1. (10 points) The graph below shows level curves $f(x,y)$ for a differentiable function $f$. At the point $P_m$ determine the sign of each of the indicated partial derivatives. (Just mark the correct answer; no penalty for guessing, and no partial credit.)

(a) $f_x$ positive negative zero, or too close to tell
(b) $f_y$ positive negative zero, or too close to tell
(c) $f_{xx}$ positive negative zero, or too close to tell
(d) $f_{xy}$ positive negative zero, or too close to tell
(e) $f_{yy}$ positive negative zero, or too close to tell
2. (65) Suppose that \( f \) is a differentiable function of 3 variables, and that \( w = f(x,y,z) \). Assume that \( f(1,2,3) = 4 \) and that \( \nabla f(1,2,3) = \langle -2,3,-1 \rangle \).

(a) Find the directional derivative \( D_u f(1,2,3) \), where \( u \) is the unit vector \( \text{u} = \begin{pmatrix} 3 & 12 & -4 \\ 13 & 13 & 13 \end{pmatrix} \).

(b) Find the unit vector \( v \) such that \( D_v f(1,2,3) \) is as large as possible.

(c) Find an equation for the plane tangent to the surface \( w = 4 \) at the point \( (1,2,3) \).

(d) Give a reasonable estimate of \( f(.97, 2.01, 3.02) \).

(e) Suppose that \( x = s^2 \), \( y = s^2 + s \), and \( z = 4-s^3 \). This makes \( w \) into a function of \( s \). Find \( \frac{dw}{ds} \) when \( s = 1 \).
3. (25) Find positive numbers x, y, and z (not necessarily integers) such that \( x + 3y + z = 9 \) and \( x^2yz \) is as large as possible.