of θ in the $\theta - r$ plane. Use it to sketch the corresponding polar curve in the $x - y$ plane.



3. (a) Sketch the curves in the same Cartesian x-y plane whose polar equations are $r = 3\sin(\theta)$ and $r = 1 + \sin(\theta)$.
(b) Find the area inside $r = 1 + \sin(\theta)$ and outside $r = 3\sin(\theta)$.

4. a) Find the limit of the infinite sequence defined by $a_n = \dfrac{n+1}{5n-1}$.

b) Does the infinite series $\sum_{n=1}^{\infty} \dfrac{n+1}{5n-1}$ converge to a sum? Why?

5. A ball bounces back half as high each time it falls. If it first falls from a height of 10 ft, find the total distance it travels.

6. If $\sum_{n=1}^{\infty} \dfrac{a_n}{6^n} = 2$, then $\sum_{n=1}^{\infty} \dfrac{3^n + a_n}{6^n}$ equals a) 2 b) 3 c) 5 d) 6 e) ∞

7. Tell whether each of the following series converges or diverges and justify :

a) $\sum_{n=1}^{\infty} \dfrac{n}{n+1}$ b) $\sum_{n=1}^{\infty} \dfrac{n}{2^n}$ c) $\sum_{n=1}^{\infty} \dfrac{n!}{2^n}$ d) $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{2n+1}$

8. Why is the series $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n^5}$ convergent? Approximate the sum accurate to within .001.

9. If $\sum_{n=0}^{\infty} c_n x^n$ converges for x = 5 and diverges for x = -7, what can be said about the following series?

a) $\sum_{n=0}^{\infty} c_n$ b) $\sum_{n=0}^{\infty} c_n 4^n$ c) $\sum_{n=0}^{\infty} (-1)^n c_n 3^n$ d) $\sum_{n=0}^{\infty} c_n 8^n$

10. Find the radius of convergence for each of the following series:

a) $\sum_{n=1}^{\infty} \frac{n(x-2)^n}{3^n}$ b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

11. a) The geometric series $1 + x + x^2 + x^3 + x^4 + \ldots = \frac{1}{1-x}$ for what values of x? What is the radius of convergence for this series?

b) Use part a) to find a series centered at 0 for $\frac{1}{1+x}$. What is the radius of convergence?

c) Use the information in b) to find a series centered at 0 for $f(x) = \ln(1+x)$.

d) Find $\int_0^{.5} \frac{\ln(1+x)}{x} dx$ accurate to within .01.

12. Sketch a function whose Taylor series centered at a = 1 could be given by $1 + 1(x-1) - 1(x-1)^2 + \ldots$.

13. Let $f(x) = x^2 + 3x - 4$. a) Write the Maclaurin series for $f(x)$. b) Write the Taylor series for $f(x)$ centered at a = 2.

14. Use the Maclaurin series for e^x to find the Maclaurin series for $f(x) = e^{-x^2}$.