

Please answer the following 3 questions on the paper provided. Additional paper is available should you need it. Show the details of your work and indicate your answers clearly. I am more interested in the process you use to discover a solution than I am in the solution itself. When you rely on your calculator for a computation, please make it clear what you asked your calculator to do.

1. (30 points)

(a) Find an equation for the plane through the point $(1, 2, 3)$ which is perpendicular to the line $\mathbf{r}(t) = \langle 1+2t, 2-3t, 4+\pi t \rangle$.

(b) Find an equation for any of the infinitely many planes that contain the point $(1, 2, 3)$ and which do not intersect the line $\mathbf{r}(t) = \langle 1+2t, 2-3t, 4+\pi t \rangle$.

2. (10) The equation of a surface in spherical coordinates is $\rho \sin(\phi) = 3$.

(a) Write an equation for the same surface in cylindrical coordinates.

(b) Write an equation for the same surface in rectangular coordinates.

3. (60) Our rocket ship is powered by the Amazing Oscillating Drive, which provides an acceleration of $\mathbf{a}(t) = \langle \cos(\pi t), 5, -5 \rangle$ (meters per second per second). We are well outside any significant gravitational fields, so you may assume this formula gives the actual acceleration of our ship. At time $t=0$, our velocity is $\mathbf{v}(0) = \langle 300, 100, 100 \rangle$ (meters per second) and our position relative to the Standard Galactic Coordinate System is $\langle 0, -2000, 4000 \rangle$.

(a) What is our location at time t ?

(b) What is our location at time $t=100$?

(c) How far did we travel between time $t=0$ and time $t=100$? (Please show me the integral which gives this value. Use $\infty+\div$ to evaluate the integral; otherwise your calculator will take too long to give you an answer.)

(c) What is our velocity at time $t=100$?

(d) If we instantaneously shut down the Amazing Oscillating Drive at time $t=100$, our ship will proceed at a constant velocity along the line tangent to our path. Give parametric equations for this tangent line.

(e) Where will we be at time $t=200$, after we've drifted along our tangent line for 100 seconds?

(f) What was the curvature of our path at time $t=0$?